



“ शिक्षा मानव को बन्धनों से मुक्त करती है और आज के युग में तो यह लोकतंत्र की भावना का आधार भी है। जन्म तथा अन्य कारणों से उत्पन्न जाति एवं वर्गगत विषमताओं को दूर करते हुए मनुष्य को इन सबसे ऊपर उठाती है।”

–इन्दिरा गांधी

VOLUME II

BLOCK 3: DIFFRACTION

BLOCK 4: LASERS AND OPTICAL FIBRE

“ Education is a liberating force, and in our age it is also a democratising force, cutting across the barriers of caste and class, smoothing out inequalities imposed by birth and other circumstances.”

– Indira Gandhi

Block

3**DIFFRACTION**

UNIT 9**Fresnel Diffraction**

7**UNIT 10****Fraunhofer Diffraction**

39**UNIT 11****Diffraction Grating**

67

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BLOCK 3 : DIFFRACTION

You have studied two blocks of this course. In Block 1, you learnt waves and their characteristic features. You also learnt how Maxwell's electromagnetic theory predicted that light is an electromagnetic wave which comprises time-varying electric and magnetic fields. The wave nature of light provides us a basis for understanding many experimentally observed phenomena such as polarisation, interference and diffraction of light. In Block 2, you learnt that when light from two coherent sources is made to superpose, redistribution of energy manifests in the formation of bright and dark fringes. This phenomenon, known as interference of light, was explained on the basis of the wave model of light. In the present block, you will study diffraction of light.

In our day-to-day experience, we observe that light casts shadow of objects which indicates that light travels in straight lines rather than bending around obstacles as we should expect from waves. This apparent contradiction was explained by Fresnel. You will learn that the ease with which a wave bends around edges depends on the size of the obstacle relative to the wavelength. The wavelength of light is about 6×10^{-7} m and the obstacles used in ordinary experiments are about a few centimetres in size. Thus, the size of the obstacles is 10^4 to 10^5 times bigger than the wavelength of light and we do not observe bending of light. However, when the size of the obstacle is comparable to the wavelength of light, we do observe diffraction of light; that is, bending of light round the corners.

The phenomenon of diffraction was first observed by Grimaldi and a systematic explanation was given by Fresnel. In diffraction phenomenon, interference takes place between secondary wavelets from different parts of the same wavefront. Diffraction is classified in two categories: Fresnel diffraction and Fraunhofer diffraction.

In Unit 9, we discuss Fresnel diffraction. In the experimental set up to observe Fresnel diffraction, the source or the observation screen or both are at a finite distance from the diffracting obstacle. You will learn Fresnel construction and the concept of half period elements (or zones) which were proposed to explain the observed diffraction pattern of some common obstacles such as a straight edge, a slit and a wire.

Unit 10 deals with Fraunhofer diffraction wherein the source of light and the observation screen are effectively at infinite distance from the obstacle. This requirement on the distance of the source and screen from the obstacle is ascertained by using suitable lenses. You will learn the salient features of the diffraction patterns caused by a single slit and a double slit. When a vertical slit is illuminated by a distant point source, the diffraction pattern consists of a series of bright spots along a horizontal line. The unique feature of the double slit diffraction pattern is that it consists of bright and dark fringes similar to those observed in interference experiments. You will learn the theoretical basis to explain all the salient features of the single slit and double slit diffraction patterns.

In **Unit 11**, we have extended our analysis of double slit diffraction to understand the Fraunhofer diffraction pattern caused due to a large number (N) of closely spaced, equidistant slits. The understanding of N slit diffraction enables us to discuss an important and useful optical device called diffraction grating. You will learn that diffraction grating, which consists of

a large number of equidistant vertical slits, produces sharp fringes. Since the direction of these fringes depends on the wavelength of light, grating is a very useful optical device for spectral analysis.

We hope you enjoy studying this Block.





UNIT 9

FRESNEL DIFFRACTION

A Fresnel zone plate is a circular device with alternating transparent and opaque circular strips called zones. This is a useful device to focus electromagnetic waves which are difficult to focus using ordinary lenses. It is designed on the basis of the concept of half period zones and works on the principle of diffraction.

(Source: wikipedia.org)

Structure

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|-----|---------------------------------|-----|--|
| 9.1 | Introduction | 9.4 | The Zone Plate |
| | Expected Learning Outcomes | 9.5 | Diffraction Patterns of Simple Obstacles |
| 9.2 | Observing Diffraction | | A Straight Edge |
| | Some Simple Experiments | | A Slit |
| | Producing a Diffraction Pattern | | A Wire |
| 9.3 | Fresnel Construction | 9.6 | Summary |
| | Half-period Elements | 9.7 | Terminal Questions |
| | Rectilinear Propagation | 9.8 | Solutions and Answers |

STUDY GUIDE

This is the first unit of the block on diffraction. Diffraction of light refers to bending of light around corners or obstacles. As such, diffraction is a property exhibited by all types of waves. Thus, observing diffraction of light is an experimental evidence of wave nature of light. For ease of mathematical analysis, diffraction of light is classified into two classes: Fresnel diffraction and Fraunhofer diffraction. While discussing Fresnel diffraction in this unit, we shall use Huygens-Fresnel principle you studied in Block 1 of the course. You should revise Huygens principle as well as its modification by Fresnel. You will also learn quite a few new concepts such as half period elements (or zones) and zone plate. To appreciate these concepts better, you should carefully look at the figures and focus on its geometrical orientation. The complete mathematical analysis of Fresnel diffraction from common obstacles/apertures is rather complex and beyond the scope of this course. We have, therefore, discussed diffraction qualitatively using the concept of secondary wavelets proposed by Huygens and the concept of half period zones proposed by Fresnel. You should focus on these concepts and how they are applied to explain Fresnel diffraction patterns. Also, you must try to solve SAQs and TQs yourself because it will give you confidence in respect of your understanding of the subject matter discussed in this unit.

“Simplicity lies concealed in this chaos, and it is only for us to discover it.”

***Augustin-
Jean Fresnel***

9.1 INTRODUCTION

Our daily life experiences have made us believe that light travels in straight line. But, as you have learnt in the previous units of this course, light is a wave and waves do not always propagate in straight line. **When waves – mechanical as well as electromagnetic – pass near a barrier, they tend to bend around the barrier and spread out in the region of geometric shadow. This phenomenon is called diffraction.** We come across such bending of waves around barriers in our day-to-day life. For example, we can hear persons talking in an adjoining room whose door is open due to the ability of sound waves to bend around the corners of obstacles in their way. You are also familiar with the ability of water waves to propagate around obstacles. Similarly light, which is an electromagnetic wave, also bend around corners of obstacles in its path. In the previous block, you have studied about manifestation of wave nature of light in the form of interference: Light from two coherent sources interferes to form fringed pattern. Diffraction is yet another phenomenon which happens because light is a wave.

But what may puzzle you is the fact that light casts shadows of objects, i.e. appears to travel in straight lines rather than bending around corners. This apparent contradiction was explained by Fresnel who argued that the ease with which a wave bends around corners is strongly influenced by the size of the obstacle (aperture) relative to its wavelength. Sound waves have wavelengths in the range 1.7 cm to 17 m and a typical door is about 1 m wide (aperture) so that long wavelength sound waves bend more readily around the door way. On the other hand, wavelength of light is about 6×10^{-5} cm and the obstacles used in ordinary experiments are about a few centimetres in size, i.e. $10^4 - 10^5$ times bigger. For this reason, the diffraction of light is not prominent and light appears to travel along straight lines and casts shadows of objects.

To observe bending of light around corners, we need to ascertain suitable conditions where size of obstacles is comparable with the wavelength of light. You can get a feel for this by closely examining shadows cast by objects. You will observe that the edges of shadows are not sharp. This happens because the light waves have diffracted in the region of geometrical shadow.

The phenomenon of diffraction was first observed by Grimaldi, an Italian mathematician. However, a systematic explanation of diffraction was given by Fresnel on the basis of Huygens-Fresnel principle. According to it, diffraction is attributed to the mutual interference of secondary wavelets from different parts of the same wavefront. And, depending upon the phase difference between these secondary wavelets, the interference or superposition of these secondary wavelets give rise to diffraction fringe pattern. **Thus, all diffraction phenomena are consequences of interference.** However, all interference phenomena do not produce diffraction. For example, the interference patterns produced in Michelson interferometer, discussed in Unit 8, is not a diffraction pattern. The difference between interference and diffraction is that, in the former, we consider the superposition of wave from two light waves which are emanating from independent but coherent sources and in the later, we

consider the superposition of secondary wavelets emanating from the same wavefront.

For mathematical convenience and ease in understanding, diffraction is classified into two categories: Fraunhofer diffraction and Fresnel diffraction. In Fraunhofer class of diffraction, the source of light and the observation screen (or human eyes) are effectively at infinite distance from the obstacle. This condition on the distances of the source and the screen is obtained most conveniently using suitable lenses. Fraunhofer diffraction is of particular practical importance in respect of the general theory of optical instruments. You will learn about Fraunhofer diffraction in the next unit.

In Fresnel class of diffraction, the source or the observation screen or both are at finite distances from the obstacle. Thus, for Fresnel diffraction, the experimental arrangement is fairly simple. But its theoretical analysis is rather difficult compared to that of Fraunhofer diffraction. Also, Fresnel diffraction, which you will study in this unit, is more general; it includes Fraunhofer diffraction as a special case.

In Sec. 9.2, we discuss the basic experimental set up to observe Fresnel diffraction. In Sec. 9.3, you will learn the concept of Fresnel half period elements. In Sec. 9.4, we have discussed the zone plate. And, based on these concepts, the Fresnel diffraction patterns of some obstacles/aperture have been discussed in Sec. 9.5.

Expected Learning Outcomes

After studying this unit, you should be able to:

- ❖ describe simple experiments which illustrate diffraction phenomenon;
- ❖ describe an experimental set-up for observing diffraction by a circular obstacle;
- ❖ discuss the concept of Fresnel half-period zones and apply it to zone plate;
- ❖ show that each half period zone has nearly the same area;
- ❖ explain the rectilinear propagation of light on the basis of the concept of half period zones;
- ❖ show that a zone plate acts like a converging lens;
- ❖ discuss diffraction pattern due to a straight edge, a slit and a wire; and
- ❖ solve numerical problems related to Fresnel diffraction.

9.2 OBSERVING DIFFRACTION

Diffraction of light refers to the bending of light around corners or edges and thereby deviating from rectilinear propagation. It refers to various phenomena associated with wave propagation such as bending, scattering and interference of waves emerging from an aperture (or encountering an obstacle). Diffraction occurs for all types of waves including sound waves, water waves, light (electromagnetic) waves and matter waves. Diffraction

always occurs but its effects are noticeable only when the wavelength is comparable to the size of the diffracting object.

Now, before we discuss how to explain diffraction of light, it will be worthwhile to tell you about some simple and easy to do experiment to observe this phenomenon.

9.2.1 Some Simple Experiments

As you know, the wavelength of visible light ranges from 400 nm to 700 nm. Thus, to see diffraction of light, careful observation has to be made because the size of the diffracting object has to be comparable to the wavelength. We will now familiarise you with some simple situations and experiments to observe diffraction of light. The pre-requisites for these are: i) a source of light, preferably narrow and monochromatic, ii) a sharp edge obstacles, and iii) an observation screen, which could be human retina as well.

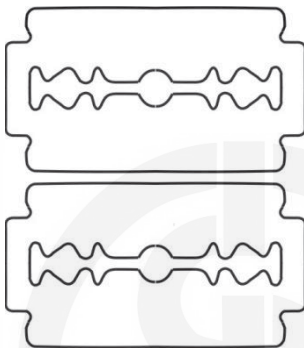


Fig. 9.1: Observing diffraction using a pair of razor blades.

French scientist, Simeon D. Poisson was a member of the committee which was appointed to judge Fresnel's dissertation. To disprove Fresnel, and hence wave theory, Poisson argued that a central bright spot should appear in the shadow of a circular obstacle. His logic, called *reduction ad absurdum*, goes as follows: Consider the shadow of a perfectly round object O being cast by a point source (S) shown in Fig. 9.2.

(Contd...)

1. Look at a distance street-light at night and nearly close your eye lashes. You will observe that the light appears to streak out from the bulb. This is because light has bent around the corners of your eyelids.
2. Stand in a dark room and look at a distance light bulb in another room. Now move slowly until the doorway blocks half of the light bulb. The light from the bulb will appear to streak out into the umbra region of the dark room due to diffraction around the doorway.
3. Take a piece of fine cloth, say fine handkerchief or muslin cloth. Stretch it flat and keep it close to the eye. Now, focus your eye on a distant street lamp (at least 100 m away) through it. Do it at night. Do you observe a regular pattern of spots arranged along a rectangle? On careful examination, you will note that the spots on the outer part of the pattern appear coloured. Now, rotate the handkerchief in its own plane. Does the pattern rotate? You will be excited to see that the pattern rotates and the speed of rotation of the pattern is same as that of the handkerchief.
4. Take a pair of razor blades and one clear glass electric bulb. Hold the blades so that the edges are parallel and have a narrow slit in-between, as shown in Fig. 9.1. Keep the slit close to your eye and parallel to the filament. (Use spectacles if you normally do.) By carefully adjusting the width of the slit, you should observe a pattern of bright and dark bands which show some colours. Now, use a blue or red filter. What do you observe? Does the pattern become clearer?
5. Mount a small ball bearing carefully on a plate of glass with a small amount of beeswax so that no wax spreads beyond the rim of the ball. Place this opaque obstacle in a strong beam of light (preferably monochromatic) diverging from a pinhole. Under suitable conditions, you will observe a bright spot, called **Poisson spot** (see the margin remark) at the centre of the shadow cast by the ball bearing. This exciting observation proved unchallengable evidence for diffraction of light.

We hope that in the situations and experiments described above, you must have observed bright and dark fringes of different shapes. These are diffraction fringes produced by the diffraction of light by the obstacles/apertures. Let us now discuss a typical experimental set-up in a laboratory to observe Fresnel diffraction.

9.2.2 Producing a Diffraction Pattern

In Fresnel class of diffraction, the source of light or the screen or both are, in general, at a finite distance from the diffracting obstacles. On the other hand, in Fraunhofer diffraction, this distance is effectively infinite. This condition is achieved by putting suitable lenses between the source, the obstacles and the screen. A large number of workers have observed and studied Fresnel and Fraunhofer diffraction patterns.

A systematic study of Fresnel diffraction pattern from obstacles of different shapes e.g. small spheres, discs and apertures – circular, elliptical, square, or triangular – of different sizes was done by Indian Physicist, Y.V. Kathvate under the guidance of Prof. C.V. Raman. Their experimental set up for photographing these patterns is shown in Fig. 9.3. It consists of a light tight box (length nearly 5 m) B with a fine pinhole at one end. The light on the pinhole from a 100 W lamp L was focussed using a convex lens. A red filter F was used to obtain monochromatic light of wavelength $6320 \text{ \AA}$. The obstacle O was placed at a suitable distance (about 2 m) from the pinhole. The photographic plate P was mounted on a movable stand so that its distance from the obstacle could be varied. They used steel ball bearings of radii 1.58 mm, 1.98 mm, 2.37 mm and 3.17 mm as spherical obstacles. They also worked with discs of the same sizes. (As such, you should not attach much significance to the exactness of these sizes.) These obstacles, in turn, were mounted on a glass plate, which was kept at a distance of about 2 m from the pinhole.

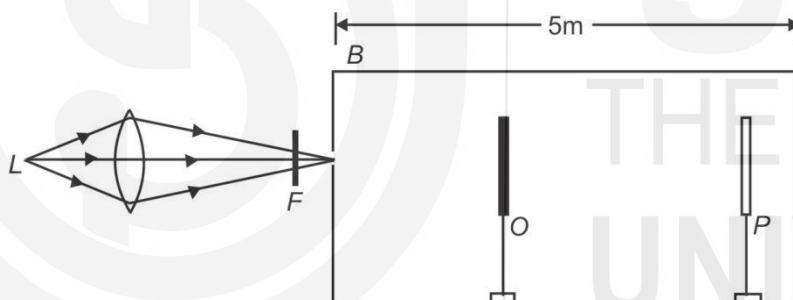


Fig. 9.3: Schematic diagram of the experimental arrangement used by Kathvate to observe Fresnel diffraction.

The photographic plate was kept at distances of 5 cm, 10 cm, 20 cm, 40 cm and 180 cm from the mounted glass plate (obstacles). The diffraction patterns obtained from these spheres when the separation between the obstacles and the photographic plate was 180 cm are shown in Fig. 9.4a. These patterns clearly show the distribution of light intensity in the region of geometrical shadow of the obstacles.

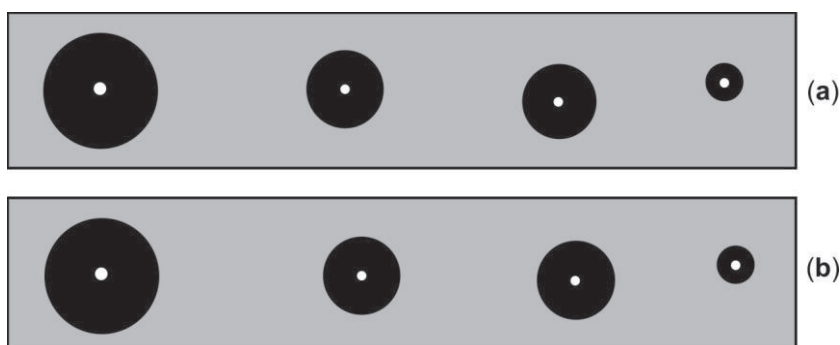


Fig. 9.4: Fresnel diffraction patterns obtained in Kathvate experiments with a) spheres; b) circular discs of four different sizes.

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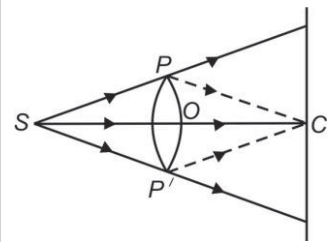


Fig. 9.2: Shadow of a circular object formed due to light from a point source.

