No. of Printed Pages : 7 **MPH–001**

M. Sc. (PHYSICS)

(MSCPH)

Term-End Examination

June, 2026

MPH-001 : MATHEMATICAL METHODS IN PHYSICS

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) Attempt any* ***five*** *questions.*

(ii) Marks are indicated against each question.

(iii) Symbols have their usual meanings.

(iv) You may use non-programmable calculator.

1. (a) Consider a rod whose ends are kept at a constant temperature and its lateral surface is insulated. The heat flow is described by one-dimensional heat equation 4

$$\frac{\partial T(x,t)}{\partial t} = \frac{K\partial^2 T(x,t)}{\partial x^2} \quad (0 < x < L, t > 0)$$

Separate the equation into two ODEs.

(b) Starting from expression for Bessel equation of the first kind of order m

$$J_m(x) = \sum_{k=0}^{\infty} (-1)^k \frac{1}{k!\,\Gamma(m+k-1)} \left(\frac{x}{2}\right)^{m+2k}$$

obtain its recurrance relations

$$J_{m-1}(x) + J_{m+1}(x) = \frac{2m}{x} J_m(x)$$

and $J_{m-1}(x) - J_{m+1}(x) = 2J'_m(x)$ 6

2. (a) The Rodrigues formula for the Legendre Polynomial is given by 4

$$P_n(x) = \frac{1}{2^n\, n!} \frac{d^n}{dx^n} (x^2 - 1)^n$$

Obtain the value of $P_4(x)$.

(b) Expand the following function in terms of Legendre polynomials : 6

$$f(x) = \begin{cases} 0 & -1 < x < 0 \\ 1 & 0 < x < 1 \end{cases}$$

3. (a) Show that for two non-zero vectors f and g 5

$$\| f + g \|^2 - \| f - g \|^2 - i \| f + ig \|^2 + i \| f - ig \|^2 = 4\langle f, g \rangle$$

(b) Show that the set of all 2 × 2 Hermitian matrices form a four-dimensional real vector space. 5

4. (a) Obtain the normalized eigenvectors of real symmetric matrix 6

$$\begin{bmatrix} a & b & 0 \\ b & a & c \\ 0 & c & a \end{bmatrix}$$

along with its diagonalizing matrix.

(b) For the following function locate and name the singularities in a finite z-plane : 2

$$\frac{z^2 - 2z}{z^2 + 2z + 2}$$

(c) Obtain the value of gamma function $\Gamma\left(\frac{3}{2}\right)$. 2

5. Using the method of residues, show that : 10

$$\int_0^{\infty} \frac{dx}{x^4+1} = \frac{\pi\sqrt{2}}{4}$$

6. (a) Obtain the Fourier sine and cosine transforms of the following function : 5

$$f(x) = \begin{cases} 1 & 0 < x < \dfrac{\pi}{2} \\ 0 & x > \dfrac{\pi}{2} \end{cases}$$

(b) Determine inverse Laplace transform of the following function : 5

$$\frac{1}{(s-1)(s^2+1)}$$

7. Consider an infinite metal plate placed in the *xy* plane. Its edge along the *y*-axis is

maintained at a temperature 0°C and the temperature in its edge along x-axis is given by

$$T(x, 0) = \begin{cases} 100^\circ\text{C} & 0 < x < 1 \\ 0^\circ\text{C} & x > 1 \end{cases}$$

The temperature of the plate satisfies Laplace equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Determine the steady state distribution of the plate. 10

8. (a) Let S be a subset of a group G. What are the properties that S should have so that it is a subgroup of G. 3

(b) Complete and verify the multiplication table for the 3 × 3 representation of

S_3. The matrix representation are given by : 7

$$D(e) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; D(p_1) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix};$$

$$D(p_2) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$D(p_3) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}; D(p_4) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix};$$

$$D(p_5) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

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