No. of Printed Pages : 4 **MMT–005**

M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [MSC (MACS)]

Term-End Examination

June, 2026

MMT-005 : COMPLEX ANALYSIS

Time : $1\frac{1}{2}$ Hours *Maximum Marks : 25*

Note : *(i) Question no. 1 is compulsory.*

*(ii) Attempt any **three** questions from question no. 2 to 5.*

(iii) Use of calculator is not allowed.

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1. State giving seasons whether the following statements are True or False : 5×2=10

   (a) The right half plane H = $\{z : \text{Re}(z) > 0\}$ is neither open nor a closed set.

(b) There exists a non-constant analytic function defined on the whole complex plane whose image is contained in a disc of radius 5.

(c) The radius of convergence of the power series $\sum_{n=0}^{\infty}\left(\frac{3n+2}{2n+5}\right)^n (z-2)^n$ is $\frac{3}{2}$

(d) The function $f(z) = e^z + 50$ is conformal everywhere.

(e) If C is the positive oriented circle $|z| = 3$, then $\int_C \frac{z^3+2z}{(z-2)^3}\,dz = 0$.

2. (a) Evaluate $\lim_{z \to 1+\sqrt{3}i}\left[\frac{z^2-2z+4}{z-1-\sqrt{3}i}\right]$ 2

(b) Let $f : \boldsymbol{c} \to \boldsymbol{c}$ be defined by $w = f(z) = z^2$. Find the image of the vertical line Re $(z) = 1$ of z-plane in the w-plane. Also draw the image. 3

3. (a) Find the conjugate harmonic function $v(x, y)$ corresponding to :

$$u(x, y) = x^2 - y^2 + x + 5$$

Also find the analytic function f whose real part is $u(x, y)$. 2

(b) Find $i^{(i+1)}$. Also find its principal value. 3

4. (a) Let f be analytic inside and on the boundary of the open disc D whose boundary is given by positively oriented circle C : $|z - z_0| = \text{R}$. Assume that there exists a positive constant M such that $|f(z)| \leq \text{M}$ in D. Show that

$$|f^{(n)}(z_0)| \leq \frac{\text{M}n!}{\text{R}^n}.$$

Use it to find an upper bound for $f^{(3)}(2i)$ where $f(z)$ is analytic inside and on the boundary of the open disc D bound by $|z - 2i| = 3$ such that $|f(z)| \leq 5$ in D. 3

(b) Find the bilinear transformation which maps the points $z = -2, 0, 2$ into the points $0, i, -i$ respectively. 2

5. (a) Find the power series expansion of the function

$$f(z) = \frac{1}{z^2 - 3z + 2}$$

which is valid in the region $1 < |z| < 2$. 2

(b) Let $f(z) = \dfrac{e^z}{z(z-1)^2}$ 3

(i) Find all the singular points of $f(z)$ and their nature.

(ii) Use Cauchy Residue theorem to evaluate $\int_C f(z)dz$ where C is the positive oriented circle $|z| = 2.5$.

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