According to the wave theory, all the waves at the periphery of the circular object O will be in phase. This is because they have covered the same distance from the source S . So, the waves starting from the rim PP' and reaching the centre C of the shadow on the screen should all be in phase. This implies that there should be constructive interference giving rise to a bright spot at the centre C of the shadow. **This was considered absurd by Poisson.** He was definitely not aware that the bright spot in question had already been discovered by Maraldi almost a century ago. Soon after Poisson's objection, Arago carried out the experiment using a disk of 2 mm diameter. To his surprise, he rediscovered the central bright spot.

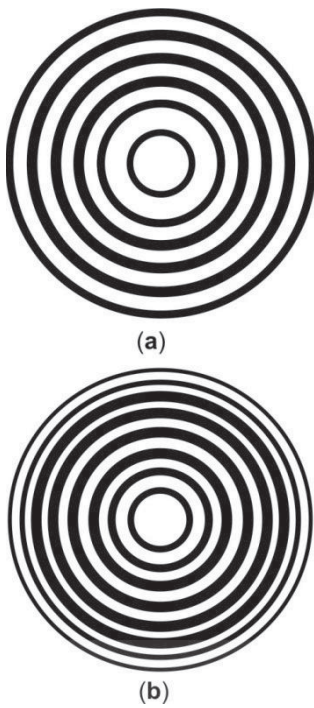


Fig. 9.5: Enlarged view of fringe patterns for discs of radii a) 1.58 mm; b) 1.98 mm.

The diffraction patterns for circular discs of the same size as that of spheres are illustrated in Fig. 9.4b. You will find that these patterns are almost similar to those for spheres. Moreover, the diffraction patterns on the left half of this figure, which correspond to bigger spheres and discs (radii 3.17 mm and 2.37 mm), show the geometrical shadow and a central bright spot within it. On the other hand, in the diffraction patterns corresponding to the smaller sphere and disc of radius 1.98 mm, the geometrical images are recognisable but have fringes appearing on edges.

The fringe pattern around the central spot becomes markedly clearer for the sphere and disc of radius 1.58 mm. An enlarged view of the pattern for discs of radius 1.58 mm and 1.98 mm are shown in Fig. 9.5. The formation of the bright central spot in the shadow and the rings around the central spot are the most definite indicators of non-rectilinear propagation of light. This suggests that light bends in some special way around opaque obstacles. These departures from rectilinear propagation come under the heading of diffraction phenomenon.

In the experiment described above as well as in Sec. 9.2, we find that the diffraction pattern produced by obstacles/apertures consist of bright and dark fringes. So, you must now be interested to understand the genesis of these patterns, at least, qualitatively. First systematic effort in this direction was made by French physicist, Augustin-Jean Fresnel. At the core of Fresnel's explanation is the concept of half period elements (or zones) and the application of Huygens principle. Let us learn about it now.

9.3 FRESNEL CONSTRUCTION

Let us consider a plane wavefront of light represented by WW' propagating towards the right, as shown in Fig. 9.6a. First, we calculate the intensity due to this plane wavefront at an external point P_0 on the screen at a distance b . Then, we will introduce an obstacle like a straight edge Q and see how intensity at P_0 changes.

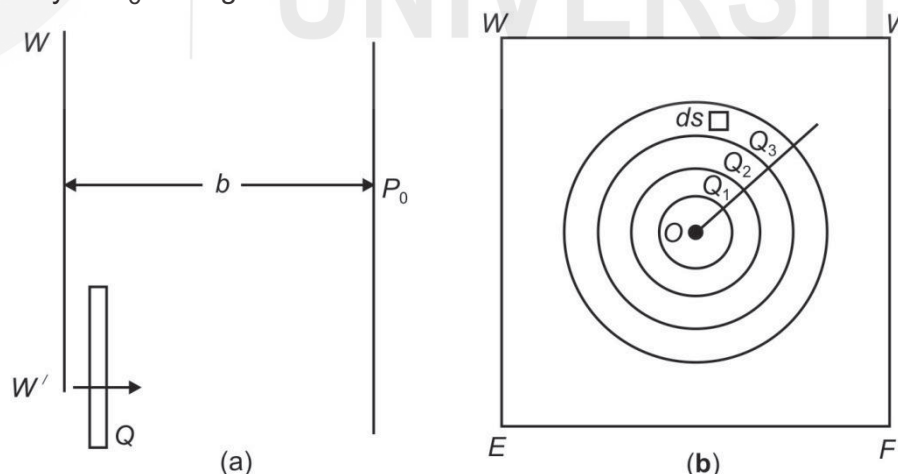


Fig. 9.6: Fresnel construction: a) propagation of a plane wavefront WW' ; b) division of wavefront into annular spaces enclosed by concentric circles.

To compute the resultant effect of the plane wavefront WW' at point P_0 , Fresnel used Huygens principle. As you have learnt in Unit 4, according to this principle, every point on the plane wavefront WW' may be thought of as a source of secondary wavelets. You also learnt that Huygens principle was

modified by Fresnel who proposed that the resultant effect at a point due to a wavefront is governed by the interference (or superposition) of the secondary wavelets emanating from the wavefront. So, we wish to compute the resultant effect at P_0 by applying Huygens-Fresnel principle. One way would be to write down the equations of the electric field vibrations at P_0 due to each secondary wavelet and then add them together. This is a cumbersome proposition. The difficulty in mathematical calculation arises on two counts: (i) There are an infinite number of points and each acts as a source of secondary wavelets, and (ii) Since the distance travelled by the secondary wavelets arriving at P_0 is different, they reach the point P_0 with different phases.

To get over these difficulties, Fresnel devised a simple geometrical method which provided useful insight and beautiful explanation of diffraction phenomenon from small obstacles. He argued that (i) it is possible to locate a series of points on the wavefront situated at the same distance from P_0 so that all the secondary wavelets originating from them travel the same distance, and (ii) we can determine the locus of those points from where the wavelets travel a distance $b + (\lambda/2)$, $b + (2\lambda/2)$, $b + (3\lambda/2)$, and so on.

So, the Fresnel construction consists of dividing the wavefront into annular spaces enclosed by concentric circles (Fig. 9.6b). The effect at P_0 will be obtained by summing contributions of wavelets from these annular spaces called **half period elements**. When an obstacle is inserted in-between the wavefront WW' and the point P_0 , some of these half period elements will be obstructed depending upon the size and shape of the obstacle. The wavelets only from the unobstructed parts will reach P_0 and resultant amplitude (and intensity) due to them can be calculated easily by Fresnel's method. Let us now learn about Fresnel method of summation of the contributions of secondary wavelets in terms of the concept of half period elements.

9.3.1 Half Period Elements

To elaborate the concept of Fresnel's half period elements and how it helps in understanding diffraction of light, we assume, for simplicity, that light comes from infinity so that the wavefront is plane. Now, refer to Fig. 9.7. It shows a

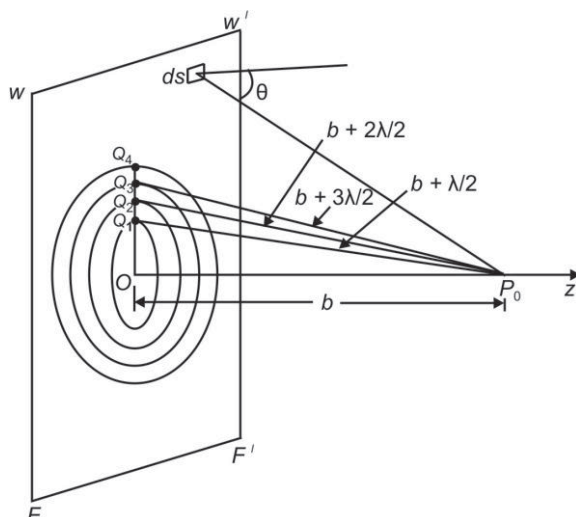


Fig. 9.7: A schematic diagram of half period element (or zones) on a plane wavefront

plane wavefront $WW'F'F$ of monochromatic light propagating along the z -direction. We wish to calculate the resultant amplitude of the field at an arbitrary point P_0 due to superposition of all the secondary Huygens wavelets originating from the wavefront. To do so, we divide the wavefront into half period zones using the following construction:

From the point P_0 we drop a perpendicular P_0O on the wavefront, which cuts it at O . **The point O is called the pole of the wavefront with respect to point P_0 .** Suppose that b is the distance between the foot of the perpendicular to P_0 , i.e. $OP_0 = b$. Now, with P_0 as centre, we draw spheres of radii $b + \frac{\lambda}{2}$, $b + \frac{2\lambda}{2}$, $b + \frac{3\lambda}{2}$, and so on. You can easily visualise that these spheres will intersect the plane wavefront in a series of concentric circles with centre O and radii $OQ_1, OQ_2, OQ_3, \dots$, as shown in Fig. 9.7. This geometrical construction divides the wavefront into circular strips called **zones**. The first zone is the space enclosed by the circle of radius OQ_1 , the second zone is the annular space between the circles of radii OQ_2 and OQ_1 . The third zone is annular space between the circles of radii OQ_3 and OQ_2 and so on. These concentric circles or annular rings are called **Fresnel zones** or **half period elements**. **This nomenclature has genesis in the fact that the path difference between the wavelets reaching P_0 from corresponding points in successive zones is $\lambda/2$.**

To compute the resultant amplitude at P_0 due to all the secondary wavelets emanating from the entire wavefront, we first consider an infinitesimal area dS of the wavefront. We assume that the amplitude of the field at P_0 due to dS is

- directly proportional to the area dS since it determines the number of secondary wavelets,
- inversely proportional to the distance of dS from P_0 , and
- directly proportional to the **obliquity factor** $(1 + \cos\theta)$, where θ is the angle between the normal drawn to the wavefront at dS and the line joining dS to P_0 .

Note from the figure (Fig. 9.7) that θ is zero for the central point O . As we go away from O , the value of θ increases until it becomes 90° for a point at infinite distance on the wavefront. Physically, it ensures that wavefront moves forward. That is, there is no reverse (or backward) wave. The obliquity factor takes the value 2 for forward direction, 1 for $\theta = 90^\circ$ and zero for $\theta = 180^\circ$. Let us denote the resultant amplitudes at P_0 due to all the secondary wavelets from the first, second, third, fourth, ..., n^{th} zone by $a_1, a_2, a_3, a_4, \dots, a_n$, respectively. So, taking into consideration all the above three factors, we can write the resultant amplitude due to the n^{th} zone as

$$a_n = \text{Const} \times \frac{A_n}{b_n} (1 + \cos\theta) \quad (9.1)$$

where A_n is the area of the n^{th} zone and b_n is the average distance of the n^{th} zone from P_0 .

Eq. (9.1) shows that, to know the amplitude of secondary wavelets arriving at P_0 from any zone, we must know A_n . This, in turn, requires knowledge of the radii of the circles defining the boundaries of the Fresnel zones. To calculate radii of various half period zones in terms of known distances, let us denote

$OQ_1 = r_1, OQ_2 = r_2, OQ_3 = r_3, \dots, OQ_n = r_n$. From Pythagoras' theorem, we find that the radius of the first circle (zone) is given by

$$r_1 = \left[\left(b + \frac{\lambda}{2} \right)^2 - b^2 \right]^{1/2} \cong \sqrt{b\lambda + \frac{1}{4}\lambda^2} \cong \sqrt{b\lambda} \quad (9.2a)$$

Note that we have neglected $(\lambda^2 / 4)$ term because we can assume $\lambda \ll b$ for practical systems using visible light. Similarly, the radius of the n th circle (zone) is given by

$$r_n = \left[\left(b + \frac{n}{2}\lambda \right)^2 - b^2 \right]^{1/2} \cong \left[nb\lambda + \frac{n^2\lambda^2}{4} \right]^{1/2} \cong \sqrt{nb\lambda} \quad (9.2b)$$

where, we have again neglected the term $(n^2\lambda^2 / 4)$ in comparison to $nb\lambda$. This approximation holds for all diffraction problems of interest to us here.

Thus, it readily follows from Eqs. (9.2a) and (9.2b) that the radii of the circles are proportional to the square root of natural numbers, i.e.

$\sqrt{1}, \sqrt{2}, \sqrt{3}, \sqrt{4}, \dots$. Therefore, if the first zone has radius r_1 , the successive zones have radii $1.41r_1, 1.73r_1, 2r_1$ and so on.

To get the feel of the numerical value of the radii of Fresnel zones for practical systems used for observing diffraction of light, solve the following SAQ.

SAQ 1 - Radii of Fresnel zones

Suppose light of wavelength 6328 Å from He-Ne laser is used in a diffraction experiment. If the distance between the wavefront and the observation screen is 30 cm, calculate the radii of the first two Fresnel zones.

Let us now calculate the area of each of the half-period zones. We can write the area of the first zone as

$$A_1 = \pi r_1^2 = \pi \left[\left(b + \frac{\lambda}{2} \right)^2 - b^2 \right] = \pi b\lambda + \frac{\pi}{4}\lambda^2 \cong \pi b\lambda \quad (9.3a)$$

The area of the second zone, i.e. the annular region between the first and the second circles (Fig. 9.7) is

$$\begin{aligned} A_2 &= \pi(r_2^2 - r_1^2) = \pi[(b + \lambda)^2 - b^2] - \pi b\lambda \\ &= 2\pi b\lambda - \pi b\lambda = \pi b\lambda \end{aligned} \quad (9.3b)$$

Similarly, you can readily verify that the area of the n th zone is given as

$$A_n = \pi(r_n^2 - r_{n-1}^2) \cong \pi b\lambda \quad (9.3c)$$

Note that we have neglected λ^2 term in comparison to $b\lambda$ terms in obtaining Eqs. (9.3). This is valid under the approximation $\lambda \ll b$.

Thus, from Eqs. (9.3a,b,c) we note that **all individual zones have nearly the same area**. The physical implication of the equality of zone areas is that the amplitude of secondary wavelets starting from any two zones will be very nearly equal. You must, however, remember that the result contained in Eq. (9.3) is approximate and is valid for cases where $b > n\lambda$. A more rigorous calculation shows that the area of a zone gradually increases with n :

$$A_n = \pi\lambda \left[b + \left(n - \frac{1}{2} \right) \frac{\lambda}{2} \right] \quad (9.3d)$$

In this equation, $b + \left(n - \frac{1}{2}\right) \frac{\lambda}{2}$ denotes the average distance of the n th zone from P_0 . The geometry of the n th zone is shown in Fig. 9.8. It is important to point out here that the effect of increase in A_n with n is almost balanced by the increase in the average distance of the n th zone from P_0 . That is, the ratio A_n / b_n in Eq. (9.1) remains $\pi\lambda$, which is constant, independent of n .

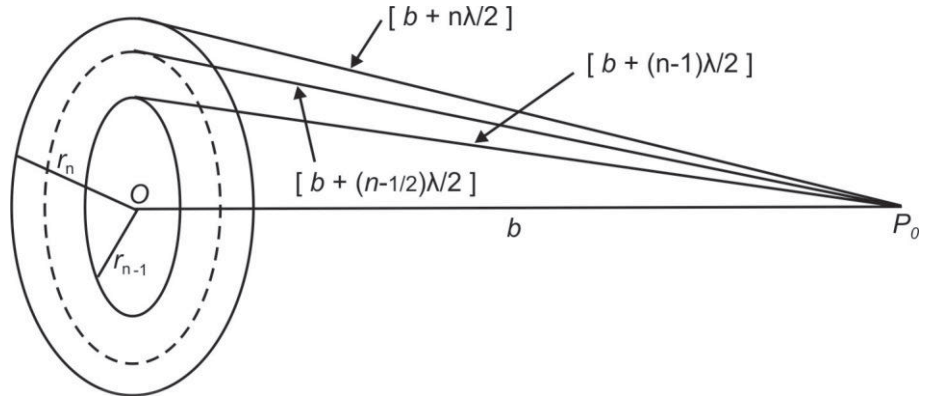


Fig. 9.8: Half period zones and their average distance from the point of observation.

Thus, from Eq. (9.1), we find that the amplitude due to any zone will be influenced by the obliquity factor $(1 + \cos\theta)$ because, (A_n / b_n) is constant for given λ . Due to $\cos\theta$ term, the obliquity factor causes monotonic decrease in the amplitudes of higher zones; that is, $a_1 > a_2 > a_3, \dots > a_n$.

Now, another parameter which will influence the amplitude of the resultant field at point P_0 (Fig. 9.7 or 9.8) is the phase difference among the wavelets emanating from the half period zones. **The phases of the wavelets from consecutive zones differ by one-half of a wavelength.** Therefore, the secondary waves from any two corresponding points in successive zones, say n th and $(n - 1)^{\text{th}}$ or $(n + 1)^{\text{th}}$, will reach point P_0 out of phase by π or half of a period. To understand why is it so, go through the following example carefully.

EXAMPLE 9.1: PHASE DIFFERENCE BETWEEN HALF PERIOD ZONES

Explain qualitatively that the phase difference between secondary waves from two successive zones is π .

SOLUTION ■

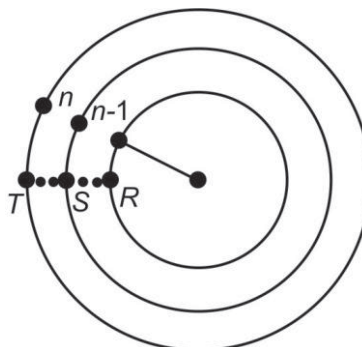


Fig. 9.9: Fresnel half period zones

Refer to Fig. 9.9 and consider the contributions from the $(n - 1)$ th and n th zones. Firstly, the areas of the two annular regions are approximately equal, i.e., the amplitudes of secondary wavelets starting from both the zones are equal. Secondly, the points on the innermost circle of $(n - 1)$ th zone e.g. points like R are situated at a distance of $b + (n - 2) \lambda/2$ from P_0 . whereas, the points on the innermost circle of n th zone e.g. points like S are situated at a distance of $b + (n - 1) \lambda/2$ from P_0 . Thus, the path difference between the secondary wavelets reaching P_0 from R and S is $\lambda/2$. This means that the waves reaching P_0 have path difference of $\lambda/2$ or phase difference of π and cancel each other. Similarly, for every point between R and S in the $(n - 1)$ th zone, we have a corresponding point between S and T in the n th zone which have a path difference of $\lambda/2$ or phase difference of π and hence cancel each other. Since the areas of the two zones are approximately equal, we can say that for every point in the $(n - 1)$ th zone, we have a point in the n th zone which is out of phase by π or half of a period. This is the reason why the Fresnel zones are called half-period zones.

