No. of Printed Pages : 8 **MMT–008**

M. SC. IN MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE [MSC(MACS)]

Term-End Examination

June, 2026

MMT-008 : PROBABILITY AND STATISTICS

Time : 3 Hours *Maximum Marks : 100*

Weightage : 50%

Note : *(i) Question No. 8 is compulsory.*

(ii) Attempt any ***six*** *questions from question nos.* ***1*** *to* ***7****.*

(iii) Use of scientific and non-programmable calculator is allowed.

(iv) Symbols have their usual meanings.

1. (a) Let $X \sim N_3(\mu, \Sigma)$, where $\mu = [5 \quad 3 \quad 4]'$ and :

$$\Sigma = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1/2 \\ 1 & 1/2 & 1 \end{bmatrix}.$$

Find the distribution of : 6

$$\begin{bmatrix} 2X_1 + X_2 - X_3 \\ X_1 + X_2 + X_3 \end{bmatrix}.$$

(b) Let (X, Y) have the joint pdf given by :

$$f(x, y) = \begin{cases} 1, & |y| < x, 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the marginal pdf's of X and Y.

(ii) Test the independence of X and Y.

(iii) Find the conditional distribution of X given $Y = y$.

(iv) Compute $E(X \mid Y = y)$ and $E(Y \mid X = x)$. 9

2. (a) Consider the Markov chain with transition probability matrix : 6

$$\rho = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0.25 & 0.125 & 0.125 & 0.5 \end{bmatrix}$$

(i) Whether the chain is irreducible ? If irreducible, classify the states of a Markov chain, i.e. recurrent, transient, periodic and mean recurrent time.

(ii) Find the limiting probability vector.

(b) Determine the principal components Y_1, Y_2 and Y_3 for the covariance matrix :

$$\Sigma = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Also, calculate the proportion of total population variance for the first principal component. 9

3. (a) If a random vector Z be N_4 (μ, Σ), where :

$$\mu = \begin{bmatrix} 1 \\ 2 \\ 5 \\ -2 \end{bmatrix} \text{ and } \Sigma = \begin{bmatrix} 3 & 3 & 0 & 9 \\ 3 & 2 & -1 & 1 \\ 0 & 1 & 6 & -3 \\ 9 & 1 & -3 & 7 \end{bmatrix}$$

Find r_{34} and $r_{34.21}$. 8

(b) Suppose life times X_1, X_2, ... are iid uniformly distributed on (0, 3) and $C_1 = 2$ and $C_2 = 8$. Find μ^T and T at which, it minimizes C (T). 7

4. (a) Define ultimate extinction in a branching process. Let $p_k = bC^{k-1}$, $k = 1, 2, \ldots\ldots$; $0 < b < c < b + c < 1$ and $p_0 = 1 - \sum_{k=1}^{\infty} p_k$. Find the probability of extinction in different cases for $E(X_i) \geq 1$ or $E(X_1) < 1$. 7

(b) Let the random vector $X' = \left[X_1\ X_2\ X_3\right]$ has mean vector $\left[-2\quad 3\quad 4\right]'$ and variance-covariance matrix as $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 9 \end{bmatrix}$. Fit the equation $Y = b_0 + b_1 X + b_2 X_2$. Also, obtain the multiple correlation coefficient between X_3 and $\left[X_1, X_2\right]$. 8

5. (a) The mean Poisson rate of arrival of planes at an airport during peak hours is 20 per hour. 60 planes per hour can land at the airport in good weather and 30 planes per hour in bad weather in Poisson fashion. Find : 8

   (i) The average number of planes flying over the field in good weather.

(ii) The average number of planes flying over the field in bad weather.

(iii) The average number planes flying over the field and landing in good weather.

(iv) The average number of planes flying over the field and landing in bad weather.

(v) The average landing time in good weather and bad weather.

(b) An equal number of balls are kept in three boxes B_1, B_2 and B_3. The boxes B_1, B_2 and B_3 contain 3%, 5% and 2% defective balls, respectively. One of the boxes is selected at random and a ball is drawn randomly. If the ball is found to be defective, what is the probability that it has come from B_2 ? 7

6. (a) Suppose interoccurrence times $\{X_n \mid n \geq 1\}$ are uniformly distributed on [0, 1]. Find $\overline{M}_t$, the Laplace transform of the renewal function M_t. Also, find $\lim_{t\to\infty} \frac{M_t}{t}$. 7

(b) Suppose $n_1 = 20$ and $n_2 = 30$ observations are made on two variables X_1 and X_2, where $X_1 \sim N_2(\mu^{(1)}, \Sigma)$ and $X_2 \sim N_2(\mu^{(2)}, \Sigma)$ where $\mu^{(1)} = [1 \quad 2]'$, $\mu^{(2)} = [-1 \quad 0]'$ and $\Sigma = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$.

Considering equal cost and equal prior probabilities, classify the observation $[-1, \ 1]'$ in one of the two populations. 8

7. (a) Differentiate between age replacement and block replacement with examples. 6

(b) Define factor analysis. Consider a data matrix of order (3 × 3) and perform factor analysis. 9

8. Which of the following statements are true or false ? Justify : 10

(i) In a Markov chain, if P is the transition matrix, all rows of $\lim_{n \to \infty} P^n$ are identical.

(ii) All the elements of a variance-covariance matrix are always positive.

(iii) If X_i are iid. $N_2(\mu, \Sigma)$, then the average $\frac{X_1 + X_2 + X_3}{3} \sim N_2\left(\mu, \frac{\Sigma}{3}\right)$.

(iv) Partial and multiple correlation coefficients always lie between –1 and 1.

(v) For a renewal function M (t), $\lim_{n \to \infty} \frac{M(t)}{t} = \frac{1}{\mu}$, where μ is the mean interarrival time.

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