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MASTER OF SCIENCE IN PHYSICS (MSCPH)

Term-End Examination

June, 2026

MPH-008 : QUANTUM MECHANICS—II

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) Answer any **five** questions.*

(ii) Symbols have their usual meanings.

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1. (a) Consider the unitary operator $\hat{S} = 1 - i\,\varepsilon\,\hat{k}$ for an infinitesimal translation or rotation of a physical system where $\hat{k}$ is the generator of the transformation and $\hat{k}$ is Hermitian. If $\hat{S}$ is a symmetry transformation show

that $[\hat{S}, \hat{H}] = 0$ where $\hat{H}$ is the Hamittonian of the system. Show also that $\frac{d<k>}{dt} = 0$. 5

(b) State how the position and momentum operators transform under time reversal. Hence show that the commutation relation $[\hat{x}, \hat{p}] = i\hbar$ is preserved under time-reversal. 5

2. (a) Explain exchange degeneracy in a quantum mechanical system of two identical particles. 2

(b) State the symmetrization postulate. Use the symmetrization postulate to establish the Pauli Exclusion principle. 4

(c) Calculate the ground state energy for a system of five identical spin $-\frac{1}{2}$ particles placed in a simple harmonic oscillator potential of frequency ω_0. 4

3. (a) For a total angular momentum operator $\hat{J} = \hat{J}_1 + \hat{J}_2$ show that $[\hat{J}^2, \hat{J}_1^2] = 0$. 7

(b) Write down the direct product basis kets $| j_1, mj_1 \rangle \otimes | j_2 > mj_2 \rangle$ for the values of $j_1 = 1, j_2 = \frac{1}{2}$, where $\hat{J}_1^2 | j_1, mj_1 \rangle = j_1(j_1 + 1)\hbar^2 | j_1, mj_1 \rangle$ and $\hat{J}_2^2 | j_2, mj_2 \rangle = j_2(j_2 + 1)\hbar^2 | j_2^2, mj_2 \rangle$. 3

4. Consider the two-dimensional infinite potential well :

$$V(x, y) = \begin{cases} 0 & \textit{for } 0 < x \le L, -0 \le y \le L \\ \infty & \text{otherwise} \end{cases}$$

with the following perturbation

$$H_1(x) = \begin{cases} \dfrac{U_0}{2} & \textit{for } 0 \le x \le L/2 \\ 0 & \text{elsewhere} \end{cases}$$

(i) Write down the eigen functions for the first excited state for the unperturbed system and the energy eigen value. 3

(ii) The eigen function and eigen energy of a one dimensional infinite potential well of width L are $\phi_n(x) = \sqrt{\dfrac{2}{L}} \dfrac{\sin \pi x}{L}$ and

$$E_n = \frac{h^2 \hbar^2 \pi^2}{2mL^2}$$ respectively.

Calculate the first order correction to the energy of the first excited state due to the perturbation. 7

5. Write the quantization condition for the energy of a particle in a bound potential in the WKB approximation. Use it to derive the energy eigen values for :

   (i) The particle in a box

   (ii) The harmonic oscillator 2+4+4

6. (i) Consider a system described by the Hamiltonian $\hat{H} = \hat{H}_0 + V(t)$ where $\hat{H}_0$ has no explicit time dependence. Given that the state ket $|\,\psi(t)\rangle$ for the system at the initial time t_0 is $|\,\psi(t_0)\rangle$, derive the equation of motion for the state

ket in the interaction picture. Use

$$| \psi(t) \rangle_I = e^{\frac{i\hat{H}_0(t - t_0)}{\hbar}} | \psi(t) \rangle \quad \text{and}$$

$$\hat{O}_I = \frac{e^{iH_0(t - t_0)}}{\hbar} \hat{O}_e - iH_0(t - t_0) / \hbar . \qquad 6$$

(ii) Explain the adiabatic approximation and the sudden approximation in time dependent perturbation theory. 4

7. (a) Consider a two state quantum system with normalized eigen kets $|1\rangle$ and $|2\rangle$, and corresponding eigen energies E_1 and $E_1 + 2\hbar$ respectively. For $t < 0$ the system is in the state $|1\rangle$. At $t = 0$, the potential is altered by a time

independent potential V. Such that $\langle 2 | V | 1 \rangle = \frac{\hbar}{2}$. Calculate the transition probability from state $|1\rangle$ to state $|2\rangle$ as a function of time. 5

(b) The differential scattering cross-section for scattering by a target is given by :

$$\frac{d\sigma}{d\Omega} = \alpha^2 + \beta^2 \cos^2 \theta$$

where α and β are constants.

Calculate the total scattering cross-section and the number of particles scattered per unit time if the incoming flux of particles is N_0. 5

8. Derive the Klein-Gordon equation from the relativistic energy momentum relation for the free particle. Obtain the charge conservation equation. Explain how the interpretation of the charge density differs from the non-relativistic interpretation. 10

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