With the background knowledge about the amplitudes and phase relation among the wavelets emanating from half period zones discussed above, you are now in a position to **obtain an expression for the amplitude of the resultant field at P_0 due to superposition of these wavelets.**

Suppose that the contribution of all the secondary wavelets in the n th zone at P_0 is denoted by a_n . Then, the contribution of $(n - 1)$ th zone, a_{n-1} , will tend to annihilate the effect of n th zone because they are having phase difference of π . Mathematically, we write the resultant amplitude, $a(P_0)$ at P_0 due to the whole wavefront as a sum of an infinite series whose terms are alternately positive and negative but the magnitude of successive terms gradually diminishes; that is,

$$a(P_0) = a_1 - a_2 + a_3 - a_4 + \dots + (-1)^{n+1} a_n \quad (9.4)$$

where, a_n represents the net amplitude produced by secondary wavelets of the n th zone. Alternate positive and negative signs represent the fact that the resultant disturbance (or wavelets) produced from two consecutive zones differ by a phase difference of π with respect to each other.

We will show (see TQ 1) that the sum of the infinite series in Eq. (9.4) is equal to $a_1 / 2$ if n is very large (that is, if we are considering infinitely large number of zones). **Thus, we find that, the resultant amplitude at P_0 due to all the secondary wavelets emanating from the entire wavefront is equal to one-half of the contribution of secondary wavelets from the first zone.**

However, if we consider a finite number of zones, say n , the resultant amplitude is given by

$$a(P_0) = \frac{a_1}{2} + \frac{a_n}{2}; \text{ if } n \text{ is odd} \quad (9.5a)$$

and
$$a(P_0) = \frac{a_1}{2} - \frac{a_n}{2}; \text{ if } n \text{ is even} \quad (9.5b)$$

So, from Eq. (9.5), we see that the resultant amplitude contributed by a wavefront that has odd number of zones is one half of the sum of amplitudes

contributed by the first and the last zones, whereas, if the number of zones is even, the resultant is one half of the difference of amplitudes contributed by the first and the last zones. If n is very, very large, contribution of a_n can be neglected in comparison with a_1 and we have

$$a(P_0) \approx \frac{a_1}{2} \quad (9.6)$$

Further, you know that the intensity of wave is equal to its amplitude squared. Therefore, if the wavefront WW' (Fig. 9.7) is unobstructed, the intensity at point P_0 can be written as

$$I(P_0) \approx \frac{a_1^2}{4} \quad (9.7)$$

The Fresnel construction and the concept of half period zones is very successful in explaining, at least qualitative, the salient features of the diffraction pattern of a variety of obstacles / apertures such as a straight edge, a slit, a wire and a circular disc. You will learn the qualitative analysis of some of these diffraction patterns later in this unit.

We now illustrate the concepts developed here by solving an example.

EXAMPLE 9.2: HALF PERIOD ZONES

A beam of parallel light of wavelength $\lambda = 5000 \text{ \AA}$ is incident normally upon a big plane metal sheet having a circular aperture of diameter 1 mm. The shadow is cast on a screen whose distance can be varied continuously. Calculate the distance of the screen from the aperture at which 1, 2, 3, ... Fresnel zones will be transmitted.

SOLUTION ■ From Eq. (9.2b), we know that

$$nb\lambda = r_n^2 \quad (i)$$

where, r_n is the radius of the n th Fresnel zone. As per the problem, we have to calculate the distance b of the screen from the aperture so that it (aperture) is covered by 1, 2, 3, ... zones. Since r , the radius of aperture is fixed, we can vary b to accommodate desired number of zones in the aperture. Thus, we can write Eq. (i) as

$$b_n = \frac{r^2}{n\lambda} \quad (ii)$$

Hence, if only one zone is to be transmitted, we take $n = 1$ in Eq. (ii) and get

$$b_1 = \frac{r^2}{\lambda} = \frac{(0.5 \text{ cm})^2}{5 \times 10^{-5} \text{ cm}} = 50 \text{ cm}$$

Similarly, we find that

$$b_2 = \frac{r^2}{2\lambda} = \frac{50 \text{ cm}}{2} = 25 \text{ cm}, \quad b_3 = \frac{50}{3} \text{ cm} = 16.7 \text{ cm}, \quad b_4 = \frac{50}{4} \text{ cm} = 12.5 \text{ cm}$$

$$b_5 = 10 \text{ cm}, \quad b_6 = 8.3 \text{ cm}, \quad b_7 = 7.1 \text{ cm}, \quad b_8 = 6.2 \text{ cm} \text{ and so on.}$$

You may now like to answer an SAQ.

SAQ 2 - Fresnel zones

A coin has a diameter of 2 cm. How many Fresnel zones does it cut off if the screen is 2 m away? If we move the screen to a distance of 4 m, how many zones will it cut off? Take $\lambda = 5 \times 10^{-7} \text{ m}$.

Before proceeding further, let us recapitulate what you have learnt till now in this unit.

Recap

- Diffraction refers to bending of light around corners or edges. As such, diffraction occurs for all kinds of waves such as mechanical waves (sound waves), electromagnetic waves and matter waves.
- Diffraction effects are observable when the size of the obstacles/aperture is of the order of the wavelength.
- Diffraction phenomenon is explained on the basis of Huygens-Fresnel principle.
- Fresnel proposed the concept of half period elements (or zones). According to this principle, a plane wavefront can be divided into a series of concentric annular rings called half period elements.
- The secondary wavelets emanating from any two consecutive half period elements have a phase difference of π .
- The resultant amplitude due to all the secondary wavelets emanating from the entire wavefront at point P_0 is approximately equal to one-half of the contribution of the secondary wavelets from the first zone.

You will agree that our day-to-day experience with light tells us that light follows a rectilinear path. Naturally, therefore, you would like to know how does the concept of half period elements and Huygens-Fresnel principle of wave propagation account for the rectilinear propagation of light waves. Let us learn it now.

9.3.2 Rectilinear Propagation

Let us consider a point source of light S from which light waves are emitted and propagate towards the aperture AB as shown in Fig. 9.10. After passing through the aperture, the light waves fall on a screen.

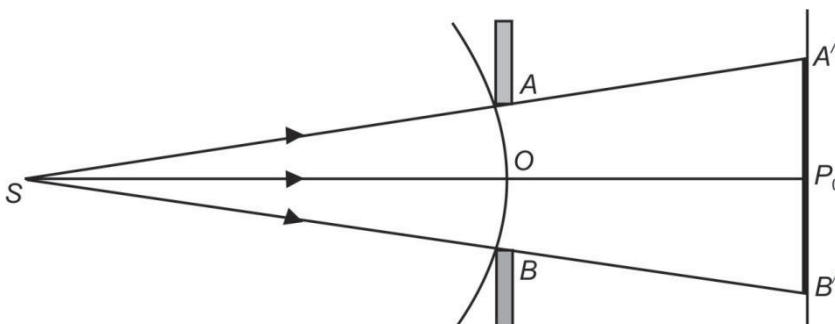


Fig. 9.10: Fresnel construction and rectilinear propagation of light.

If the size of the aperture AB is large compared to the wavelength of light, we expect a shadow $A'B'$ to be formed on the screen and that there should be practically no illumination beyond the geometrical shadow $A'B'$. It would then demonstrate rectilinear propagation of light.

To see whether such a conclusion can be arrived at using Fresnel construction, let us consider a practical situation. Suppose that the source is 1 m away from the aperture AB . Since the point source is 1 m away from the aperture, we may consider the spherical wave incident on the aperture as nearly a plane wave. (The radius of curvature of the incident spherical wave will not qualitatively change the argument.) Let us work out the sizes of Fresnel half period elements for the typical case where the screen is 30 cm away from the aperture. Taking $\lambda = 5 \times 10^{-5} \text{ cm}$, we get

$r_1 = \sqrt{(30 \text{ cm}) \times (5 \times 10^{-5} \text{ cm})} = 3.87 \times 10^{-2} \text{ cm}$ using Eq. (9.2a). This means that the diameter of the first zone is less than 1 mm. Let us consider the 100th zone. Its radius

$$r_{100} = \sqrt{bn\lambda} = \sqrt{30 \text{ cm} \times 100 \times 5 \times 10^{-5} \text{ cm}} = 3.87 \times 10^{-1} \text{ cm}$$

Thus, the diameter of the 100th zone will be a little less than 1 cm. Therefore, if the aperture AB is about 1 cm in diameter, it will accommodate over 100 Fresnel zones. So, from Eq. (9.5b), we can write the resultant amplitude at P_0 due to the exposed part of the wavefront as:

$$a(P_0) = \frac{a_1}{2} - \frac{a_{100}}{2}$$

Since a_{100} will be fairly small, it can be neglected. Thus, the intensity at P_0 is

$$I(P_0) = \frac{a_1^2}{4}$$

that is, the intensity at P_0 is one-fourth of that due to the first half period zone.

Now, if there was no aperture or the aperture radius goes to infinity, the amplitude at P_0 would be

$$a(P_0) = \frac{a_1}{2}$$

So that,

$$I(P_0) = \frac{a_1^2}{4}$$

We thus see that, even in the presence of a small aperture, the intensity at P_0 remains unaltered. That is, **light travels in a straight line** for all practical purposes.

Formation of Shadow

Let us now try to understand the formation of shadow and illuminated regions on the basis of Fresnel construction. Refer to Fig. 9.11 which depicts a plane wavefront WW' propagating along z -direction. Let an obstacle T is placed in the path of the wavefront.

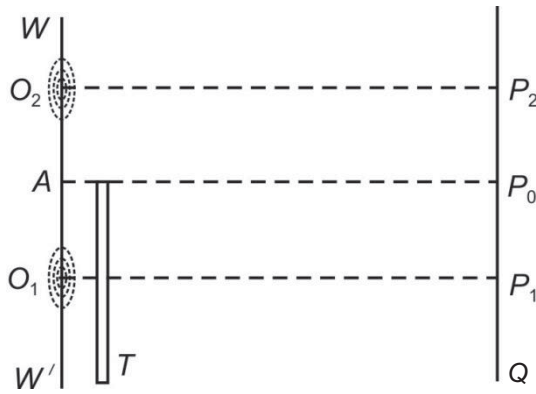


Fig. 9.11: Fresnel construction and formation of shadow and illuminated regions.

Consider the point P_2 on the screen Q whose pole is O_2 . If the distance between O_2 and the edge A of the obstacle T is nearly 1 cm, over 100 half period elements will be accommodated in it (refer to discussion above). And, as we discussed above, the intensity at P_2 will be nearly equal to $\left(\frac{a_1}{2}\right)^2$. In

other words, the obstacle T will have no effect at the point P_2 . Similarly, for point P_1 , which is taken 1 cm inside the geometrical edge of the shadow, over 100 half period elements around O_1 are obstructed. Thus, the intensity at P_1 will be less than $\left(\frac{a_{100}}{2}\right)^2$, which is almost negligible. This implies almost complete darkness at P_1 . In other words, the obstacle has completely obstructed the light from the source and the region around point P_1 is in the shadow.

You may ask: What happens around point P_0 , the geometrical edge of the shadow? We find fluctuations in intensity depending on how many half period elements have been allowed to pass or have been obstructed. In other words, we obtain diffraction fringes around point P_0 . You will learn the details of the diffraction pattern of a straight edge later in the unit.

Thus, we find that Fresnel construction explains the observed rectilinear propagation of light since Fresnel zones are obstructed or allowed through by obstacles of the size of a few mm for these typical distances between the aperture/obstacles and the screen.

A special optical device, designed to obstruct light from alternate half-period elements is known as **zone plate**. It provides experimental evidence in favour of Fresnel's theory and the concept of half period zones. Let us learn about it now.

9.4 THE ZONE PLATE

The zone plate is a special optical device constructed by applying the concept of Fresnel half period zones. It is designed to block light from alternate half-period zones. You can easily make a zone plate by drawing concentric circles on a white paper, with their radii proportional to the square roots of natural

numbers and shading alternate zones. Fig. 9.12 shows two zone plates. Note that each zone plate comprises several Fresnel zones, where all even numbered or odd numbered zones are blacked out. Now, photograph these pictures. The photographic transparency (negative) in reduced size acts as a Fresnel zone plate. Lord Rayleigh made the first zone plate in 1871. Now-a-days, zone plates are used to form images using X-rays and microwaves for which conventional lenses do not work.

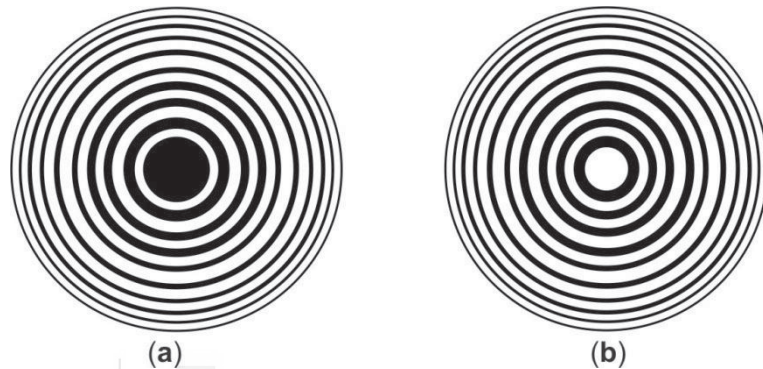


Fig. 9.12: Zone plate: a) with odd zones opaque; b) with even zones opaque.

Note that in the zone plate shown in Fig. 9.12a, odd zones are opaque (i.e. black) and in Fig. 9.12b, even zones are opaque.

The working of a zone plate is a practical verification of Fresnel construction and the concept of half period zones. According to Fresnel theory, there is a phase difference of π between the secondary wavelets emanating from two consecutive half period zones; that is, they are out of phase. **Therefore, the superposition of wavelets from consecutive zones will lead to destructive interference.** However, if we cover or make the alternate zones opaque to light, we can obtain a very bright illumination at the central point P_0 (refer Fig. 9.7) because, in this situation, all the secondary waves reaching P_0 are in phase and gives rise to constructive interference.

To elaborate the above point, consider that plane wavefront of light is incident on a zone plate whose even zones are opaque (Fig. 9.12b). Since secondary wavelets from all the even zones are blocked, only the secondary wavelets emanating from the transparent odd zones will reach the central point P_0 on the screen. Thus, from Eq. (9.4), we can write the resultant amplitude at P_0 (refer to Fig. 9.7 and assume that the unobstructed wavefront WW' has been replaced by the zone plate) as

$$\begin{aligned} a(P_0) &= a_1 - 0 + a_3 - 0 + a_5 - 0 + \dots \\ &= a_1 + a_3 + a_5 + \dots \end{aligned} \quad (9.8)$$

Eq. (9.8) shows that by obstructing light from passing through alternate zones (which, in the instant case, is all the even zones), the resultant amplitude at P_0 is enhanced manifold giving rise to an intense bright point at P_0 .

To check your understanding of the working of the zone plate, solve the following SAQ.

SAQ 3 - Intensity of light passing through a zone plate

A plane wavefront of light is incident normally on a zone plate whose ten odd zones are transparent to light. Calculate the intensity of light at the central point on a screen kept in front of the zone plate. If the zone plate is removed so that unobstructed wavefront is incident on the screen, what will be the intensity at the central point? Compare the two intensities and comment on your result.

Solving SAQ 3, you must have realised that a zone plate enhances the intensity of light at a point. This function of a **zone plate is similar to a converging lens**. You should go through the following example to learn the mathematical proof of the fact that a Fresnel zone plate acts like a converging lens.

EXAMPLE 9.3: ZONE PLATE AS A CONVERGING LENS

Show that a zone plate acts like a converging lens.

SOLUTION ■ Refer to Fig. 9.13. It shows a section AB of a zone plate perpendicular to the plane of the paper. The odd zones of the zone plate are opaque. S is a point source of light at a distance u from the zone plate and emits spherical waves. We wish to find the effect at point P_0 at a distance v from the plane of the zone plate.

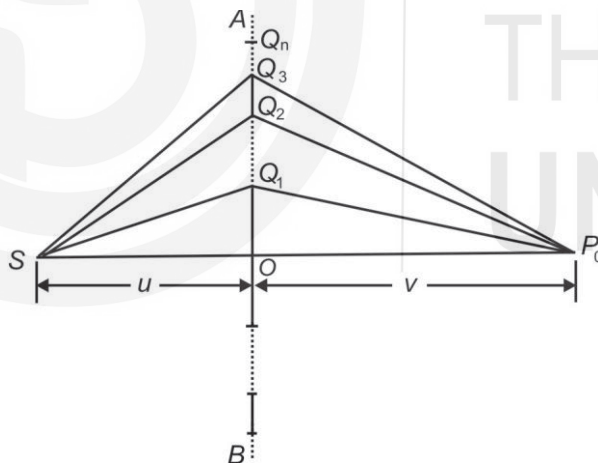


Fig. 9.13: Action of a zone plate as a converging lens.

