No. of Printed Pages : 8 **MMTE–005**

M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

[M. SC. (MACS)]

Term-End Examination

June, 2026

MMTE-005 : CODING THEORY

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) There are six questions in this paper.*

*(ii) The **sixth** question is compulsory.*

*(iii) Do any **four** questions from Question Nos. **1** to **5**.*

(iv) Show all the relevant steps. Do the rough work at the bottom or at the side of the page only.

*(v) Calculators are **not** allowed.*

---

1. (a) Consider the codes :

$\mathbf{C}_1 = \{(0, 0, 0, 0), (1, 0, 1, 0),$

$(0, 1, 1, 0), (1, 1, 1, 0)\}$ ..(A)

$\mathbf{C}_2 = \{(0, 0, 0), (1, 1, 0),$

$(1, 0, 1), (0, 1, 1)\}$ ..(B)

(i) Which of the above binary codes are linear ? Justify your answer. 4

(ii) Find the minimum distance of $\mathbf{C}_1$ and $\mathbf{C}_2$. 4

(b) Let $n \in \mathrm{N}$, q be a power of a prime and $0 \leq s < n$. Define the q-cyclotomic coset of s modulo n. Find the 13-cyclotomic set of 1 modulo 17. 2

2. (a) Define permutation equivalence of linear codes. Check whether the codes $\mathbf{C}_1$ and $\mathbf{C}_2$ with generator matrices

$$G_1 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \text{ and } G_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

respectively are permutation equivalent.

2

(b) Define a cyclic code. Check whether the code {1110, 0111, 1011} is cyclic. 2

0000	0	1000	α^3	1011	α^7	1110	α^{11}
0001	α	0011	α^4	0101	α^8	1111	α^{12}
0010	α	0110	α^5	1010	α^9	1101	α^{13}
0100	α^2	1100	α^6	0111	α^{10}	1001	α^{14}

Table 1 : $\mathbf{F}_{16}$ with primitive element α, where $\alpha^4 = \alpha + 1$

(c) Let $\mathbf{C}_1$ be the [4, 3, 2] binary linear code generated by :

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

and let $\mathbf{C}_2$ be the [4, 1, 4]-binary linear code generated by [1101]. Let $\mathbf{C}$ be the code obtained through using $(\mathbf{u} \mid \mathbf{u} + \mathbf{v})$ construction on the codes $\mathbf{C}_1$ and $\mathbf{C}_2$. Find the generator matrix of $\mathbf{C}$. Also, give the length and the dimension of $\mathbf{C}$.

3

(d) If a polynomial generator matrix of an $[n, k]$ convolutional code $\mathbf{C}$ is basic and reduced, then prove that it is canonical.

3

3. (a) Let **C** be a narrow-sense binary BCH code of designed distance $\delta = 5$, which has the defining set $\{1, 2, 3, 4, 6, 8, 9, 12\}$. Using the primitive 15th root of unity α, where $\alpha^4 = \alpha + 1$, the generator polynomial is $g(x) = 1 + x^4 + x^6 + x^7 + x^8$. Suppose **C** is used to transmit a codeword and $y(x) = x^6 + x^{10} + x^{12}$ is received. Find the transmitted codeword. You may find Table 1 useful. 8

(b) Show by a counter example that integers modulo n do not form a field if n is not a prime. 2

4. (a) Let **C** be the [5, 2] binary code generated by : 6

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

(i) Find the weight distribution of **C**.

(ii) Find the weight distribution of $\mathbf{C}^{\perp}$ using MacWilliams identity.

(b) Let **C** be the $\mathbf{Z}_4$-linear code of length with generator matrix $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix}$

(i) List all the 16 codewords in **C**.

(ii) List the 16 codewords in the Gray image of **C**. 4

5. (a) Find the generator matrix and the parity check matrix for the binary cyclic code of length 7 with generator polynomial $x^3 + x + 1$. 5

(b) Let $\alpha = x + \mathbf{F}_2[x]/ < x^2 + x + 1 >$. Make a table that gives the powers of α as a linear combination of 1, α and α^2. Use it to write $(\alpha^5 - \alpha^2 + 1)(\alpha + 1)/(\alpha^2 + 1)$ as a linear combination of 1, α and α^2. 5

6. Which of the following statements are true and which are false ? Justify your answer with short proof or a counter example : 10

(i) If the weight of each element in the generating matrix of a linear code is at least r, the minimum distance of the code is at least r.

(ii) There is no linear self orthogonal code of odd length.

(iii) There is no 3-cyclotomic coset modulo 121 of size 25.

(iv) There is no duadic code of length 15 over $\mathbf{F}_2$.

(v) There is no LDPC code with parameters $n = 16$, $c = 3$ and $r = 5$.

× × × × ×