No. of Printed Pages : 8 **MMT–007**

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

[M. SC. (MACS)]

Term-End Examination

June, 2026

MMT–007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 Hours *Maximum Marks : 50*

Weightage : 50%

Note : *(i) Question No.* ***1*** *is compulsory.*

(ii) Attempt any ***four*** *questions out of question nos. 2 to 7.*

(iii) Use of scientific (non-programmable) calculator is allowed.

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1. State whether the following statements are True *or* False. Justify your answer with the help of a short proof or a counter-example. No marks are awarded for a question without justification : 2×5=10

(a) The partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + 4x \frac{\partial^2 u}{\partial x \partial y} + (1 - y^2) \frac{\partial^2 u}{\partial y^2} = 0$$

is elliptic inside the ellipse $4x^2 + y^2 = 1$.

(b) $f(x, y) = x^2 |y|$ satisfies the Lipschitz condition on the rectangle $|x| \leq 1$ and $|y| \leq 1$.

(c) On any interval $e^x \sin x$ and $e^x \cos x$ are linearly independent solutions of equation $y'' - 2y' + 2y = 0$.

(d) $\sin x \ln|\sec x + \tan x| - 1$ is the particular integral of $\dfrac{d^2y}{dx^2} + y = \sec^2 x$.

(e) The series $\sum_{n=0}^{\infty} nx^n$ converges for $|x| < 1$.

2. (a) Find a series solution about $x = 0$, of the equation $2x^2y'' - xy' + (1 - x^2)y = 0$. 5

(b) Using Rodrigue's formula, show that : 2

$$\int_{-1}^{1} f(x)\mathrm{P}_n(x)dx = \frac{(-1)^n}{2^n n!}$$

$$\int_{-1}^{1} (x^2 - 1)^n f^{(n)}(x)\, dx$$

(c) Express $f(x) = x^4 + 3x^3 + 4x^2 - x + 2$ in terms of Legendre polynomials. 3

3. (a) Find the solution of the heat equation : 5

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

subject to the conditions, $u\ (x,\ 0) = 0$, $u\ (0,\ t) = 0$, $u\ (1,\ t) = t$, using implicit Crank-Nicolson method with $h = \frac{1}{2}$ and $k = \frac{1}{8}$. Integrate for two time levels.

(b) Construct Green's function for the boundary value problem : 5

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = 0,\ 0 < x < \frac{\pi}{2},$$

$$y\ (0) = 0,\ y\left(\frac{\pi}{2}\right) = 0.$$

4. (a) Using Laplace transform solve the initial boundary value problem $\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial t} = 0$, $u\ (x,\ 0) = 1$, $u\ (0,\ t) = t$. 3

(b) Using inverse sine transform, find $f\ (x)$ if $F_s(\alpha) = \frac{1}{\alpha}e^{-a\alpha}$, $a > 0$. 3

(c) Using the substitution $Z = \sqrt{x}$, reduce the given equation to Bessel equation and hence find its solution : 4

$$xy'' + y' + \frac{y}{4} = 0$$

5. (a) Solve the initial value problem $y' = x^2 + y^2$, $y(1) = 2$ on the interval [1, 1.4] using second order Runge-Kutta method with $h = 0.2$. 3

(b) Find the constants in the explicit method : 4

$$y_{i+1} = a_1 y_i + h[b_1 y_i' + b_2 y_{i-1}' + b_3 y_{i-2}']$$

Also, determine the truncation error and order of the method.

(c) Solve the boundary value problem

$$y'' = xy, \; y(0) + y'(0) = 1, \; y(1) = 1$$

using second order finite difference method with $h = \frac{1}{3}$. 3

6. (a) For the initial value problem $2y' = x + y$, $y(0) = 2$ find $y(2)$ using Milne's fourth order prediction-corrector method.

Where $y(0.5) = 2.636$, $y(1) = 3.595$, $y(1.5) = 4.968$. Perform two corrector iterations. 5

(b) Derive the method : 5

$$y_{i+1} = a_1 y_i + a_2 y'_{i-1} + h b_0 y'_{i+1}$$

for solving the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$. Find the truncation error and the order of the method.

7. (a) Find the solution of $\nabla^2 u = x + y$ in R subject to the boundary conditions $u\ (x,\ y) = (x^3 + y^3)/6$ on Γ, where R is the triangle $0 \le x \le 1$, $0 \le y \le 1$, $0 \le x + y \le 1$. Using the five point formula. Assume uniform step length $h = \frac{1}{4}$ along the axes. 4

(b) Use Laasonen method to find the solution of the heat conduction equation, subject to the given initial and boundary conditions $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 \le x \le 1$, $u\ (x,\ 0) = \sin\ (\pi x)$, $0 \le x \le 1$, $u\ (0,\ t) = 0 = u\ (1,\ t)$. 3

(c) Apply Picard's method of successive approximation to the initial value problem $y' = x(y - x^2 + 2)$, $y(0) = 1$. 3

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