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# M. Sc. (PHYSICS)

## (MSCPH)

## Term-End Examination

## June, 2026

### MPH-006 : CLASSICAL MECHANICS—II

*Time : 2 Hours* *Maximum Marks : 50*

***Note :*** *(i) Answer any **five** questions.*

*(ii) The marks for each question are indicated against it.*

*(iii) Symbols have their usual meaning.*

*(iv) You may use a calculator.*

1. (a) Define the Routhian function.

   (b) Write the Lagrangian of a 2-dimensional isotropic simple harmonic oscillator in polar coordinate.

(c) Obtain the Routhian of the simple harmonic oscillator in polar coordinate from the Lagrangian obtained in (b).

(d) Obtain the equations of motion.

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2. State and prove Liouville's theorem for a two-dimensional phase space. 10

3. (a) Write down the condition for canonical transformation when the functional dependence is given. Using this condition, show for what value of A the transformation 5

$Q = -p$, $P = 2Aq + p^2$ is canonical.

(b) Obtain the generating function $F_1(q, Q)$ for the given transformation. 5

4. Consider a charged particle in an electric field where the Hamiltonian is given as :

$$H = \frac{p^2}{2m} + e\,\varepsilon_0\,q\,\sin\omega t$$

where $e$ is the charge, $\varepsilon_0$ and $\omega$ are constants. Using the generating function $F_2 = \left( qP - \frac{e\varepsilon_0}{\omega} q \cos\omega t \right)$, obtain the new Hamiltonian K(Q, P, $t$) and hence the equations of motion in the new variables. 10

5. Consider a particle of mass $m$ in a potential V($r$, $\theta$). The Hamiltonian is given as :

$$H = \frac{1}{2m}\left( p_H^2 + \frac{p_\theta^2}{r^2} + \frac{p_\phi^2}{r^2 \sin^2\theta} \right) + b_1(r) + \frac{b_2\theta}{r^2}$$

Obtain the Hamilton's principle function(s) in its integral form. 10

6. State Hamilton's Principle. Hence derive Hamilton's equations of motion from the variational principles, where the variation in $\mathrm{L}(q_i, \dot{q}_i, t)$ is given as 2+8

$$\delta \mathrm{L} = \dot{q}_i \delta p_i + p_i \delta \dot{q}_i - \frac{\partial \mathrm{H}}{\partial q_i} \delta q_i - \frac{\partial \mathrm{H}}{\partial p_i} \delta p_i + \frac{\partial \mathrm{H}}{\partial t} \delta t.$$

7. (a) Using appropriate diagram, obtain the centre of mass of a : 3+3

   (i) Uniform semi-circular wire of radius R

   (ii) Semi-circular plate of radius R.

   (b) Obtain the moment of inertia of a right circular cone of height $h$ and radius R about its axis. 4

8. Consider a charged particle with mass $m$, charge $q$ moving in a magnetic field $\vec{\mathrm{B}}$. The Lagrangian of the system is given as :

$$\mathrm{L} = \frac{1}{2} m v^2 - q\phi + q\vec{\mathrm{A}} \cdot \vec{\mathrm{V}}$$

where $\phi = (r, t), \vec{A} = \vec{A}(r, t)$ are scalar and vector potentials respectively.

Determine the adiabatic invariant if the circular path of the charge is perpendicular to $\vec{B}$. 10

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