M. SC. (APPLIED STATISTICS) (MSCAST)

Term-End Examination

June, 2026

MST-018 : MULTIVARIATE ANALYSIS

Time : 3 Hours *Maximum Marks : 50*

Note : *(i) Question No.* ***1*** *is compulsory.*

(ii) Attempt any ***four*** *questions from the remaining question nos.* ***2*** *to* ***6****.*

(iii) Use of scientific calculator (non-programmable) is allowed.

(iv) Symbols have their usual meanings.

1. State whether the following statements are True *or* False. Give reasons in support of your answer : 5×2=10

(a) The dendrograms obtained using single linkage, complete linkage and average linkage methods are same.

(b) Canonical correlation is a statistical multivariate technique that identifies and quantifies the association between the two sets of variables.

(c) Factor loadings of an orthogonal factor model are uniquely determined.

(d) If the covariance matrix of a random vector $\underset{\sim}{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ is $\begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$, then X_1 and X_2 are independent.

(e) The sample mean vector $\underset{\sim}{\bar{X}}$ and sample covariance matrix S are correlated.

2. (a) Let $\Sigma = \begin{pmatrix} A & B \\ 0 & C \end{pmatrix}$ be the partitioned matrix such that A and C are non-singular, then the inverse of Σ is given by

$$\Sigma^{-1} = \begin{pmatrix} A^{-1} & -A^{-1}BC^{-1} \\ 0 & C^{-1} \end{pmatrix}.$$ 5

(b) Let $\underset{\sim}{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ has the following joint density function. 5

$$f(x_1, x_2) = \begin{cases} k(x_1 + x_2 - 4x_1^3 x_2), & 0 < x_1 < 1, 0 < x_2 < 1 \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant.

Then find :

(i) the value of k

(ii) marginal distribution of X_1 and X_2

(iii) $E(X_1)$, $E(X_2)$, $Var(X_1)$ and $Var(X_2)$

3. (a) Define canonical correlation. Prove that multiple correlation is a particular case of canonical correlation. 5

(b) If $\underset{\sim}{X} \sim N_4 (\mu, \Sigma)$ with $\underset{\sim}{\mu} = \begin{pmatrix} 3 \\ 1 \\ -4 \\ 2 \end{pmatrix}$ and

$$\Sigma = \begin{pmatrix} 4 & 0 & 3 & 0 \\ 0 & 5 & 0 & 2 \\ 3 & 0 & 9 & 0 \\ 0 & 2 & 0 & 6 \end{pmatrix}.$$ 5

(i) Obtain the marginal distribution of $\begin{pmatrix} X_1 \\ X_3 \end{pmatrix}$ and $\begin{pmatrix} X_2 \\ X_4 \end{pmatrix}$.

(ii) Obtain the conditional distribution of $\begin{pmatrix} X_1 \\ X_3 \end{pmatrix}$ given $\begin{pmatrix} X_2 \\ X_4 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

4. (a) Define Hotelling's T^2 statistic and prove that the distribution of Hotelling's T^2 statistic remains preserved after performing a non-singular linear transformation. 5

(b) If $\underset{\sim}{X} \sim N_p(\underset{\sim}{\mu}, \Sigma)$, then prove that

$$\underset{\sim}{Y}_{q\times 1} = C_{q\times p}\,\underset{\sim}{X}_{p\times 1} \sim N_q(C\underset{\sim}{\mu}, (\Sigma C^1),$$

where C is any $q \times p$ matrix of rank $q\ (\leq p)$ of constant elements. 5

5. (a) Let $\underset{\sim}{X} \sim N_4(\underset{\sim}{\mu}, \Sigma)$, where $\mu = \begin{pmatrix} -5 \\ 2 \\ 3 \\ -1 \end{pmatrix}$ and

$$\Sigma = \begin{pmatrix} 9 & 1 & 2 & 8 \\ 1 & 7 & 1 & -4 \\ 2 & 1 & 1 & 0 \\ 8 & -4 & 0 & 14 \end{pmatrix}.$$ 5

(i) Find the joint distribution of

$Y_1 = 2X_1 - X_2 + X_3 + X_4$ and

$Y_2 = -X_1 + 2X_2 + X_3 - 3X_4$

(ii) Find the joint distribution of

$Y_1 = X_1 + 1$, $Y_2 = X_1 + X_2 + 0$,

$Y_3 = X_1 + X_2 + X_3 + 1$ and

$Y_4 = X_1 + X_2 + X_3 + X_4 + 0$.

(b) Prove the following :

(i) χ^2 distribution is a univariate analogue of the Wishart distribution.

(ii) If $A_i \sim W(n_i, p, \Sigma), i = 1, 2$ are independent, then

$$\sum_{i=1}^{2} A_i \sim W\left(\sum_{i=1}^{2} n_i, p, \Sigma\right)$$ 2+3=5

6. For the following, distance matrix D :

$$D = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & & & \\ 2 & 0 & & \\ 10 & 3 & 0 & \\ 4 & 1 & 5 & 0 \end{bmatrix} \end{array}$$

cluster the four samples using complete linkage method. 10

× × × × ×