No. of Printed Pages : 8 **MPH–004**

M. Sc. (PHYSICS)

(MSCPH)

Term-End Examination

June, 2026

MPH-004 : QUANTUM MECHANICS—I

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) Attempt any* ***five*** *questions.*

(ii) Symbols have their usual meanings.

(iii) The values of physical constants are given at the end.

(iv) You may use a calculator.

---

1. (a) A photon and an electron each have an energy of 6.0×10^3 eV. What are their de Broglie wavelengths ? Which of these

would you use to probe atomic structures and why ? 5

(b) Starting from the uncertainty principle explain the concept of the zero point energy. Calculate the zero-point energy for a proton confined to move in a one-dimensional box of length 0·1 nm. 5

2. (a) Write the expressions for the probability density P(x, t) and the probability current density J(x, t) for a one-dimensional quantum mechanical system. Calculate the probability current density for the wave function $\psi(x) = f(x) \exp[ig(x)]$, where $f(x)$ and $g(x)$ are real functions. 5

(b) Derive Ehrenfest theorem for the time-derivative of the expectation values of the position and momentum operators for a system defined by $\hat{H} = \frac{\hat{p}_x^2}{2m} + V(x)$. 5

3. (a) Show that for a symmetric potential $V(x) = V(-x)$, the parity operator commutes with the Hamiltonian. 5

(b) Use the result of 3a to derive the eigen function and eigen energies for a symmetric infinite potential well defined as :

$$V(x) = \begin{cases} \infty \text{ for } x < -L \\ 0 \text{ for } -L < x < L \\ \infty \text{ for } x > L \end{cases}$$

State the boundary conditions. 5

4. (a) Write down the Schrödinger equation for the one-dimensional harmonic oscillator potential $\frac{1}{2} m\omega^2 x^2$. Derive the equation in terms of the dimensionless variables λ and ξ, where $E = \frac{\lambda \hbar \omega}{2}$ and $\xi = \sqrt{\frac{m\omega}{\hbar}}\, x$, E being the energy of the system. 5

(b) Suppose the harmonic oscillator is in the state $|\psi\rangle = A\,|\phi_0\rangle + B\,|\phi_1\rangle + C\,|\phi_2\rangle$ where A, B, C are constants and $|\phi_n\rangle$ are the eigen kets with the corresponding eigen energies E_n. Calculate the probability that a measurement of the energy would yield the value E_1 given that $\langle H \rangle = \frac{3\hbar\omega}{2}$ and $\langle \hat{H}^2 \rangle = \frac{11\hbar^2\omega^2}{4}$. 5

5. Consider an electron in the hydrogen atom in the state

$$\psi(\vec{r}) = N(\psi_{100} + 2\sqrt{2}\, i\, \psi_{210} + 4\, \psi_{21-1})$$

Determine the normalization constant N and the expectation values of the $\hat{H}, \hat{L}^2$ and $\hat{L}_z$. 4+6

6. (a) Show that a Hermitian operator $\hat{\Omega}$ with eigen kets $|\phi_i\rangle, i = 1, ..., n$ and corresponding eigenvalues ω_i can be written as 5

$$\hat{\Omega} = \sum_{i=1}^{n} \omega_i \,|\, \phi_i > < \phi_i \,|$$

(b) Explain collapse of a state vector in measurement. What are compatible and incompatible operators ? What is the consequence of a pair of operators $\hat{A}$ and $\hat{B}$ being compatible or incompatible on the simultaneous measurement of the observables A and B ? 5

7. (a) Starting from the Hamiltonian $\hat{H} = \hbar\omega\left(\hat{a} + \hat{a} + \frac{1}{2}\right)$ for a simple harmonic oscillator of frequency ω, show that the minimum possible value of its eigen energy is $\frac{\hbar\omega}{2}$. 5

(b) Defining the raising and covering operators as $\hat{J}_+ = \hat{J}_x + i\,\hat{J}_y$; $\hat{J}_- = \hat{J}_x - i\,\hat{J}_y$, show that for an angular momentum eigenstate $\left| j, m_j \right\rangle$:

$$\hat{J}_+ \left| j, m_j \right\rangle = \hbar \left[j(j+1) - m_j^2 - m_j \right]^{\frac{1}{2}} \left| j, m_j + 1 \right\rangle$$

Use $\hat{J}_- \hat{J}_+ = \hat{J}^2 - \hat{J}_3^{\,2} - \hbar \hat{J}_z$. 5

8. (i) Write the z-component of the spin operator $\hat{S}_z$ for the electron in the spectral representation using the eigenkets $\left|\uparrow\right\rangle$ and $\left\langle\downarrow\right|$ for the spin up and spin down states respectively.

(ii) Determine the matrix S_z.

(iii) Determine the eigen values and eigen vectors of S_z. 2+2+6

Physical Constants :

$h = 6.626 \times 10^{-34}$ Js;

$\hbar = 1.015 \times 10^{-34}$ Js.

$m_p = 1.67 \times 10^{-27}$ kg;

$m_e = 9.1 \times 10^{-31}$ kg

× × × × ×