To do so, we divide the plane AB into circular zones by drawing circles with centre at O and radii $OQ_1, OQ_2, OQ_3, \dots, OQ_n$ such that the path of the ray from S to P_0 increases by $(\lambda/2)$ from successive zones. Then we can easily write

$$SQ_1 + Q_1P_0 = u + v + (\lambda/2)$$

$$SQ_2 + Q_2P_0 = u + v + (2\lambda/2)$$

$\vdots$

$$SQ_n + Q_nP_0 = u + v + (n\lambda/2)$$

Now, to determine the radius r_n of the n th zone, we can use Pythagoras' theorem for $\triangle SQ_nO$ and write

$$\begin{aligned} SQ_n &= \sqrt{SO^2 + OQ_n^2} = \sqrt{u^2 + r_n^2} \\ &= u \left(1 + \frac{r_n^2}{u^2} \right)^{1/2} \approx u + \frac{r_n^2}{2u} \end{aligned} \quad (i)$$

Similarly, we can write

$$Q_nP_0 = v + \frac{r_n^2}{2v} \quad (ii)$$

Note that in writing Eqs. (i) and (ii), we have used binomial theorem and neglected terms containing higher powers of (r_n^2/u^2) or (r_n^2/v^2) under the assumption that $r_n \ll u$ and $r_n \ll v$. From Eqs. (i) and (ii), we can write

$$SQ_n + Q_nP_0 = u + \frac{r_n^2}{2u} + v + \frac{r_n^2}{2v} \quad (iii)$$

Also, the geometry of the figure gives us

$$SQ_n + Q_nP_0 = u + v + \frac{n\lambda}{2} \quad (iv)$$

Thus, from Eqs. (iii) and (iv), we can write

$$\begin{aligned} u + \frac{r_n^2}{2u} + v + \frac{r_n^2}{2v} &= u + v + \frac{n\lambda}{2} \\ \therefore r_n^2 \left(\frac{1}{u} + \frac{1}{v} \right) &= n\lambda \end{aligned} \quad (v)$$

Eq. (v) shows that the external radii of the various zones for given values of u and v are proportional to the square root of natural numbers.

If we identify $\frac{r_n^2}{n\lambda}$ as f_n , the focal length of the zone plate, we find that

$$\frac{1}{u} + \frac{1}{v} = \frac{n\lambda}{r_n^2} = \frac{1}{f_n} \quad (vi)$$

which is identical to the **lens equation**. Therefore, we can conclude that zone plate behaves like a converging lens with a focal length $f_n = r_n^2 / n\lambda$ and forms a real image of source S at point P_0 .

From Example 9.3, we find that a zone plate behaves like a converging lens having focal length, f_n given as

$$f_n = \frac{r_n^2}{n\lambda} \quad (9.9)$$

You may ask: What does the subscript n in f_n signify in Eq. (9.9)?

It indicates that the **zone plate has several foci**. To understand this, let us assume that the observation screen is at a distance of one focal length from the diffracting aperture. Then, it readily follows from the above example that

the most intense (first order) focal point is situated at $f_1 = r_1^2 / \lambda$. To give you a feel for numerical values, let us calculate f_1 for a zone plate with radii $r_n = 0.1\sqrt{n}$ cm and illuminated by a monochromatic light of wavelength $\lambda = 5500 \text{ \AA}$. You can easily see that

$$f_1 = \frac{r_1^2}{\lambda} = \frac{(0.1 \text{ cm})^2}{5500 \times 10^{-8} \text{ cm}} = 182 \text{ cm}$$

To locate higher order focal points, we note from Eq. (9.2b) that for r_n fixed, n increases as b decreases. Thus, for $b = f_1/2, n = 2$. That is, as P_0 moves towards the zone plate along the axis, the same zonal area of radius r_1 encompasses more half-period zones. For $n = 2$, therefore, each of the original zones covers two half-period zones and all zones cancel. When $b = f_1/3, n = 3$ and in this case, each of the original zone (in zonal area of radius r_1) will contain three zones. Of these, two zones will cancel out but one is left to contribute and we have a point of maximum intensity. Thus, we get

maximum intensity points along the axis situated at $f_n = \frac{r_1^2}{n\lambda}$ for n odd. For the above numerical example for which we calculated $f_1 = 182 \text{ cm}$, we have , $f_3 = \frac{182}{3} \text{ cm}, f_5 = \frac{182}{5} \text{ cm}, f_7 = \frac{182}{7} \text{ cm}$ and so on. And, between any two consecutive foci, there will be dark points.

To check your understanding of converging lens like behaviour of a zone plate, solve the following SAQ.

SAQ 4 - Zone plate as a converging lens

Suppose the radius of the first half period zone of a zone plate is 0.5 mm. When a light of wavelength $6 \times 10^{-5} \text{ cm}$ is incident on the zone plate, it brings the rays of light to focus at its brightest point. Calculate the focal length of the zone plate.

Till now, you studied about the concepts of half period zones proposed by Fresnel to explain the phenomenon of diffraction of light. You also learnt how to calculate the resultant amplitude at a point arising due to the superposition of secondary wavelets emanating from the half period zones. **The most significant aspect of the Fresnel's zones is that the secondary waves emanating from any two consecutive zones are out of phase; that is, there is a phase difference of π between them.** With this understanding of the effect of half period zones on the secondary wavelets, you are now in a position to investigate the diffraction pattern produced by some simple obstacles/apertures such as straight edge, a slit and a wire. This is the subject matter of the next section.

9.5 DIFFRACTION PATTERNS OF SIMPLE OBSTACLES

You may recall from Sec. 9.2 that by using experimental set up similar to Kathvate's experimental arrangement, the Fresnel diffraction pattern of various apertures and obstacles could be obtained by varying distances

It is instructive to compare the action of a converging lens and a zone plate in forming a real image of an object. Refer to Fig. 9.14 which shows a converging lens.

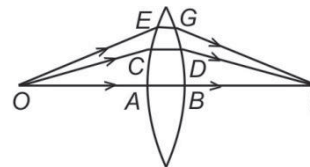


Fig. 9.14: Image formation by a converging lens.

Consider two rays $OABI$ and $OCDI$. Taking the refractive index for air and glass as μ_a and μ_g respectively, the optical path lengths of these rays will be

$$\mu_a OA + \mu_g AB + \mu_a BI \text{ and } \mu_a OC + \mu_g CD + \mu_a DI.$$

The lens is so designed that these optical paths are equal. This is true for all other rays, e.g. $OEGI$. Thus, different rays starting from O reach I in the same phase and form a bright image.

In a zone plate (Fig. 9.13), alternate zones are blocked. Therefore, rays from the source S , after passing through the second, fourth, sixth, etc. zones reach the point P_0 with a path difference of $\lambda, 2\lambda, \dots$ and hence reinforce each other. This results in the formation of a bright image at P_0 . Obviously, for a zone plate, we will get several positions of bright images if the path difference between the successive exposed zones is $n\lambda$.

between the source, the object and the photographic plate. We will now use the concepts discussed and the results obtained in Sec. 9.3 to explain the observed diffraction patterns of simple obstacles.

Let us first take up the Fresnel diffraction pattern of a straight edge.

9.5.1 A Straight Edge

Refer to Fig. 9.15a. Let S be a slit source of light perpendicular to the plane of the paper. The source will emit cylindrical wavefront which propagate towards the obstacle which is a straight edge perpendicular to the paper. You can think of a thin metal sheet or a razor blade with the sharp edge parallel to the slit as a straight edge obstacle. **Fig. 9.15a shows a cross-sectional view of the slit source and the straight edge such that the cross-section is perpendicular to the length of the slit.** The line joining S and E , the point on the wavefront touching the edge of the straight edge, when produced meets the screen at P_0 which is the geometrical boundary of the shadow.

Now, let us consider any point P in the illumination region on the screen. A line joining it to S cuts the wavefront WW' at R . We wish to know what will be the intensity of light at this point as per Fresnel half period zone analysis. This calculation is somewhat complicated because the wavefront is cylindrical and the obstacle does not possess an axial symmetry.

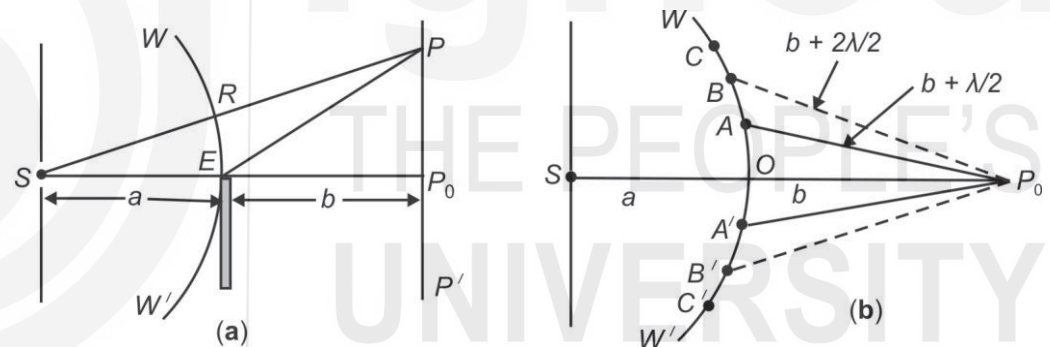


Fig. 9.15: a) Cross-sectional view of the geometry to observe diffraction due to a straight edge; b) Fresnel construction divides the cylindrical wavefront from the slit source in half period strips.

You have learnt how to construct Fresnel half period zones for a plane wavefront in Sec. 9.2. To construct half period zones for a cylindrical wavefront we divide it into strips. As before, we make sure in the construction that the amplitudes of the wavelets from any two consecutive strips arrive at P_0 with a phase difference of π . This is achieved by drawing a set of circles with P_0 as centre and radii $b, b + \frac{\lambda}{2}, b + \frac{2\lambda}{2}, \dots$ cutting the circular sections of the cylindrical wavefront at points $O, AA', BB', CC', \dots$ as shown in Fig. 9.15b. If lines are drawn through $A, A', B, B', \dots$ etc. normal to the plane of the paper, the upper as well as the lower half of the wavefront gets divided into a set of **half-period strips**. These half period strips stretch along the wavefront perpendicular to the plane of the paper and have widths $OA, AB, BC, \dots$ in the upper half and $OA', A'B', B'C', \dots$ in the lower half. **You may recall from Sec. 9.2 that Fresnel zones are of nearly equal area. For half period**

strips, this does not hold. The areas of half-period strips are proportional to their widths and it decreases rapidly as we go out along the wavefront from O .

Further, from the geometry of the arrangement (Fig. 9.15a), it is obvious that on the screen, there will be no intensity variation along the direction parallel to the length of the slit. **Therefore, the bright and dark fringes will be straight lines parallel to the edge.**

A plot of theoretically calculated intensity distribution in the diffraction pattern of a straight edge obtained on the screen is shown in Fig. 9.16. The salient features of the intensity distribution are as follows:

- As we go from a point P' deep inside the shadow (Fig. 9.15a) towards the point P_0 defining the edge of the shadow, the intensity gradually rises. At P' , the intensity is almost zero.
- At P_0 , the intensity is one-fourth of what would have been the intensity on the screen with the unobstructed wavefront (that is, when the straight edge is removed).
- On moving further towards P (see Fig. 9.15a), the intensity rises sharply and goes through an alternating series of maxima and minima of gradually decreasing magnitude before approaching the value for the unobstructed wave. Beyond these alternate maxima and minima, there is uniform illumination. This is expected since effect of the edge at far off distances from point P_0 will be almost negligible.
- The intensity of the first maximum (marked by dotted line in Fig. 9.16) is greater than the intensity of unobstructed wave, i.e. it is 4 times greater than the intensity at P_0 .
- The diffraction fringes are not of equal spacing (as in interference experiments); the fringes gradually come closer together as we move away from the point P_0 .

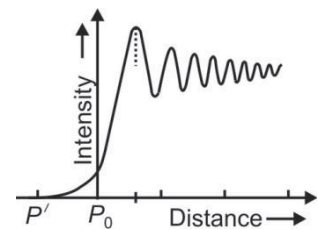


Fig. 9.16: Intensity distribution in the diffraction pattern due to a straight edge.

You may now like to know, at least, a qualitative explanation of the above mentioned features of the intensity distribution. Let us discuss it now.

Illumination Region

Let us first consider the intensity at point P . From Fig. 9.15a, we note that the line joining P and S divides the wavefront into two parts WR and RE . Note that R is pole of the wavefront corresponding to point P . The amplitude of light wave at P is due to the part WE of the wavefront, which is completely unaffected by the straight edge. The amplitude at P will be maximum if RE contains odd number of half strips. This will happen if $EP - RP = (2n + 1) \lambda/2$. On the other hand, when $EP - RP = n\lambda$, the portion RE of the wavefront will contain even number of strips and the amplitude at P will be minimum. It is so because, as pointed out earlier, the amplitudes due to strips are alternately positive and negative and, even number of strips will, in pair, cancel each other's contribution to the amplitude at P . **Therefore, as point P moves away from P_0 , the illumination on the screen will pass alternately through maxima and minima** when the number of half period strips in RE is 1, 2, 3, 4, ... etc.

At P_0 , only EW part of the wavefront, which is half of the wavefront WW' contributes. **Therefore, the amplitude is halved and the intensity is one-fourth of the unobstructed wavefront.**

It is worthwhile to ponder as to what will be the nature of the bright and dark fringes in the diffraction pattern. As explained earlier, we expect dark and bright bands parallel to the edge. However, the dark bands will not be completely dark because the upper part RW of the wavefront always contributes light to this part of the screen.

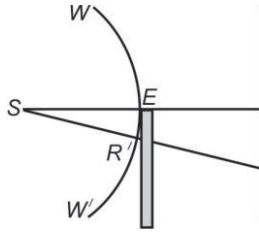


Fig. 9.17: Cross-sectional view of geometry shown in Fig. 9.15a when the observation point P' is in the geometrical shadow.

Shadow Region

Let us now consider the intensity at a point P' inside the geometrical shadow. Refer to Fig. 9.17. You may note that the joining P' to S cuts the wavefront at point R' which is below the edge. Note that R' is the pole of the wavefront for P' . So, the illumination at P' is due entirely to the wavelets from the upper half WR' of the wavefront; the lower portion RW' having been blocked by the edge. Even the upper half is exposed only in part. If the edge cuts off r strips of the upper half of the wavefront, the effect at P' will be due to $(r + 1)$, $(r + 2)$, $(r + 3)$ etc. strips which may be taken to be equal to one-half of that due to $(r + 1)$ th strip. This will rapidly diminish to zero as shown in Fig. 9.16 because the effectiveness of higher order strips goes on decreasing.

So, the qualitative arguments given above explains the salient features of the intensity distribution in the diffraction pattern of a straight edge. Let us now obtain expression for the position of the diffraction bands produced by the straight edge.

Position of Diffraction Fringes

Obtain an expression for the position of diffraction fringes, let us again refer to Fig. 9.15a. Let us assume that the portion RE of the wavefront contain even number of half period strips. Then, we have dark band (minima) at P because the contribution of the even number half period strips will cancel in pairs. So, we can write

$$EP - RP = n\lambda \quad (9.10)$$

From the $\triangle EPP_0$, if $PP_0 = x$, we have

$$\begin{aligned} EP &= (b^2 + x^2)^{1/2} = b \left(1 + \frac{x^2}{b^2} \right)^{1/2} \\ &= b \left(1 + \frac{1}{2} \frac{x^2}{b^2} \right) = b + \frac{1}{2} \frac{x^2}{b} \end{aligned} \quad (9.11)$$

where we have used binomial theorem and retained only first two terms in the binomial series.

From the $\triangle SPP_0$, we can similarly write

$$SP = (a + b) + \frac{1}{2} \frac{x^2}{(a + b)}$$

Since point R is close to E , we can assume that $SE \cong SR$. Thus, we can write

$$RP = SP - SR = SP - SE = b + \frac{1}{2} \frac{x^2}{(a + b)} \quad (9.12)$$

So, from Eqs. (9.11) and (9.12) for EP and RP , we can write

$$EP - RP = \left(b + \frac{1}{2} \frac{x^2}{b} \right) - \left(b + \frac{1}{2} \frac{x^2}{(a + b)} \right) = \frac{x^2 a}{2b(a + b)} \quad (9.13)$$

Substituting Eq. (9.13) in Eq. (9.10), we get for the n th dark band

$$\frac{x^2 a}{2b(a+b)} = n\lambda$$

or
$$x = \sqrt{\frac{2b(a+b)}{a} n\lambda} \quad (9.14)$$

For obtaining the location of bright bands, we note that the condition given by Eq. (9.10) for dark bands will modify to the following for bright bands:

$$EP - RP = (2n+1) \frac{\lambda}{2} \quad (9.15)$$

So, substituting Eq. (9.13) in Eq. (9.15), we get the position of bright bands from the edge of the geometrical shadow as

$$x = \sqrt{(2n+1) \frac{b(a+b)}{a} \lambda}; \quad n = 0, 1, 2, \dots \quad (9.16)$$

We, therefore, discover from Eqs. (9.14) and (9.16) that the position x of the dark or bright bands from the edge of the geometrical shadow (that is, point P_0 in Fig. 9.15a) are proportional to the square root of natural numbers. Consequently, the bands will get closer together as we go out from the shadow. This fact distinguishes the diffraction bands from the interference bands, which are equidistant.

To enable you to grasp these ideas, we now give a solved example.

EXAMPLE 9.4: MINIMA IN THE DIFFRACTION PATTERN OF STRAIGHT EDGE

In the experimental set up to observe diffraction pattern of a straight edge, the distance of the edge from the source is 30 cm and that from the screen is 30 cm. If the wavelength of the light used is 5×10^{-5} cm, calculate the position of the 1st, 2nd, 3rd and 4th minima from the edge of the geometrical shadow.

SOLUTION ■ From Eq. (9.14) we know that the position of n th minima from the edge of the shadow is given by

$$x = \sqrt{\frac{2b(a+b)}{a} n\lambda}$$

As per the problem, $a = b = 30$ cm and $\lambda = 5 \times 10^{-5}$ cm. So, the position of the first four minima can be obtained by taking $n = 1, 2, 3$, and 4. Thus, we get

$$x_1 = \left[\frac{2 \times (30 \text{ cm}) \times (60 \text{ cm})}{30 \text{ cm}} \times (5 \times 10^{-5} \text{ cm}) \right]^{1/2} = 7.75 \times 10^{-2} \text{ cm}$$

$$x_2 = \sqrt{2} x_1 = 1.09 \times 10^{-1} \text{ cm}$$

$$x_3 = \sqrt{3} x_1 = 1.34 \times 10^{-1} \text{ cm}$$

and, $x_4 = 2 x_1 = 1.55 \times 10^{-1} \text{ cm}$

From these values for the position of minima, we find that the distance between consecutive minima decreases continuously as we move away from the edge of the shadow.

