No. of Printed Pages : 4 **MST–022**

M. SC. (APPLIED STATISTICS)

(MSCAST)

Term-End Examination

June, 2026

MST-022 : LINEAR ALGEBRA AND MULTIVARIABLE CALCULUS

Time : 3 Hours *Maximum Marks : 50*

Note : *(i) Question number **1** is compulsory.*

*(ii) Attempt any **four** questions from question numbers **2** to **6**.*

(iii) Use of scientific calculator (non-programmable) is allowed.

(iv) Symbols have their usual meanings.

1. State whether the following statements are True or False. Give reason in support of your answer : 5×2=10

(a) If $X \in \mathbf{R}^n$, then $\nabla f = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$.

(b) If $x = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, y = \begin{bmatrix} -1 \\ 5 \\ 6 \end{bmatrix}$, then inner product of x and y is 37.

(c) If $x = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, y = \begin{bmatrix} -1 \\ 5 \\ 6 \end{bmatrix}$, then outer product of x and y is $\begin{bmatrix} -2 & 10 & 12 \\ -3 & 15 & 18 \\ 4 & -20 & -24 \end{bmatrix}$.

(d) A function $f : \mathbf{R}^2 \to \mathrm{R}$ is defined by

$$f(x, y) = x + y - 3, (x, y) \in \mathbf{R}^2$$

then $x + y = 5$ is a level surface of this function.

(e) Taylor's theorem for function of two variables x and y in terms of gradient and Hessian matrix upto terms of second order derivatives can be written as follows :

$$f(x+h_1, y+h_2) =$$

$$f(x, y) + \nabla^{T} f(X)h + \frac{1}{2} h \, H_f(X) h^{T} + \ldots\ldots$$

where $X = \begin{bmatrix} x \\ y \end{bmatrix}, h = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$ and H denotes Hessian matrix of f.

2. Find the point on the plane

$$f(x, y, z) = 2x + 3y + 4z - 24$$

which is closest to the origin O(0, 0, 0). Hence, find the minimum distance of the origin from this plane. 10

3. Find extreme values of the function

$$f(x, y) = x^4 - 4xy + y^4 + 8$$

and examine them as local minima or maxima or saddle points. 10

4. Write the Spectral Decomposition form of the matrix $S = \begin{bmatrix} 3 & 6 \\ 6 & -2 \end{bmatrix}$. 10

5. Let $(V = \mathbf{R}^3, <, >)$ be an inner product space over the field F = **R**, and W is a subspace of V spanned by the vectors 10

$w_1 = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$ and $w_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$. Find the orthogonal projection matrix that projects onto subspace W. Also, find the orthogonal projection of the vector $\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ on the subspace W.

6. State and prove Cauchy Schwarz inequality of linear algebra. 10

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