No. of Printed Pages : 6 **MSTE-012**

M. SC. (APPLIED STATISTICS)

(MSCAST)

Term-End Examination

June, 2026

MSTE-012 : STOCHASTIC PROCESSES

Time : 3 Hours *Maximum Marks : 50*

Note : *Question No.* ***1*** *is compulsory. Attempt any* ***four*** *questions from the remaining question nos. 2 to 6. Use of scientific (non-programmable) calculator is allowed. Symbols have their usual meanings.*

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1. State whether the following statements are True *or* False. Give reasons in support of your answers : 5×2=10

   (a) If parametric space 'T' is continuous in a Markov process then, it is called a Markov chain.

   (b) If birth rate 'λ' is greater than the death rate 'μ' i.e., $\lambda > \mu$, then mean of the population size $E[x(t)] \to 0$ as $t \to \infty$.

   (c) The average waiting time of a customer in the system (W_s) is $\left(\frac{\mu}{\lambda}\right)$ times of the average waiting time of a customer in the queue (W_q).

   (d) A random process $\{x(t)\}$ with infinite first and second-order moments is called a wide sense stationary or weakly stationary process.

(e) The sum of all the elements of any row of the transition probability matrix is always 0, because the transition from one state to another states is uncertain event.

2. (a) Explain a Transition Probability Matrix (TPM) in context to a Markov Chain. What are the elements of such a matrix and what properties must they satisfy ? 5

(b) A Markov Chain with three states 0, 1 and 2 has the following transition probability matrix :

$$P = \begin{bmatrix} 0.60 & 0.40 & 0.0 \\ 0.35 & 0.30 & 0.35 \\ 0.0 & 0.75 & 0.25 \end{bmatrix}$$

Determine the probabilities :

(i) $P[X_1 = 1, X_2 = 1 \mid X_0 = 0]$ and

(ii) $P[X_0 = 1, X_1 = 0, X_2 = 2]$

The initial probability distribution is

$P[X_0 = 0] = [0.30, 0.30, 0.40]$ 5

3. (a) What are Inter-Arrival Times (IATs) in Poisson process ? Derive the probability distribution of it. 5

   (b) A fire and emergency rescue service receives calls for assistance at a rate of ten per day. Teams run the service in twelve hour shifts. Assume that requests for help form a Poisson process.

   (i) What is the probability that a team would receive six requests for help in a shift ?

   (ii) What is the probability that a team has no requests for assistance in a shift ? 5

4. (a) Explain the *three* types of renewal processes :

   (i) Ordinary renewal process

   (ii) Modified renewal process

   (iii) Equilibrium renewal process. 6

(b) Find the mean and variance of the stationary process $\{X(t)\}$, whose Autocorrelation Function is given by

$$R_{XX}(\tau) = 16 + \frac{9}{1 + 6\tau^2}$$ 4

5. (a) A duplicating machine maintained for office use is operated by an office assistant who earns Rs. 5 per hour. The time to complete each job varies according to an exponential distribution with mean 6 minutes. Assume a Poisson input with an average arrival rate of 5 jobs per hour. If an 8 hour per day schedule is used as a base, determine :

(i) The percentage idle time to machine,

(ii) The average time a job in the system

(iii) The average earning per day of the assistant. 2+1+2

(b) What do you mean by Transient state and Steady-state queueing system ? Also give the formulae for the average number of customers :

(i) in the system,

(ii) in the queue,

(iii) in the non-empty queues for the (M/M/1) : {α/FIFO) model. 2+3

6. (a) A fair dice is tossed repeatedly. If X_n denotes the maximum of the numbers occurring in the first n tosses, find the Transition Probability Matrix P of the Markov Chain $\{X_n\}$. Find also P^2. 3+2

(b) Define Autocorrelation Function of a stationary process. If $R(\tau)$ is the Autocorrelation function of a stationary process $\{X(t)\}$, prove that $\lim_{\tau \to \infty} [R(\tau)] = \mu_X^2$. 2+3

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