You may now like to answer an SAQ.

SAQ 5 - Maxima in the diffraction pattern of straight edge

For the experimental set-up given in Example 9.4 above for observing diffraction from a straight edge, calculate the position of the first and second maxima from the edge of the geometrical shadow.

Let us now learn the diffraction pattern produced by a slit.

9.5.2 A Slit

Refer to Fig. 9.18 which depicts the cross-sectional view of the experimental set-up to obtain Fresnel diffraction pattern of a slit. The cylindrical wavefronts emitted by a narrow slit source S of monochromatic light are incident on a rectangular aperture (slit) AB . Note that both the slit source S and the slit aperture AB are perpendicular to the plane of the paper. WW' is the cylindrical wavefront incident on the slit aperture AB .

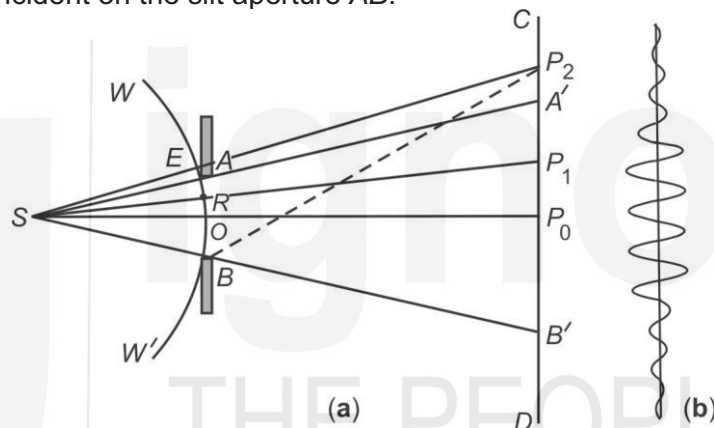


Fig. 9.18: a) Cross-sectional view of the schematic diagram for Fresnel diffraction from a slit; b) resulting diffraction pattern.

The slit aperture AB will form a geometrical shadow above A' and below B' on a screen CD kept perpendicular to the line OP_0 . The region between $A' B'$ should have uniform illumination as per geometrical optics.

If the slit aperture is narrow so that it can accommodate only a few half period strips with respect to the midpoint P_0 of the image $A' B'$ of the slit, we obtain a diffraction pattern on the screen whose intensity distribution is depicted in Fig. 9.18b. So, the question is: **How do we explain the bright and dark diffraction fringes within the region $A' B'$ on the basis of Fresnel construction and the concept of half period zones?**

Let us first consider the situation at point P_0 , the mid point of the image $A' B'$ of the slit. You may recall from Sec. 9.2 that the point P_0 will be bright or dark depending upon the number of half period zones in each half of unobstructed wavefront below and above point O which is the pole on the wavefront with respect to point P_0 . If each half contains odd number of zones, we will have maximum illumination at P_0 and minimum illumination if the number of zones in each half is even.

Illumination Region

Now, let us consider a point P_1 within the geometrical image $A' B'$. To know the resultant amplitude of light wave at this point, we need to know the number

of zones in each half of the wavefront above and below point R on the wavefront. Note that, since point R , the pole for P_1 is away from the centre O of the slit aperture, the upper portion, AR of the unobstructed wavefront will contain lesser number of zones as compared to the lower portion RB . However, if both the portions of the unobstructed wavefront AR and RB contain odd numbers of zones each, we will have maximum illumination (bright band) at P_1 . Similarly, if both the portions contain even number of zone each, we will have minimum illumination (dark band) at P_1 . As we move further from P_0 towards A' , we will have a maximum or a minimum depending upon whether the number of zones contained in each portion of the unobstructed wavefront is odd or even, respectively. The lower portion P_0B' of the geometrical image $A' B'$ will also have bright and dark bands on the basis of similar arguments.

Thus, we find that the geometrical image $A' B'$ of the slit aperture AB has a fringe pattern comprising bright and dark fringes. These are diffraction fringes formed due to the mutual interference (or superposition) of the secondary waves emanating from half period zones of the same wavefront.

Shadow Region

Now, let us examine the effect at any point P_2 in the geometrical shadow (see Fig. 9.18). The line joining this point with S will cut into the wavefront WW' at point E which is above A , the edge of the slit aperture. So, point E is the pole of point P_2 . For point P_2 , the upper half of the wavefront (that is, EW) is completely cut off and point P_2 will not receive any light from this portion. Even, a portion of the lower half EW' of the wavefront is partially obstructed (portion EA of the wavefront). So, P_2 will receive light from some of the zones in the lower half. Thus, the illumination at P_2 will be maximum or minimum depending upon whether AB contain odd or even number of zones with respect to P_2 . If the path difference $(BP_2 - AP_2)$ is equal to $(2n + 1)(\lambda/2)$, that is, an odd multiple of $(\lambda/2)$, there will be an odd number of zones in AB and P_2 will be a bright point. It is so because of an excess of one zone in AB after the mutual cancellation of other zones in pairs. On the other hand, if the path difference $(BP_2 - AP_2)$ is equal to $n\lambda$, point P_2 will be a dark point due to mutual cancellation of the even number of zone in pairs.

Thus, we find that bright and dark bands within the geometrical shadow beyond A' and B' are also accounted for by the Fresnel half period zones.

Above qualitative explanation of the diffraction from a slit holds when the slit is narrow and contains only a few half period zones with respect to the central point P_0 (Fig. 9.15a). Before we close our discussion of Fresnel diffraction from a slit, let us see how the width of slit aperture will influence the diffraction pattern. If the slit is wide and contains many zones, say about one hundred or so, the edges A and B of the slit will behave like two separate straight edges independent of each other. So, in such a situation, the nature of diffraction pattern around A' and B' will be just similar to those due to two separate straight edges.

Let us now discuss Fresnel diffraction due to a wire.

9.5.3 A Wire

Let us consider a narrow wire AB of thickness d held perpendicular to the plane of the paper in front of a slit source S as shown in Fig. 9.19a. A screen is placed in front of the wire at a distance D . The light from the source will form a shadow $A'B'$ of the wire on the screen. The diffraction of light by the thin wire gives rise to a fringe pattern as depicted in Fig. 9.19b.

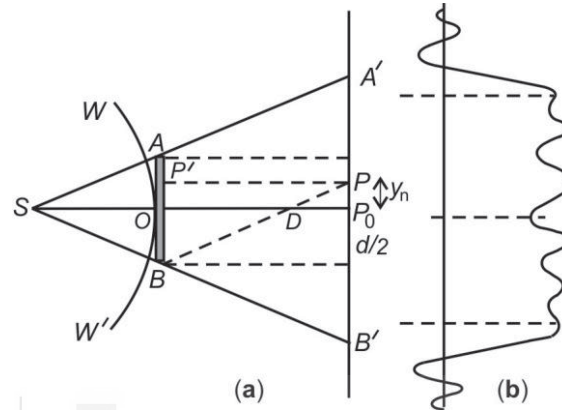


Fig. 9.19: a) Cross-sectional view of the experimental arrangement for diffraction by a wire; b) intensity distribution in the diffraction pattern of the wire.

The fringe patterns outside the geometrical shadow (above A' and below B') of the wire are called **exterior fringes** whereas fringes within the geometrical shadow $A'B'$ are called **interior fringes**. The genesis of exterior fringes can be understood in terms of diffraction due to a straight edge. It is so because the edges A and B of the wire behave like two straight edges. That is why, in the intensity distribution of the diffraction pattern of a thin wire, shown in Fig. 9.19b, the pattern just outside the geometric shadow $A'B'$ is same as due to straight edges at A or B . **These exterior fringes are diffraction fringes.**

The interior fringes formed within the shadow $A'B'$ are interference fringes formed due to the superposition of waves bending into the shadow region from both sides of the wire. The effect due to the upper portion AW of the wavefront can be regarded as due to a source of light at A and that due to the lower portion BW' as a source of light at B . **That is, wire edges A and B behave as two coherent sources.**

Now, as you have learnt in Unit 6, Block 2 of the course, the effect of superposition of waves emanating from two coherent sources A and B at any point on the screen will depend on the path difference. Since centre P_0 is equidistance from A and B , the waves will arrive at this point in phase and the point will always be bright.

Now, let us consider an off-centre point, say P within the shadow region $A'B'$ located at a distance y_n from the central point P_0 . The waves from A and B will arrive at P with a path difference ($BP - AP$). So, we will have either a dark or a bright fringe at P depending on whether the path difference

$$BP - AP = (2n + 1)(\lambda / 2); \quad n = 0, 1, 2, \dots \quad (9.17)$$

$$\text{and} \quad BP - AP = n\lambda \quad ; \quad n = 1, 2, 3, \dots \quad (9.18)$$

respectively. From Fig. 9.19a, we note that

$$BP^2 = D^2 + \left(y_n + \frac{d}{2}\right)^2$$

and $AP^2 = D^2 + \left(\frac{d}{2} - y_n\right)^2$

Thus, we can write

$$BP^2 - AP^2 = \left(y_n + \frac{d}{2}\right)^2 - \left(\frac{d}{2} - y_n\right)^2$$

$$(BP + AP)(BP - AP) = 2 \cdot \frac{d}{2} \cdot 2y_n \quad (9.19)$$

If the wire is thin, we can approximate, $BP = AP = D$ so, $BP + AP = 2D$. Thus, we can write Eq. (9.19) as

$$BP - AP = \frac{dy_n}{D} \quad (9.20)$$

So, from Eqs. (9.17), (9.18) and (9.20), we can write the position of the dark fringes as

$$y_n = \left(n + \frac{1}{2}\right) \frac{D}{d} \lambda; \quad n = 0, 1, 2, \dots \quad (9.21)$$

and the position of the bright fringes as

$$y_n = \frac{nD\lambda}{d}; \quad n = 1, 2, 3, \dots \quad (9.22)$$

Thus, the fringe width, β can be written as

$$\beta = y_{n+1} - y_n = \frac{\lambda D}{d} \quad (9.23)$$

So, from Eq. (9.23), we note that by measuring the fringe width of the interference fringes in the shadow region $A' B'$, one can determine either the thickness d of the wire if λ is known.

Further, from similar triangles ABS and $A' B'S$ (Fig. 9.19a), we have

$$\frac{AB}{A'B'} = \frac{SO}{SO + OR_0} = \frac{a}{(a + D)}$$

where a is the distance between the source S and wire AB . So, we can write,

$$A'B' = \frac{a + D}{a} \cdot d \quad (9.24)$$

Thus, from Eqs. (9.23) and (9.24) we can write the number of fringes inside the geometrical shadow $A' B'$ as

$$N = \frac{A'B'}{\beta} = \frac{(a + D)d^2}{a\lambda D} \quad (9.25)$$

The fringe pattern in the geometrical shadow $A' B'$ is due to interference of waves from edges A and B of the wire can be checked by obstructing light from either A or B . When you do that, the fringe pattern in the shadow region disappears. However, the diffraction pattern due to light from one of the edges of the wire do persists.

Now, to concretise your understanding of what you have learnt above, solve the following SAQ.

SAQ 6 - Fresnel diffraction from a thin wire

A thin wire of diameter d is placed 100 cm away from a slit source and the diffraction pattern is observed on a screen at 100 cm from the wire. The fringe width of the interference fringes in the geometrical shadow region is measured to be 0.04 mm. Calculate the diameter of the wire. How many fringes would be visible inside the shadow? Take wavelength, $\lambda = 6 \times 10^{-5}$ cm.

Let us now summarise what you have learnt in this unit.

9.6 SUMMARY

Concept	Description
Fresnel diffraction	When the distance between the source of light or the observation screen or both from the diffracting aperture/obstacle is finite, the diffraction pattern belongs to Fresnel class. When the screen is very close to the slit aperture/obstacle, the illumination on the screen is governed by rectilinear propagation of light.
Fresnel construction	Fresnel construction involves dividing the wavefront into half period zones.
Half period zones	The path difference between the wavelets emanating from the corresponding point of two consecutive zones is $(\lambda/2)$. The area of each Fresnel half-period zone is nearly equal to $\pi b\lambda$. The resultant amplitude due to nth zone at any axial point is given by $a_n = \text{Constant} \times \frac{A_n}{b_n} (1 + \cos \theta)$ The resultant amplitude at an axial point due to all the secondary wavelets emanating from the entire wavefront is equal to one-half of the contribution from the first zone: $a = \frac{a_1}{2}$
Zone plate	Zone plate is an optical device designed to block light from alternate half period zones. Due to its unique construction, a zone plate enhances the resultant amplitude at an axial point manifold. A zone plate act as a converging lens. The focal length of a zone plate is given as $f_n = \frac{r_n^2}{n\lambda}$ Thus, a zone plate has several foci.
Straight edge	The diffraction pattern of a straight edge consists of alternate bright and dark bands. The spacing between minima (or maxima) decreases as we move away from the edge of the shadow. The position of the diffraction dark bands is given as $x = \sqrt{\frac{2b(a+b)}{a}} n\lambda$ The position of the diffraction bright bands is given as $x = \sqrt{(2n+1) \frac{b(a+b)}{2}} \lambda;$

- Diffraction from a slit** ■ The Fresnel diffraction pattern due to a narrow slit comprises bright and dark diffraction fringes in the illumination region on the screen.
There are bright and dark bands in the geometrical shadow regions on both sides of the illumination region on the screen.
- Diffraction from a wire** ■ The Fresnel diffraction pattern of a thin wire comprises of exterior fringes and interior fringes.
The exterior fringes are diffraction fringes above and below the geometrical shadow region of the wire.
The interior fringes are the interference fringes in the geometrical shadow region of the wire. These interference fringes are formed due superposition of the waves from the two edges of the wire which act like two coherent sources. The fringe width of the interference fringes is given as

$$\beta = \frac{\lambda D}{d}.$$

9.7 TERMINAL QUESTIONS

- Starting from Eq. (9.4) prove the result obtained in Eq. (9.5). Assume that the obliquity factor is such that each term in Eq. (9.4) is less than the arithmetic mean of its preceding and succeeding terms.
- The eighth boundary of a zone plate has a diameter of 6 mm. Where is its principal focal point located for light of wavelength 5000 Å?
- How many Fresnel zones will be obstructed by a sphere of radius 1 mm if the screen is 20 cm away? Take $\lambda = 5000$ Å. If the distance of the screen is increased to 200 cm, what will be the size of the sphere which will cut off 10 zones?
- A straight edge is illuminated by a narrow slit source kept at a distance of 15 cm. Calculate the distance between the first and second dark bands of the diffraction pattern on a screen kept at a distance of 1.0 m from the edge. Take the value of wavelength, $\lambda = 6 \times 10^{-5}$ cm.
- A metal wire of diameter 3.0 mm is illuminated by light of wavelength 5.0×10^{-5} cm from a slit source. If the distance between the source and the wire is 1.0 m and the distance between the wire and the screen is 2.5 m, calculate the distance between two consecutive fringes formed in the shadow region of the diffraction pattern on the screen. Also calculate the total number of fringes in the shadow region.

9.8 SOLUTIONS AND ANSWERS

Self-Assessment Questions

- The radii of Fresnel zones is given by Eq. (9.2b):

$$r_n = \sqrt{n b \lambda}$$

As per the problem, $b = 30$ cm, $\lambda = 6.328 \times 10^{-5}$ cm. So, radius of the first ($n = 1$) zone is

$$r_1 = \sqrt{(30 \text{ cm}) \times (6.328 \times 10^{-5} \text{ cm})} = 0.44 \text{ mm}$$

and $r_2 = \sqrt{2 \times (30 \text{ cm}) \times (6.328 \times 10^{-5} \text{ cm})} = 0.62 \text{ mm}$

- The radius of the coin is equal to 1 cm. To know the number of zones being obstructed, we use the relation

$$n = \frac{r_n^2}{b\lambda}$$

where $r_n = 1\text{ cm}$, $b = 200\text{ cm}$ and $\lambda = 5 \times 10^{-5}\text{ cm}$.

$$\therefore n = \frac{(1\text{ cm})^2}{(200\text{ cm}) \times (5 \times 10^{-5}\text{ cm})} = 100$$

Since the number of zones cut off is large, we expect to see a very dim spot at the centre.

When the screen is moved to 4 m, the number of zones being obstructed is given by

$$n = \frac{(1\text{ cm})^2}{(400\text{ cm}) \times (5 \times 10^{-5}\text{ cm})} = 50$$

The central spot is expected to be somewhat brighter.

3. As per the problem, ten odd zones of the zone plate are transparent to the plane wavefront of light incident on it normally. Thus, from Eq. (9.8), we can write the resultant amplitude at the central point on a screen kept in front of the zone plate as

$$(a)_{\text{zone plate}} = a_1 + a_3 + a_5 + \dots + a_{19}$$

If we assume that the obliquity factor $(1 + \cos\theta)$ does not produce much change, we can write $a_1 \approx a_3 \approx \dots \approx a_{19}$. So,

$$(a)_{\text{zone plate}} = 10a_1$$

So, the intensity at the central spot

$$(I)_{\text{zone plate}} = a^2 = 100a_1^2 \quad (\text{i})$$

If the unobstructed wavefront is incident on the screen, the intensity at the central spot is given by Eq. (9.7):

$$(I)_{\text{unobstructed}} = \frac{a_1^2}{4} \quad (\text{ii})$$

So, from Eqs. (i) and (ii), we find that

$$(I)_{\text{zone plate}} = 400(I)_{\text{unobstructed}}$$

Thus, the zone plate enhances the intensity 400 times compared to the unobstructed wavefront.

