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# M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. SC. (MACS)]

## Term-End Examination

## June, 2026

### MMT-006 : FUNCTIONAL ANALYSIS

*Time : 2 Hours* *Maximum Marks : 50*

*Weightage : 70%*

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***Note :*** *(i) Question No.* ***6*** *is compulsory.*

*(ii) Attempt any* ***four*** *of the remaining questions.*

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1. (a) Give an example of a discontinuous linear functional on a normed linear space. 3

(b) If X, Y are Banach spaces, prove that X × Y is a Banach space with norm $\| (x, y) \| = \| x \|_X + \| y \|_Y$. 4

(c) Check whether the operator on $l^2$ defined by $Ax = (0, x(1), x(2), ...,)$ has eigen values. 3

2. (a) Check whether $\| x \| = \max\{| x |_1, | x |_2, | x |_3, | x |_4, | x |_5)$ is a norm on $\mathbf{R}^5$. Is it different from all the norms $\| \cdot \|_p, 1 \leq p \leq \infty$. Justify your answer. 5

(b) On $(c_{00}, \| \cdot \|_\infty)$, find a sequence of bounded linear functionals that is pointwise bounded but is not uniformly bounded. Does this contradict the Uniform Boundedness principle ? 3+2

3. (a) Find the orthogonal complement of the subspace $M = \{(x, y, z) \in \mathbf{R}^3 : x + y + z = 0\}$.

2

(b) On L²[0, 2] define $\phi(f) = \int_0^1 xf(x)\,dx$. Prove that $\phi$ is a bounded linear functional and verify the Riesz Representation theorem in this case.

2+3

(c) Give an example of an incomplete normed linear spaces with the proof of incompleteness. 3

4. (a) Show that the standard unit vectors $e_n$ in $l^2$ form an orthonormal basis for $l^2$. Find a compact operator A with $\{e_n\}$ as eigen vectors. 2+3

(b) On $M = \{(x, y, z) \in \mathbf{R}^3 : x + y + z = 0\}$, define $f(x, y, z) = x + y - 2z$. Find two linear functionals on $\mathbf{R}^3$ that extend $f$, of which one is a Hahn-Banach extension and the other is not. 5

5. (a) If M, N are mutually orthogonal closed subspaces of a Hilbert space H, prove that M + N is closed. 4

(b) For $x \in \mathbf{C}$, let $\lim x_n = a$ where $x = \{x_n\}$. Define $P(x_n) = x_n - a, \forall n$. Show that $\mathrm{P} : c \to c_0$ is a bounded operator with $\mathrm{P}^2 = \mathrm{P}$. 3

(c) Suppose $(a_n)$ is a sequence of scalars such that $(a_n x_n) \in l^1$ for every $(x_n) \in l^1$. Prove that $(a_n) \in l^\infty$. 3

6. State, with justification, whether the following statements are True *or* False : 5×2=10

(a) If $\phi$ is a bounded linear functional on C[0, 1] vanishes on all polynomials, then $\phi = 0$.

(b) If a Banach space X has an uncountable family $\{x_i\}$ of vectors such that $\| x_i - x_j \| = 1,\ \forall i \neq j$, then X is not separable.

(c) A Hilbert space cannot be isomorphic to a proper subspace.

(d) 0 is never an eigenvalue of a compact operator.

(e) If A is a bounded operator on a sssBanach space with $\| A \| = 1$, then $\| Ax \| = \| x \|, \forall x$.

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