No. of Printed Pages : 6 **MST–012**

M. SC. (APPLIED STATISTICS)

(MSCAST)

Term-End Examination

June, 2026

MST-012 : PROBABILITY AND PROBABILITY DISTRIBUTIONS

Time : 3 Hours *Maximum Marks : 50*

Note : *(i) Q. No. 1 is compulsory.*

*(ii) Attempt any **four** questions from the remaining Question Nos. 2 to 6.*

(iii) Use of scientific calculator (non-programmable) is allowed.

(iv) Symbols have their usual meanings.

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1. (a) If $P(E_1) = 0.3$, $P(E_2) = 0.7$, $P(A|E_1) = 0.02$, $P(A|E_2) = 0.05$, $E_1 \cup E_2 = \Omega$, $E_1 \cap E_2 = \phi$, then find $P(E_2|A)$. 2

(b) Suppose you are interested in the distribution formed by the last digits of all the mobile numbers of a particular company. Without collecting data what distribution the last digit of mobile number may be supposed to follow. Write PMF of this distribution. 3

(c) If X ~ Exp. (λ), then $P(X > x) = e^{-\lambda x}$. Comment whether this statement is true *or* false. Justify your answer. 2

(d) It is known that sum of 1000 numbers is 100000 and sum of their squares is 10025. On the basis of this information find an upper bound on how many numbers out of these 1000 numbers are ≥ 110. 3

2. (a) Some entries of the CDF of a discrete bivariate random variable (X, Y) have lost. Available entries are given as follows :

Values of X	**Values of Y**					
	1	2	3	4	5	6
1	NA	0.05	NA	0.12	0.19	NA
2	0.04	NA	0.20	NA	0.38	NA
3	NA	0.19	NA	0.46	NA	0·71
4	0.08	0.23	0.39	NA	0.69	NA
5	NA	0.32	NA	0.68	NA	1

Find the following probabilities : 6

(i) $P(2 < X \leq 4, 3 < Y \leq 5)$

(ii) $P(3 < X \leq 5, 2 < Y \leq 6)$

(iii) $P(1 < X \leq 4, 2 < Y \leq 5)$

(b) If Ω be the sample space of a random experiment and $\omega \in \Omega$ be a fixed element of Ω. Let F be a σ-field on Ω. Show that the function $\delta_\omega : F \rightarrow [0, 1]$ defined by :

$$\delta_\omega (E) = \begin{cases} 0; \text{ if } \omega \notin E \\ 1; \text{ if } \omega \in E \end{cases}$$

is a probability measure. 4

3. Suppose in a bag there are 2 red, 4 blue and 5 black balls. Three balls are drawn from this bag randomly. Let X, Y denote the number of red and blue balls, respectively out of the three drawn balls.

Find :

(a) E (X) (b) E (Y), (c) V(X), (d) V(Y), (e) Cov (X, Y),(*f*) SD(X), (g) Interpret results of parts,(e) and (*f*). 10

4. State and prove weak and strong laws of large numbers. 10

5. (a) In a city concentration of pollutants (X-in parts per ten lakhs) is such that $e^{X} \sim N\ (2.9,\ 2.25)$. Write PDF of the random variable X. Find mean and variance of X. Also, find the probability that the concentration is less than 5 parts per ten lakhs. 5

(b) If X ~ Cauchy (10, 5), then find probabilities that the random variance X is : 5

(i) Less than 8

(ii) Greater than 10

6. (a) Let X be a random variable with E (X) = 3. Then which one of the following is true and why ? 4

$$E\left(\frac{1}{X+2}\right) \geq \frac{1}{5} \text{ or } E\left(\frac{1}{X+2}\right) < \frac{1}{5}\ \forall x > -2$$

(b) Let (Ω, F, P) be a probability space and we have a random variables X and a sequence of random variable X_1, X_2, X_3, ... such that $X_n \xrightarrow{\text{m.s.}} X$. Does it change implies that $X_n \xrightarrow{\text{a.s.}} X$. If your answer is yes, then prove your claim and if your answer is no, then give a counter example. 6

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