4. We know that the foci of zone plate is given by Eq. (9.9):

$$f_n = \frac{r_n^2}{n\lambda}$$

As per the problem, $n = 1$; $r_1 = 0.05\text{ cm}$ and $\lambda = 6 \times 10^{-5}\text{ cm}$.

$$f_1 = \frac{(0.05\text{ cm})^2}{(6 \times 10^{-5}\text{ cm})} = 41.6\text{ cm}$$

5. The position of the maxima in the diffraction pattern of a straight edge from the edge of the geometrical shadow of the edge is given by Eq. (9.16):

$$x = \sqrt{(2n+1) \frac{b(a+b)}{a} \lambda}$$

As per the problem, $a = 30\text{ cm}$, $b = 30\text{ cm}$ and $\lambda = 5 \times 10^{-5}\text{ cm}$.

$$x = \sqrt{\frac{(30 \text{ cm}) \times (60 \text{ cm}) \times (5 \times 10^{-5} \text{ cm})}{(30 \text{ cm})}} = 0.55 \text{ mm.}$$

For second maxima, $n=1$ and we have

$$x = \sqrt{\frac{3 \times (30 \text{ cm}) \times (60 \text{ cm}) \times (5 \times 10^{-5} \text{ cm})}{(30 \text{ cm})}} = 0.94 \text{ mm.}$$

6. The fringe width of the interference fringes in the shadow region of a wire is given by Eq. (9.23):

$$\beta = \frac{\lambda D}{d}$$

As per the problem, $\lambda = 6 \times 10^{-5} \text{ cm}$, $D = 100 \text{ cm}$, and $\beta = 0.004 \text{ cm}$. So, the diameter of the wire is

$$d = \frac{\lambda D}{\beta} = \frac{(6 \times 10^{-5} \text{ cm}) \times (100 \text{ cm})}{(0.004 \text{ cm})} = 1.5 \text{ cm}$$

The number of fringes visible inside the shadow is given by Eq. (9.25):

$$N = \frac{(a+D)d^2}{a\lambda D} = \frac{(100 \text{ cm} + 100 \text{ cm}) \times (1.5 \text{ cm})^2}{(100 \text{ cm}) \times (6 \times 10^{-5} \text{ cm}) \times (100 \text{ cm})} = 750$$

Terminal Questions

1. We rewrite Eq. (9.4) as

$$a(P_0) = \frac{a_1}{2} + \left(\frac{a_1}{2} - a_2 + \frac{a_3}{2} \right) + \left(\frac{a_3}{2} - a_4 + \frac{a_5}{2} \right) + \frac{a_5}{2} + \dots \quad (\text{i})$$

when n is odd, the last term would be $\frac{a_n}{2}$. We are told that obliquity is such

that each term in Eq. (i) is less than the arithmetic mean of its preceding and succeeding terms i.e., $a_n < \frac{1}{2}(a_{n-1} + a_{n+1})$. Then the quantities in the

parentheses in Eq. (i) will be positive. So when n is odd, the minimum value of the amplitude produced by consecutive zones is given by

$$a(P_0) > \frac{1}{2}(a_1 + a_n)$$

To obtain the upper limit, we rewrite Eq. (9.4) as

$$a(P_0) = \left(a_1 - \frac{a_2}{2} \right) - \left(\frac{a_2}{2} - a_3 + \frac{a_4}{2} \right) - \left(\frac{a_4}{2} - a_5 + \frac{a_6}{2} \right) - \dots - \frac{a_{n-1}}{2} + a_n \quad (\text{ii})$$

Following the argument used in obtaining the lower limit on the amplitude, we find that the upper limit is

$$a(P_0) < \left(a_1 - \frac{a_2}{2} - \frac{a_{n-1}}{2} + a_n \right) \quad (\text{iii})$$

Since the amplitude for any two adjacent zones are nearly equal, we can take $a_1 \approx a_2$ and $a_{n-1} \approx a_n$. Within this approximation

$$a(P_0) < \frac{a_1 + a_n}{2} \quad (\text{iv})$$

The results contained in Eqs. (ii) and (iv) suggest that when n is odd, the resultant amplitude at P_0 is given by

$$a(P_0) = \frac{a_1 + a_n}{2} \quad (\text{v})$$

Following the same method, you can readily show that if n were even

$$a(P_0) = \frac{a_1 - a_n}{2} \quad (\text{vi})$$

2. As per the problem, the diameter of the 8th boundary of the zone plate is 0.6 cm. So, $r_8 = 0.3$ cm. From Eq. (9.9):

$$f_n = \frac{r_n^2}{n\lambda}$$

$$f_8 = \frac{r_8^2}{8\lambda} = \frac{(0.3 \text{ cm})^2}{8 \times (5 \times 10^{-5} \text{ cm})} = 2.25 \times 10^2 \text{ cm} = 225 \text{ cm}$$

3. a) The radius of a Fresnel zone is given by

$$r_n = \sqrt{n\lambda b}$$

Here, we are told that $r_n = 0.1$ cm, $b = 20$ cm and $\lambda = 5 \times 10^{-5}$ cm.

So,

$$n = \frac{r_n^2}{b\lambda} = \frac{10^{-2} \text{ cm}^2}{(20 \text{ cm}) \times (5 \times 10^{-5} \text{ cm})} = 10$$

- b) In this part, we have to calculate r_n for given values of

$n = 10$, $b = 200$ cm and $\lambda = 5 \times 10^{-5}$ cm. So,

$$r_n = \sqrt{10 \times (200 \text{ cm}) \times (5 \times 10^{-5} \text{ cm})} = 0.32 \text{ cm}$$

4. The position of the dark band in the diffraction pattern of straight edge outside the geometrical shadow is given by Eq. (9.14):

$$x = \sqrt{\frac{2b(a+b)}{a} n\lambda}$$

As per the problem, $a = 15$ cm, $b = 100$ cm, $\lambda = 6 \times 10^{-5}$ cm. For the first dark band, $n = 1$; so

$$x = \sqrt{\frac{2 \times (100 \text{ cm}) \times (15 \text{ cm} + 100 \text{ cm}) \times (6 \times 10^{-5} \text{ cm})}{(15 \text{ cm})}} = 0.30 \text{ cm}$$

For second dark band, $n = 2$. So,

$$x = \sqrt{\frac{2 \times (100 \text{ cm}) \times (15 \text{ cm} + 100 \text{ cm}) \times 2 \times (6 \times 10^{-5} \text{ cm})}{(15 \text{ cm})}} = 0.43 \text{ cm}$$

5. Interference fringes are formed in the shadow region of the diffraction pattern of a thin wire. The fringe width is given by Eq. (9.23):

$$\beta = \frac{\lambda D}{d}$$

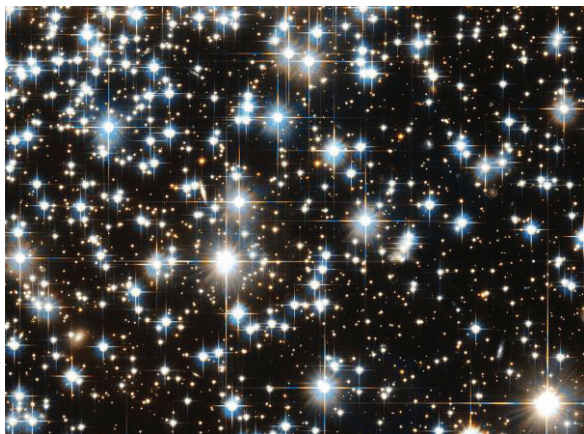
As per the problem, $d = 0.3$ cm, $\lambda = 5.0 \times 10^{-5}$ cm, $D = 250$ cm. So,

$$\beta = \frac{(5 \times 10^{-5} \text{ cm}) \times (250 \text{ cm})}{(0.3 \text{ cm})} = 0.04$$

The total number of fringes is given as

$$N = \frac{(a + D)d^2}{a\lambda D}$$

$$= \frac{(100 \text{ cm} + 250 \text{ cm}) \times (0.3 \text{ cm})^2}{(100 \text{ cm}) \times (5 \times 10^{-5} \text{ cm}) \times (250 \text{ cm})} = 25$$



UNIT 10

FRAUNHOFER DIFFRACTION

Diffractions spikes are seen in photographs of bright sources of light such as stars. They appear to radiate from the light source and are created when light from a distant star passes through the aperture of a telescope and undergoes diffraction. (Source: commons.wikimedia.org)

Structure

10.1	Introduction	10.4	Diffraction from a Double Slit
	Expected Learning Outcomes		Observed Pattern
10.2	Evolution from Fresnel to Fraunhofer Diffraction		Intensity Distribution
10.3	Diffraction from a Single Slit: Point Source	10.5	Summary
	Observed Pattern	10.6	Terminal Questions
	Intensity Distribution	10.7	Solutions and Answers

STUDY GUIDE

You know that diffraction of light is classified into two types: Fresnel diffraction and Fraunhofer diffraction. This classification is based on the distance of source and observation screen from the diffracting aperture or obstacle. In this unit, we will discuss Fraunhofer diffraction which is characterised by the fact that the source and the screen are effectively at infinite distance from the diffracting object. This condition for observing Fraunhofer diffraction pattern is ascertained by using suitable lenses between the source and diffracting object and between the diffracting object and the observation screen. The analysis of Fraunhofer diffraction is based on the concepts of secondary wavelets given by Huygens and their superposition. Thus, you should revise the superposition of secondary wavelets discussed in Unit 9. To understand the mathematical treatment given in this unit better, you should revise the basic concepts of trigonometry and complex numbers. Also, you are advised to revise Unit 6 on interference by division of wavefront because we will refer to some of the concept discussed there such as superposition of secondary waves while explaining the salient features of the double slit diffraction pattern. You should also 'study' the graphs very carefully to follow the discussions and explanations given in the text better. To master the concepts discussed in the unit, you should try to solve SAQs, Examples and TQs yourself. If you are able to solve them, it will give you an instant feedback about your level of understanding of the topics.

"There is nothing that living things do that cannot be understood from the point of view that they are made of atoms acting according to the laws of physics."

***Richard P.
Feynman***

10.1 INTRODUCTION

In the previous unit, you have learnt that diffraction phenomena can be classified into two categories, namely, Fresnel diffraction and Fraunhofer diffraction. In Fresnel class of diffraction, the source or the observation screen or both are kept at a finite distance from the diffracting object. In the previous unit, you learnt the characteristics features of Fresnel diffraction patterns of a straight edge, a slit and a thin wire. You also learnt the qualitative explanation of the salient features of these diffraction patterns on the basis of the concept of Fresnel half period zones. In the present unit, we shall study Fraunhofer diffraction in which the source and the screen (or the telescope in which the diffraction pattern is observed) are effectively at infinite distance from the diffracting object. Under these experimental conditions, we can assume that the wavefronts of light incident on the aperture/obstacles are plane wavefronts.

We begin this unit with a discussion, in Sec. 10.2, on the effect of varying the distance between aperture and screen on the diffraction pattern of an aperture or obstacle. You will discover that, as the observation screen is moved away from the aperture, the diffraction pattern passes from the forms predicted in turn by geometrical optics, Fresnel diffraction and Fraunhofer diffraction. When a plane wavefront is incident at the diffracting aperture, the transition from Fresnel to Fraunhofer pattern is determined by the ratio of the size of the diffracting obstacle to its distance from the source and/or the observation screen.

In Sec. 10.3 we have described the experimental arrangement and salient features of the observed Fraunhofer diffraction pattern from a single slit illuminated by a point source. This is followed by a simple theoretical analysis of the salient features of the observed diffraction pattern. When we deal with plane wavefronts, you will find that theoretical analysis is fairly simple. In Sec. 10.4 we have listed qualitative features of the observed double slit diffraction pattern and compared it with that of a single slit pattern. You will discover that the double slit pattern consists of dark and bright fringes similar to those observed in interference experiments. You will also learn in this section to obtain an expression for the intensity distribution in the double slit pattern. This mathematical analysis is extension of the analysis for single slit. You will discover that the intensity of the central maxima in the double slit pattern is four times the intensity due to either slit.

Expected Learning Outcomes

After going through this unit, you should be able to:

- ❖ explain that Fraunhofer diffraction is a special case of Fresnel diffraction;
- ❖ describe an experimental arrangement for observing Fraunhofer diffraction pattern from a narrow vertical slit;
- ❖ derive an expression for intensity distribution in the single slit diffraction pattern;
- ❖ discuss the position of minima and secondary maxima in single slit pattern;

- ❖ describe the salient features of Fraunhofer diffraction pattern of a double slit;
- ❖ derive an expression for intensity distribution in the double slit pattern;
- ❖ discuss conditions for interference maxima and minima and diffraction minima for a double slit pattern; and
- ❖ explain the missing order fringes in the double slit pattern.

10.2 EVOLUTION FROM FRESNEL TO FRAUNHOFER DIFFRACTION

In Unit 9, you have studied the Fresnel diffraction pattern for some typical diffracting objects such as a straight edge, a slit and a wire. You may recall that for Fresnel diffraction, the source or the screen or both must be at a finite distance from the diffracting object. You may ask: Are these diffraction patterns unique for a given source and obstacle? What happens if we vary the distance of the source and the screen from the obstacle? Let us try to answer these questions by a simple experiment.

Refer to Fig. 10.1a. It shows a point source S located at the focal point of a converging lens L . The spherical waves originating from the source S are changed into plane waves by this lens and the wavefront is now parallel to the diffracting screen with a narrow opening in the form of a long narrow slit. These waves pass through the slit. The diffracted waves produce a diffraction pattern on a screen. A cross-sectional view of this experimental arrangement is shown in Fig. 10.1b. You may now like to know the shape, size and intensity distribution in the diffraction pattern on the screen and how the nature of the diffraction pattern is affected when the distance between the slit and screen is changed. You will learn it now.

A slit is a rectangular opening whose width (0.1 mm or so) is much smaller than its length (1 cm or more).

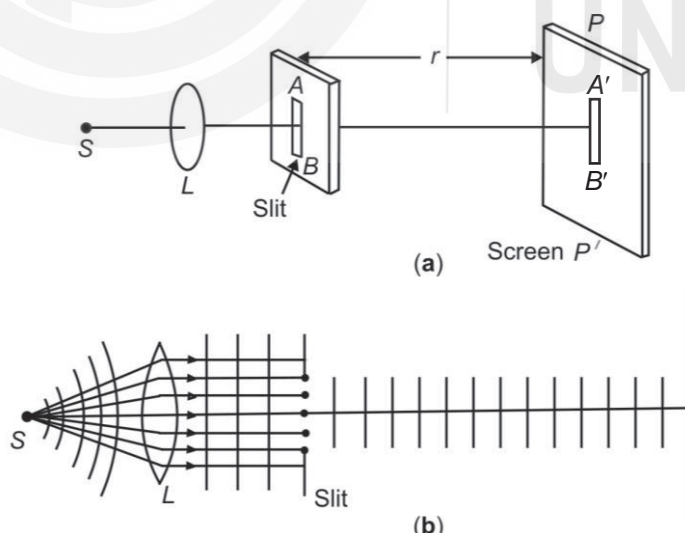


Fig. 10.1: a) Experimental set up for observing transition in Fresnel diffraction pattern; b) Cross-sectional view of the geometry shown in (a) above.

1. When the incident wavefront is strictly parallel to the diffracting slit, we get a vertical patch of light on the screen when it is kept **immediately behind the slit**. That is, we get a region $A'B'$ of uniform illumination on the observation screen. The size of this region is equal to the size of the slit

both in width and height. The remaining portion of the screen is absolutely dark. A plot of this intensity distribution is shown in Fig. 10.2a. From P to A' , the intensity is zero. At A' , it abruptly rises to I_0 and remains constant from A' to B' . At B' , it again drops to zero. We can say that $A'B'$ represents the edges of the geometrical shadow (and the law of rectilinear propagation holds).

- As the screen is moved away from the slit, a careful observation shows that the patch of light seen in (1) above begins to lose sharpness. **If the distance between the slit and the observation screen is large compared to the width of the slit, some fringes start appearing at the edges of the patch A' to B' of light.** But, this patch still resembles the shape of the slit. The intensity distribution shows diffraction rippling effect somewhat like that shown in Fig. 10.2b. From this, we can say that the intensity distribution in the pattern depends on the distance between the observation screen and the slit.
- When the distance r (~ 1 m) between the slit and the screen is much greater than the width of the slit (~ 0.1 mm), the fringes seen in (2) above – close to edge of the patch – now spread out and the **geometrical image** of the slit can no longer be recognised. As distance is increased further, diffraction effects become progressively more pronounced (Fig. 10.2c).
- When r is very large, that is, once we have moved into the Fraunhofer region, the diffraction pattern spreads out considerably bearing little or no resemblance to the actual slit. The diffraction pattern for this situation is shown in Fig. 10.2d. The condition for Fraunhofer diffraction, that is, screen at a very large distance from the slit, is obtained by putting a convex lens after the slit. The observation screen should be arranged so that it is at the second focal plane of the lens. You will learn the details of the Fraunhofer pattern in the next section of the unit.

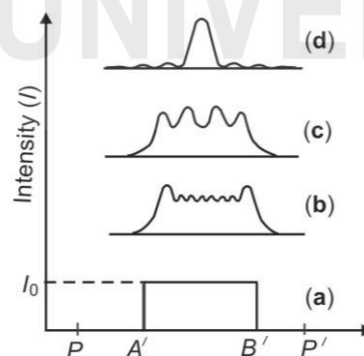


Fig. 10.2: Spatial evolution of a diffraction pattern.

On the basis of above discussion, we may conclude that the Fresnel diffraction pattern can change significantly as the distance of the observation screen from the aperture is varied. In addition to the distance r between the aperture and screen, the nature of diffraction pattern is also dependent on the distance between the source and the aperture. Note that, by placing the source S at the focal point of a convex lens in Fig. 10.1, we have ensured that the source is effectively at a large distance and the spherical wavefronts emitted by the point source S have been made plane wavefronts. So, if we remove the lens and bring the source

closer to aperture so that spherical wavefronts are incident on the aperture, we will obtain Fresnel pattern even for large values of r .

