No. of Printed Pages : 7 **MMT–004**

M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

[MSC (MACS)]

Term-End Examination

June, 2026

MMT-004 : REAL ANALYSIS

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) Question No.* ***1*** *is compulsory.*

(ii) Attempt any ***four*** *questions out of Question Nos.* ***2*** *to* ***6****.*

(iii) Use of calculators is not allowed.

(iv) Notations as in the study material.

1. State whether the following statements are True *or* False. Give reasons for your answers : 5×2=10

(a) The sequence $\left\{e^{1/n}\right\}_{n=1}^{\infty}$ is not a Cauchy sequence in $\mathbb{R}$ under the Euclidean metric on $\mathbb{R}$.

(b) If C is the unit circle in $\mathbb{R}^2$ and $f : \mathrm{C} \to \mathbb{R}$ is a continuous function, then $f(\mathrm{C})$ is a connected set in $\mathbb{R}$.

(c) The function $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by $f(x, y) = (x + 2y, x^2 - y^2)$ is locally invertible at (0, 0).

(d) If a function from $\mathbb{R}$ to $\mathbb{R}$ is Lebesgue measurable, then it is continuous.

(e) $\{0\} \cup \left[\frac{1}{n}, n = 1, 2, 3.......\right]$ is compact in **R**.

2. (a) State the Cantor's intersection theorem. Prove that the assumption $d(\mathrm{F}_n) \to 0$ cannot be dropped in the theorem. 3

(b) Let X, Y be metric spaces with X compact. Prove that a continuous function from X to Y is uniformly continuous. 4

(c) Which of the following sets are connected sets in the Euclidean spaces :

(i) $\mathrm{L}' = \left\{(x, 4) \in \mathbb{R}^2 : x \geq 1 \text{ and } y = 1\right\}$

(ii) $\mathbb{R} \setminus \{0\}$

(iii) $[0, 2] \cup [1, 4] \cup \left\{\frac{3}{2}\right\}$ 3

3. (a) Consider $\mathbb{R}^2, \mathbb{R}^3$ with the usual metric. Prove that the map $f : \mathbb{R}^3 \to \mathbb{R}^2$ given by $p(x, y, z) = (x, y)$ is uniformly continuous. 3

(b) Let $f : \mathbb{R} \to \mathbb{R}^2$ defined by $f(t) = (t, t^2)$ and $g : \mathbb{R}^2 \to \mathbb{R}^3$ defined by

$$g(x_1, x_2) = (x_1^2, x_1 x_2, x_2^2 - x_1^2).$$

Find the derivation of the composite function of $g \text{ o } f$. 3

(c) Define a Lebesgue measurable set. Show that if E_1 and E_2 are measurable and $E_1 \cap E_2 = \phi$, then $E_1 \cup E_2$ is measurable. 4

4. (a) Obtain the Taylor's series expansion of the function

$$f(x_1, x_2) = x_1 + 2x_2 + x_1 x_2 - x_1^2 - x_2^2$$

at the point (1, 1). 3

(b) State the Inverse Function Theorem. Check the local inevertibility of the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by 4

$$f(x, y) = (x + y, e^x \cos y)$$

(c) When is a system called causal ? Check whether the system R : $f \to g$. given by

$$g(t) = \mathrm{R} f(t) = \int_{-\infty}^{t/2} f(t)\, dt$$

is causal or not. 3

5. (a) State the Implicit Function theorem and verify the theorem for the function $f(x, y, z) = x + y + z - \sin xyz$ at (0, 0, 0). 4

(b) Compute the Fourier series of the function f given by $f(t) = t^2, -\pi \leq t \leq \pi$. 3

(c) Suppose $\{f_n\}$ is a sequence of non-negative measurable functions defined on a measurable set E. Prove that 3

$$\int_E \sum_{n=1}^{\infty} f_n \, dm = \sum_{n=1}^{\infty} \int_E f_n \, dm .$$

6. (a) Prove that the components of the set of rationals with the usual metric are singletons. 3

(b) Find the critical points of

$$f(x, y, z) = x^2y + y^2z + z^2 - 2x$$

and check whether they are extreme points. 4

(c) For a function $f \in L^1(\mathbb{R})$ define its Fourier transform $\hat{f}$. Prove the Fourier transform $\hat{f}$ is a continuous function on $\mathbb{R}$. 3

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