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MMTE-001

**M. SC. (MATHEMATICS WITH
APPLICATIONS IN COMPUTER
SCIENCE) [M. SC. (MACS)]**

Term-End Examination

June, 2026

MMTE-001 : GRAPH THEORY

Time : 2 Hours

Maximum Marks : 50

Note : *Question No. 1 is compulsory. Answer
any **four** questions from Q. Nos. 2 to 7.*

Use of calculators is not allowed.

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1. State whether the following statements are true or false. Justify your answers with a short proof or a counter-example : $2 \times 5 = 10$

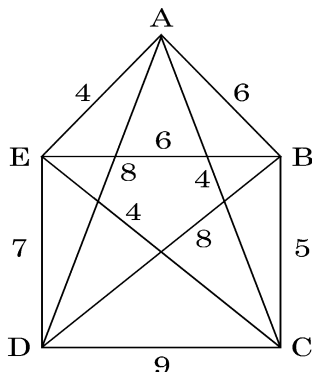
- (i) Every Hamiltonian graph has a perfect matching.
 - (ii) The girth of the Petersen graph is 4.
 - (iii) Every tree is bipartite.
 - (iv) If $\chi(G) = n$, for some n , then $G = (V, E)$ contains the complete graph K_n as a subgraph.
 - (v) For all values of m and n , $K_{m, n}$ is Eulerian.
2. (a) Let v be a cut-vertex of a graph G . Then prove that $\overline{G} - v$ is a connected graph. 5
- (b) Prove that the following sequence is a graphic sequence : 5
- (5, 5, 4, 3, 2, 2, 2, 1)

3. (a) If u, v are the only vertices of odd degrees in a graph G , then prove that G contains a (u, v) path. 4

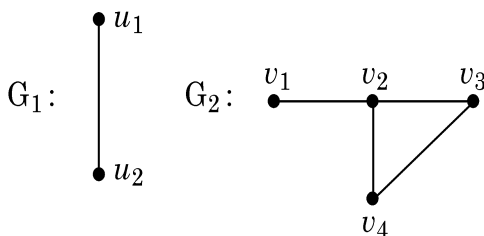
(b) Prove that for any labelled complete graph K_n , $\tau(K_n) = n^{n-2}$. 6

4. (a) Prove that in any group of n persons ($n \geq 2$), there are at least two persons with the same number of friends. 4

(b) Find the minimum spanning tree for the following weighted graph using Prim's algorithm : 6



5. (a) What is the connectivity of the wheel graph W_n , for $n \geq 4$? 3
- (b) State and prove Euler's formula for a connected planar graph. 7
6. (a) If G is a bipartite graph, then prove that the maximum size of a matching in G equals the minimum size of a vertex cover of G . 5
- (b) Find the Cartesian product of the following graphs :



Also, check whether the Cartesian product of G_1 and G_2 is Eulerian or not.

5

7. (a) Prove that every graph $G \neq K_2$ has fewer cut vertices than blocks. 5
- (b) Prove that for every graph G with order n , $\chi(G) > \frac{n}{\alpha(G)}$. Also, give an example of a graph for which equality holds. 5

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