So, after having learnt the qualitative difference between Fresnel and Fraunhofer diffraction patterns and the conditions under which they are obtained, let us now discuss Fraunhofer diffraction from some typical diffracting objects. Let us first take up a single slit.

10.3 DIFFRACTION FROM A SINGLE SLIT: POINT SOURCE

To observe Fraunhofer diffraction pattern, it is required that the source of light and the observation screen should be at very large (effectively, at infinity) distance from the diffracting object. Let us consider a point source, which is far away (almost at infinity) from the diffracting aperture – a single slit. Thus, the wavefronts of light approaching the diffracting aperture can be assumed to be essentially plane, because the point source is far away. The observation screen should also be at infinite distance from the aperture. You may now like to ask: **Is it practical to put the source of light and the observation screen at infinite distance from the diffracting aperture?** This definitely is not practical because (i) the intensity of diffracted light reaching the observation screen would be reduced infinitesimally (inverse square law) and (ii) we will require infinitely big laboratory rooms. Do these limitations suggest that we cannot observe Fraunhofer diffraction? No, it is not so. These difficulties are readily overcome by using converging lenses in an actual experiment.

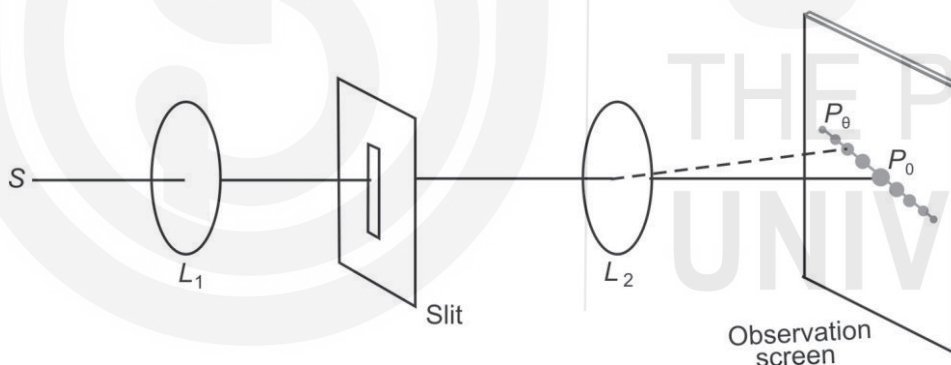


Fig. 10.3: Producing Fraunhofer diffraction pattern

The experimental arrangement for producing Fraunhofer diffraction pattern is shown in Fig. 10.3. A point source (S) of monochromatic light is placed in the focal plane of a converging lens L_1 , so that a plane wave is incident on a long narrow slit. Another converging lens L_2 is placed on the other side of the slit. The observation screen is placed at the second focal point of this lens (L_2). Then, light focussed at any point on the observation screen is due to parallel diffracted wavelets emanating from different portions of the slit. You must note that the observation screen and diffraction screen are kept parallel. Moreover, both the screens are perpendicular to the common axis of L_1 and L_2 . The slit is so adjusted that the common axis of these lenses is perpendicular to the length of the slit and passes through the middle of the slit, both in height and width.

In a physics laboratory, this arrangement is easily achieved by using an ordinary spectrometer. You will get an opportunity to work with a spectrometer

when you do the experiments of the laboratory course (BPHCL-138) of the fourth semester. To observe the diffraction from a point source, the slit of collimator of the spectrometer should be replaced by a fine pinhole, which should be carefully positioned at the focal point of the collimator lens. The observation screen can be placed at the back focal plane of the telescope. Alternatively, we may observe the back focal plane of lens L_2 with an eyepiece. The diffracting screen with slit aperture is placed between the two lenses suitably on the turn table.

10.3.1 Observed Pattern

Let us pause for a moment and think what would diffraction pattern of the vertical slit look like? Or what would be the distribution of intensity in this pattern? You may think that the diffraction pattern would be a single vertical line or a series of vertical lines on the observation screen. This line of thought is wildly off-target. The actual diffraction pattern is astonishingly different; **it consists of a horizontal streak of light composed of bright elongated spots connected by faint streaks**. In other words, after passing through the vertical slit, light spreads along a horizontal line. **This means that diffraction pattern is along a line perpendicular to the length of the diffracting slit.** You may interpret this horizontal diffraction as a spread out image of the point source. The extent of horizontal spreading is controlled by the width of the slit; as the width increases, the spreading decreases. And in the extreme case of a very wide slit, the (horizontal) diffraction streak reduces to a bright point. Physically, very wide slit means that the slit has effectively been removed.

The observed Fraunhofer diffraction pattern of a single vertical slit due to light incident from a point source and its intensity distribution shown in Fig. 10.4. The salient features of the pattern are summarised below:

- i) The diffraction pattern (Fig. 10.3) consists of a **horizontal streak of light** along a line perpendicular to the length of the slit.

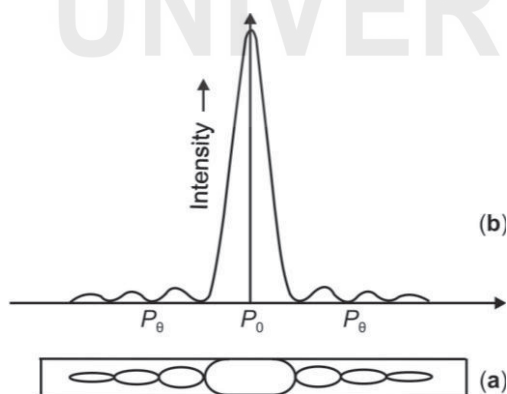


Fig. 10.4: a) Observed pattern; b) intensity distribution pattern Fraunhofer diffraction of a single slit.

- ii) The horizontal pattern is a **series of bright spots**. The spot at the central point P_0 , which lies at the intersection of the axis of L_1 and L_2 with the observation screen (refer also to Fig. 10.3) is the brightest. On the either side of this spot, we observe a few more bright spots symmetrically situated with respect to P_0 .
- iii) The intensity of the central spot is maximum and its peak is located at P_0 (Fig. 10.4b). The peak intensities of other spots, on either side of the

The width of a spot is specified by the distance between two consecutive minima.

central spot, decrease rapidly as we move away from P_0 . The central maximum is called **principal maxima** and other maxima are called **secondary maxima**.

- iv) The **width of the central spot** is twice the width of other spots.
- v) A careful examination of the diffraction pattern shows that the **central peak is symmetrical**. But on either of the central maximum, **secondary maxima are asymmetrical**. In fact, the positions of the secondary maxima are slightly shifted towards the observation point P_0 .

So, we find that the diffraction pattern comprises bright spots with central spot brightest and spots on its both sides having diminishing intensities. Let us now learn the theoretical basis of these features of the diffraction pattern of a single slit by obtaining an expression for the intensity of diffracted light.

10.3.2 Intensity Distribution

The first step in obtaining an expression for intensity of diffracted light on the observation screen is to realise that the observed diffraction pattern is focussed on the observation screen placed at the back focal plane of lens L_2 .

We know that only parallel rays are brought to focus in the back focal plane of the lens. Therefore, diffracted light must be emerging as a series of parallel rays. You know that the rays parallel to the axis of the lens are focussed at the focal point. However, the ray inclined to the axis of the lens are brought to focus on the back focal plane but away from the focal point. We can as well describe this observation in terms of the wavefront; the two (that is, the ray and the wavefront) being perpendicular to each other. Since diffraction pattern lies on a horizontal line (which is at right angles to the common axis of L_1 and L_2), the diffracted wavefronts will be vertical planes perpendicular to the plane of the paper. **That is, after passing through the vertical slit, the incident plane waves are replaced by a system of vertical plane waves which proceed in different directions.**

Now, refer to Fig. 10.5a which shows the geometry for the irradiance at point P_θ (on the distant screen) which makes an angle θ with the axis. For our theoretical analysis, it is sufficient to assume that when a plane wavefront falls on the diffracting slit, each point of the aperture such as $A, A_1, A_2, A_3, \dots B$

You should realise that diffracted waves have no existence in the domain of geometrical optics. The diffracted waves arise due to interaction between light and matter. In the present case, the interaction is between light and the jaws of the slit.

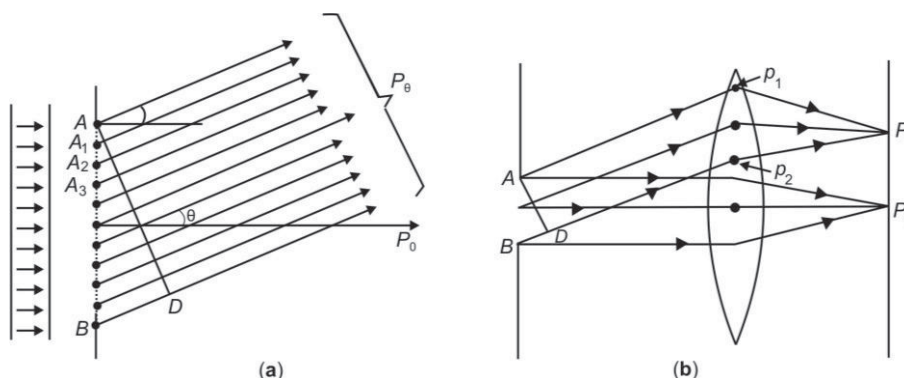


Fig. 10.5: a) Cross-sectional view of the geometry for single slit diffraction; b) trace of optical paths between slit and screen.

becomes a source of secondary wavelets, which propagate in the direction of the point P_θ under consideration. These are diffracted plane waves. In order

to sum up the contributions of different wavelets at P_θ , we must know their amplitudes and phases. The amplitudes of the disturbances (electric field associated with light) from $A, A_1, A_2, \dots$ will be very nearly equal. Can you guess why? This is because the distance of point P_θ from the diffracting screen is very large compared to the width (b) of the aperture.

Now, let us consider the phases of the disturbances reaching the point P_θ . You will agree that the points $A, A_1, A_2, A_3, \dots, B$ within the aperture form a series of coherent sources emitting secondary waves (wavelets) since they have originated from the same point source. Also, points $A, A_1, A_2, \dots, B$ are in the same phase since they lie on the same plane wavefront. The phase difference between different diffracted rays reaching the point P_θ arises due to the difference in path lengths travelled by them to reach P_θ .

To know the phase difference, we draw a plane normal to the parallel diffracted rays directed towards P_θ . The trace of this plane in the plane of the paper is AD (Fig. 10.5a). Though the disturbances are in phase at points $A, A_1, A_2, \dots, B$ when they start, they reach the trace AD in different phases because of the unequal path lengths travelled by them. The optical paths of diffracted waves from the plane AD to the focal point P_θ are equal. This is because in a well corrected converging lens the optical paths of all rays between any plane intersecting the parallel beam of light perpendicularly and the point where rays converge after traversing the lens, are equal. That is, optical paths Ap_1P_θ and Dp_2P_θ are equal, as may be seen from Fig. 10.5b. Therefore, the wavelets arrive at P_θ with the same relative phase difference as the ones existing at the trace AD .

Two sources are said to be coherent if they emit in-phase waves of the same frequency.

Let us now consider that the width $AB (= b)$ of the aperture be divided into n equal parts so that $AA_1 = A_1A_2 = A_2A_3 = b/n = \Delta$. It means that we have a series of $(n + 1)$ point sources between A and B . Actually, the aperture contains a continuous distribution of points from A to B . Therefore, in the limiting case, $n \rightarrow \infty$ and $\Delta \rightarrow 0$ we have $n\Delta \rightarrow b$. Consider the two rays starting from two neighbouring points A and A_1 . The path difference between them is $AA_1 \sin \theta$, where θ is the angle between the diffracted rays and the normal to the slit. Hence, the corresponding phase difference is given by

$$\phi = \frac{2\pi}{\lambda}(AA_1 \sin \theta) = \frac{2\pi}{\lambda}\left(\frac{b}{n} \sin \theta\right) = \frac{2\pi}{\lambda} \Delta \sin \theta \quad (10.1)$$

Let the field at P_θ due to the (electric field) disturbance originating from A be $a_0 \cos \omega t$. Then, the field due to the disturbance from A_1 is $a_0 \cos(\omega t - \phi)$. Here, we have assumed that the amplitudes of disturbances from different points are equal. The field due to disturbances from successive points $A_2, A_3, \dots, B$ are $a_0 \cos(\omega t - 2\phi), a_0 \cos(\omega t - 3\phi), \dots, a_0 \cos(\omega t - n\phi)$, respectively. Thus, the magnitude E of resultant field at P_θ is equal to the sum of these disturbances. Hence, we can write

$$E = a_0 \cos \omega t + a_0 \cos(\omega t - \phi) + a_0 \cos(\omega t - 2\phi) + \dots + a_0 \cos(\omega t - (n-1)\phi) \quad (10.2)$$

The sum of the series on the RHS of Eq. (10.2) can be written as (see the margin remark):

$$E = a_0 \left[\frac{\sin \frac{n\phi}{2}}{\sin \left(\frac{\phi}{2} \right)} \right] \cos \left(\omega t - \frac{n\phi}{2} \right) = E_\theta \cos \left(\omega t - \frac{n\phi}{2} \right) \quad (10.3)$$

where, E_θ is the amplitude of the resultant field at P_θ :

$$E_\theta = a_0 \frac{\sin \frac{n\phi}{2}}{\sin \left(\frac{\phi}{2} \right)} \quad (10.4)$$

In the limit $n \rightarrow \infty$ and $\Delta \rightarrow 0$, $n\Delta \rightarrow b$. Thus, using Eq. (10.1), we can write

$$\frac{n\phi}{2} = \frac{n}{2} \frac{2\pi}{\lambda} \Delta \sin \theta = \frac{\pi}{\lambda} (n\Delta) \sin \theta = \frac{\pi}{\lambda} b \sin \theta$$

so that $\phi = \frac{2\pi}{\lambda} \frac{b \sin \theta}{n}$ will be very small for $n \rightarrow \infty$. We may, therefore, write

$$\sin \left(\frac{\phi}{2} \right) \cong \frac{\phi}{2} = \frac{\pi b \sin \theta}{n\lambda}$$

Substitute this result in Eq. (10.4) and on simplification, we can write

$$\begin{aligned} E_\theta &= a_0 \frac{\sin(n\phi/2)}{(\phi/2)} = na_0 \frac{\sin \left(\frac{\pi b \sin \theta}{\lambda} \right)}{\left(\frac{\pi b \sin \theta}{\lambda} \right)} \\ &= na_0 \left(\frac{\sin \beta}{\beta} \right) = A \left(\frac{\sin \beta}{\beta} \right) \end{aligned} \quad (10.5)$$

Where, we have written

$$A = na_0$$

$$\text{and } \beta = \frac{\pi b \sin \theta}{\lambda} \quad (10.6)$$

You will note that for a given wavelength, β signifies half of the phase difference between disturbances originating from the extreme points A and B . Thus, the expression [Eq. (10.3)] for resultant field at P_θ takes the form

$$E = A \frac{\sin \beta}{\beta} \cos(\omega t - \beta) = E_\theta \cos(\omega t - \beta) \quad (10.7)$$

You know that the intensity of a wave is its amplitude squared. Thus, the intensity of light at point P_θ can be written as The corresponding intensity at P_θ is given by

$$I_\theta = A^2 \left(\frac{\sin \beta}{\beta} \right)^2 \quad (10.8)$$

Let us pause for a while and ponder as to what we have achieved. Eq. (10.8) suggests that the intensity is maximum at $\theta = 0$. This readily follows by noting that when we substitute $\theta = 0$ in Eq. (10.8), we have both β and $\sin \beta$ equal to zero but

$$\lim_{\beta \rightarrow 0} \frac{\sin \beta}{\beta} = 1$$

The series given in Eq. (10.2) can be summed conveniently by using the method of complex numbers. You know that, in the complex number rotation, we write

$$\exp(i\theta) = \cos \theta + i \sin \theta$$

with $\cos \theta = \text{Re}[\exp(i\theta)]$

and $\sin \theta = \text{Im}[\exp(i\theta)]$

So, we can write Eq. (10.2) as

$$\begin{aligned} E &= a_0 e^{i\omega t} + a_0 e^{i(\omega t - \phi)} + \\ & a_0 e^{i(\omega t - 2\phi)} + \dots + \\ & a_0 e^{i[\omega t - (n-1)\phi]} \\ &= a_0 e^{i\omega t} [1 + e^{-i\phi} + \\ & e^{-2i\phi} + \dots + e^{-(n-1)i\phi}] \end{aligned}$$

The series in the square bracket is in geometric progression. And, the sum of the finite geometric series is

$$S = \frac{1 - r^n}{1 - r}$$

where, r is the common ratio. In the present case, common ratio is

$e^{-i\phi}$. So, we can write

$$\begin{aligned} E &= a_0 e^{i\omega t} \left[\frac{1 - (e^{-i\phi})^n}{1 - (e^{-i\phi})} \right] \\ &= a_0 e^{i\omega t} \cdot \frac{e^{(-in\phi/2)} [e^{(+in\phi/2)} - e^{(-in\phi/2)}]}{e^{(-i\phi/2)} [e^{(i\phi/2)} - e^{(-i\phi/2)}]} \\ &= a_0 e^{i\omega t} \left[\frac{\sin(n\phi/2)}{\sin(\phi/2)} \right] \cdot \frac{e^{(i\phi/2)} \cdot e^{(-in\phi/2)}}{e^{(i\phi/2)} \cdot e^{(-i\phi/2)}} \\ &= a_0 \exp(i\omega t) \cdot \frac{\sin(n\phi/2)}{\sin(\phi/2)} \cdot \exp \left[\frac{i\phi(1-n)}{2} \right] \\ &= a_0 \frac{\sin(n\phi/2)}{\sin(\phi/2)} \cdot \exp[i\{\omega t - (n\phi/2) + (\phi/2)\}] \end{aligned}$$

For n very large, ϕ becomes very small and we can ignore $(\phi/2)$ in respect of $(n\phi/2)$. So,

$$E = a_0 \frac{\sin(n\phi/2)}{\sin(\phi/2)} \cdot \exp[i\{\omega t - (n\phi/2)\}]$$

So, taking the real part of E , we can write magnitude of the resultant field as

$$E = a_0 \left[\frac{\sin(n\phi/2)}{\sin(\phi/2)} \right] \cos(\omega t - (n\phi/2))$$

which is Eq. (10.3).

Therefore, $I_{\theta=0} = A^2$

This result is expected on geometrical considerations. In the limit of a distant screen, the central point ($\theta = 0$) on the screen becomes equidistance from each point on the slit. This implies that all diffracted waves arrive in phase at P_0 and interfere constructively. A^2 is then the value of the maximum intensity at the centre of the pattern. This maximum is also termed **principal maximum**.

For brevity, we write $I_{\theta=0} = A^2 = I_0$. Then, intensity at any point at an angle θ with the horizontal axis is given by

$$I_{\theta} = I_0 \left(\frac{\sin \beta}{\beta} \right)^2 \quad (10.9)$$

Recall from Sec. 10.3.1 that the diffraction pattern of a single slit has bright and dark spots on both sides of the central bright maximum. These are called secondary maxima and minima. Let us now discuss how to locate their positions.

Positions of maxima and minima

A plot of Eq. (10.8) for intensity distribution in the diffraction pattern of a single slit is shown in Fig. 10.6a. You will note that the intensity is maximum for $\theta = 0$; $I_{\theta=0} = I_0 = A^2$. The intensity gradually falls on either side of the principal maximum and becomes zero when $\beta = +\pi$ or $\beta = -\pi$ since $\sin(\pm\pi)$ is zero. This specifies the **first minimum**. So, we can say that the **angular half width of principal maximum** is from $\beta = 0$ to $\beta = \pi$. The second minimum on either side occurs at $\beta = \pm 2\pi$. Thus, we can write the condition for minima as

$$\begin{aligned} \beta &= \pm \pi, \pm 2\pi, \pm 3\pi, \dots \\ &= m\pi \quad m = \pm 1, \pm 2, \pm 3, \dots \end{aligned} \quad (10.10)$$

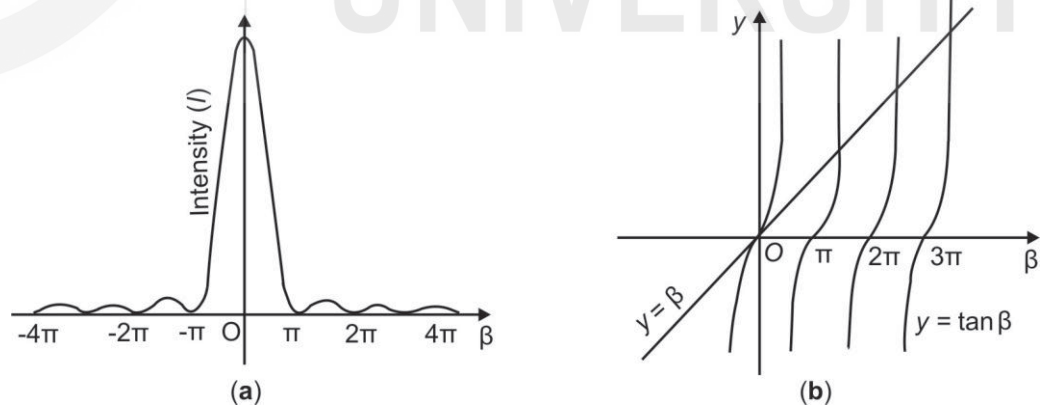


Fig. 10.6: a) Amplitude and intensity curves for Fraunhofer diffraction of a single slit showing positions of maxima and minima; b) plot of $y = \tan \beta$ for determining the position of secondary maxima.

Note that the value $m = 0$ is excluded because it corresponds to the principal maximum (for $\beta = 0$). Substituting the value of β from Eq. (10.10) in Eq. (10.6) we find that the condition for minima can also be expressed as

$$\begin{aligned} b \sin \theta &= \pm \lambda, \pm 2\lambda, \pm 3\lambda, \dots \\ &= m\lambda, \quad m = \pm 1, \pm 2, \pm 3, \dots \end{aligned} \quad (10.11)$$

Eq. (10.11) shows that points on the observation screen will be minima for which the path difference, $b \sin \theta$ between waves diffracted by extreme points (edges A and B in Fig. 10.5) of the slit is an integral multiple of the wavelength. We can arrive at this conclusion for the single slit diffraction pattern on the basis of simple qualitative arguments also (read the margin remark).

Eq. (10.11) also indicates that the location of minima, determined by θ depends on the wavelength of light and the slit width. So, for a given slit width, the spread in diffraction pattern depends directly on the wavelength.

Accordingly, you should expect that red light will be diffracted through a larger angle than the blue or violet light.

Now, you would like to know the direction and position of other maxima, called secondary maxima, in the diffraction pattern. Eq. (10.9) gives the diffraction intensity in different directions. From basic calculus, you know that we can differentiate this equation with respect to β and equate the result to zero to obtain the directions (and positions) of **secondary maxima**. This gives

$$\begin{aligned}\frac{dI_\theta}{d\beta} &= 2I_0 \left(\frac{\sin \beta}{\beta} \right) \left[\frac{\beta \cos \beta - \sin \beta}{\beta^2} \right] \\ &= 2I_0 \sin \beta \left[\frac{\cos \beta}{\beta^2} - \frac{\sin \beta}{\beta^3} \right] = 0\end{aligned}$$

so that

$$\sin \beta (\beta - \tan \beta) = 0 \quad (10.12)$$

From Eq. (10.12), we get the conditions $\sin \beta = 0$ and $\beta - \tan \beta = 0$. The condition $\sin \beta = 0$ implies that $\beta = \pm m\pi$, where m is any integer. This condition signifies minima [Eq. (10.10)].

The condition

$$\beta = \tan \beta \quad (10.13)$$

gives the positions of **secondary maxima**. Eq. (10.13) is a transcendental equation which can be solved using graphical method. To do that, we plot straight line $y = \beta$ and the curve $y = \tan \beta$ as shown in Fig. 10.6b. Note that the condition $\beta = \tan \beta$ will be satisfied at the intersection of these two plots. The points of intersection of these two plots correspond to the solution of Eq. (10.13). These points, excluding the one for $\beta = 0$ (which corresponds to principal maximum) occurs at $\beta = 1.43\pi, 2.46\pi, 3.47\pi$ etc. and give the position, respectively, of the first, second, third maxima on either side of the central maximum. **You should note that these maxima do not fall midway between two minima.** For instance, the first secondary maximum occurs at 1.43π rather than 1.50π . Similarly, the second maximum occurs at 2.46π rather than 2.50π and so on. This means that the intensity curves for these secondary maxima are asymmetrical. The plot clearly shows that the positions of maxima are slightly shifted towards the centre of the pattern. You may recall that this asymmetry is observed experimentally as well.

Let us now calculate the **intensities at these positions of secondary maxima**. Substituting the values of β ($= 1.43\pi$) in Eq. (10.9), the intensity of first secondary maximum as

$$I_0 \left(\frac{\sin \beta}{\beta} \right)^2 = I_0 \left(\frac{\sin 1.43\pi}{1.43\pi} \right)^2 = I_0 (0.0496)$$

This means that the intensity of the first secondary maxima (nearest to the central peak) is about 4.96% of the central maximum. Similarly, you can

The single slit diffraction pattern can be understood on the basis of the following simple qualitative arguments. The path difference between waves diffracted by extreme points A and B of the slit is $BD = b \sin \theta$ as shown in

Fig. 10.7.

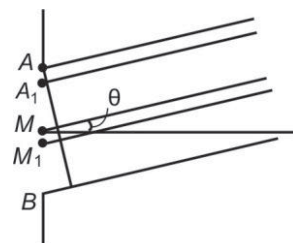


Fig. 10.7: Position of minima in single slit pattern.

Suppose that light is focused at some point P_0 . If BD is an integral multiple of λ , we will show that the resultant intensity at P_0 will be zero. For $m = 1$, Eq. (10.11) gives $b \sin \theta = \lambda$. Now, let us divide the slit into two equal halves AM and MB as shown in Fig. 10.7. Consider the waves starting from the two point sources A and M . The path difference between them is $AM \sin \theta = (b/2) \sin \theta = \lambda/2$. So, the corresponding phase difference will be π . Therefore, the two waves, on superposition, will lead to zero intensity at P_0 . Similarly, for a point A_1 just below A , there will be a corresponding point M_1 just below M such that the path difference between disturbances generated by them is again $\lambda/2$. On superposition, this pair also leads to zero intensity at P_0 . We can thus pair off all the points in the upper half (AM) with corresponding points in the lower half (MB) and the disturbances due to upper half of the slit will be cancelled by disturbances due to the lower half. So the resultant intensity at P_0 will be zero. This explains why we get a minimum intensity at P_0 when the path difference between the rays from extremes is equal to λ .

Contd...

Contd....

Let us check this whether or not Eq. (10.11) is true for the case $m = 2$. In this case, the path difference $b \sin \theta$ between the rays from extreme points A and B is equal to 2λ . We can now imagine that the slit is divided into four equal parts. We can, by similar pairing, show that the first and second quarters from top have a path difference of $\lambda/2$ and cancel each other. Third and fourth quarters cancel each other by the same argument so that the resultant intensity in the focal plane at P_θ is again zero. For $m = 3$, the path difference between the two extreme rays is $b \sin \theta = 3\lambda$. In this case, the slit should be divided into six equal parts to show similar pairing and cancellation and then leading to zero intensity. By this simple qualitative argument, we have shown that when the path difference between the extreme parallel diffracted rays in a particular direction is an integral multiple of λ , the resultant diffracted intensity in that direction is zero.

calculate and convince yourself that the intensities of the second and third secondary maxima are about 1.68% and 0.83% of the central maximum. We thus see that most of the light is concentrated in the central maximum.

Another important characteristic of single slit diffraction pattern is that the width of the principal maximum is double the width of secondary maximum. We have left the mathematical proof of this characteristic as an exercise for you in the form of an SAQ (SAQ 1). Before you proceed further, you should solve it.

SAQ 1 - Width of maxima in the single slit diffraction pattern

Show that the width of the principal (central) maximum in the diffraction pattern of a single slit is twice the width of secondary maxima.

To give you a feel for numerical values and fix the ideas developed in this section, we now give a few solved examples. You should go through them carefully.

EXAMPLE 10.1: POSITION OF MAXIMA AND MINIMA IN DIFFRACTION PATTERN OF SINGLE SLIT

In the experimental set up for obtaining Fraunhofer diffraction pattern of a vertical slit of width 0.3 mm, the focal length of lens kept between the slit and the screen is 30 cm. Calculate (a) the diffraction angles and positions of the first, second and third minima, and (b) the positions of the first, second and third secondary maxima on either side of the central spot. Take $\lambda = 6000 \text{ \AA}$.

SOLUTION ■ a) You have learn it [Eq. (10.11)] that the conditions for minima are given by $b \sin \theta = m\lambda$; $m = \pm 1, \pm 2, \pm 3, \dots$ For small values of θ , we may write $\sin \theta \cong \theta$. Then

$$\theta = m \frac{\lambda}{b}$$

and the distance $P_0 P_\theta$ is $f\theta$, where P_0 and P_θ are the positions of the central spot and the minima at angle θ and f is the focal length (see Fig. 10.3 for reference). Therefore, the diffraction angles $\theta_1, \theta_2, \theta_3$ for the

first, second and third minima are $\frac{\lambda}{b}$, $2\frac{\lambda}{b}$ and $3\frac{\lambda}{b}$, respectively.

On substituting the numerical values of λ and b we get

$$\theta_1 = \frac{6000 \times 10^{-8} \text{ cm}}{0.3 \times 10^{-1} \text{ cm}} = 2 \times 10^{-3} \text{ rad}$$

$$\theta_2 = 2\theta_1 = 4 \times 10^{-3} \text{ rad}$$

$$\theta_3 = 3\theta_1 = 6 \times 10^{-3} \text{ rad}$$

The distances d_1, d_2, d_3 of these minima from the central spot are

$$d_1 = f \theta_1 = (30 \text{ cm}) \times 2 \times 10^{-3} = 6 \times 10^{-2} \text{ cm} = 0.06 \text{ cm}$$

$$d_2 = 2f \theta_1 = 2 \times 0.06 \text{ cm} = 0.12 \text{ cm}$$

$$d_3 = 3f \theta_1 = 3 \times 0.06 \text{ cm} = 0.18 \text{ cm}$$

You may note that these minima are separated by a distance of 0.06 cm on the focal plane of the lens.

b) We know that the first three secondary maxima occur at $\beta = 1.43\pi$, 2.46π and 3.47π , respectively. Thus, from Eq. (10.6), we have (under small angle approximation; $\sin\theta \approx \theta$). The corresponding diffraction angles in radians for these three maxima as

$$(\theta_1)_{\max} = 1.43 \frac{\lambda}{b}, (\theta_2)_{\max} = 2.46 \frac{\lambda}{b} \text{ and } (\theta_3)_{\max} = 3.47 \frac{\lambda}{b}$$

$$\therefore (\theta_1)_{\max} = 1.43(2 \times 10^{-3}), (\theta_2)_{\max} = 2.46(2 \times 10^{-3}),$$

$$(\theta_3)_{\max} = (3.47)(2 \times 10^{-3})$$

and the corresponding distances from the central point (P_0) are

$$d_1 = f(\theta_1)_{\max} = (30 \text{ cm}) \times 1.43 \times 2 \times 10^{-3} = 0.086 \text{ cm}$$

$$d_2 = f(\theta_2)_{\max} = (30 \text{ cm}) \times 2.46 \times 2 \times 10^{-3} = 0.16 \text{ cm}$$

$$d_3 = f(\theta_3)_{\max} = (30 \text{ cm}) \times 3.47 \times 2 \times 10^{-3} = 0.21 \text{ cm}$$

EXAMPLE 10.2: RELATION BETWEEN SLIT WIDTH AND WIDTH OF CENTRAL MAXIMUM

In the experimental set up given in Example 10.1 above, we change slit widths to 0.2 mm, 0.1 mm and 0.06 mm. Calculate the positions of the first and second minima.

SOLUTION ■ For slit width $b = 0.2$ mm, we have

$$d_1 = f\theta_1 = (30 \text{ cm}) \times \frac{6000 \times 10^{-8} \text{ cm}}{0.2 \times 10^{-1} \text{ cm}} = 0.09 \text{ cm}$$

Similarly, $d_2 = f\theta_2 = 2 \times 0.09 \text{ cm} = 0.18 \text{ cm}$

These minima are separated by 0.09 cm. Recall from Example 10.1 above that the corresponding value for the separation between first and second minima for a slit of width 0.03 cm was 0.06 cm. This means that, for a given wavelength, the spread of secondary maximum increases as slit width decreases. This conclusion is brought out in the following calculation as well.

For a slit of width $b = 0.1$ mm, we have

$$d_1 = (30 \text{ cm}) \times \frac{6000 \times 10^{-8} \text{ cm}}{0.1 \times 10^{-1} \text{ cm}} = 0.18 \text{ cm}$$

and $d_2 = 2 \times 0.18 \text{ cm} = 0.36 \text{ cm}$

For slit width $b = 0.06$ mm, we have

$$d_1 = (30 \text{ cm}) \times \frac{6000 \times 10^{-8} \text{ cm}}{0.06 \times 10^{-1} \text{ cm}} = 0.3 \text{ cm}$$

and $d_2 = 2 \times 0.3 \text{ cm} = 0.6 \text{ cm}$

We thus find that for slits of widths 0.3 mm, 0.2 mm, 0.1 mm and 0.06 mm, the first minimum on either side of the principal maximum occurs at distances of 0.06 cm, 0.09 cm, 0.18 cm, 0.3 cm.

From the solution of above example, you may note that as the slit becomes narrower, the spread of central maximum increases. Conversely, the wider the slit, the narrower is the central diffraction maximum.

Let us now consider an interesting case where width of the slit is varied in comparison to the wavelength of light.

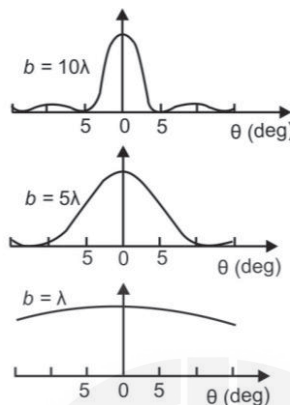


Fig. 10.8: Intensity distribution in the single-slit diffraction pattern as the slit width varies.

EXAMPLE 10.3: SLIT WIDTH AND THE NATURE OF DIFFRACTION PATTERN

In the experimental set up described in Example 10.1, consider that the slit width b has values $b = 10\lambda$, 5λ and λ . Calculate the spread of the central maximum for each value of slit width.

SOLUTION ■ From Eq. (10.11), we note that for a slit of width $b = 10\lambda$, the first minimum is located at

$$10\lambda \sin\theta = \lambda \Rightarrow \sin\theta = 0.10 \Rightarrow \theta = 5.7^\circ$$

For a slit of width 5λ , we have

$$5\lambda \sin\theta = \lambda \Rightarrow \theta = 11.5^\circ$$

That is, as the width (aperture) of the slit changes from 10λ to 5λ , the diffraction pattern spreads out about twice as far. For $b = \lambda$,

$$\sin\theta = 1 \Rightarrow \theta = 90^\circ$$

Thus, we find that the first minimum falls at 90° . That is, the central maximum spreads out and the diffraction pattern shows no ripple (rise and fall in intensity of light). These features are shown in Fig. 10.8.

You may now like to answer an SAQ.

SAQ 2 - Effect of wavelength of light on single slit pattern

In the experimental set up described in Example 10.1, suppose the slit is illuminated with violet light of wavelength $4358 \text{ \AA}$ from a mercury lamp. Show that the diffraction pattern shrinks correspondingly.

White Light Source

You may now like to know: What will happen if monochromatic source is replaced by a white light source in single slit diffraction set up? We expect that each constituent wavelength of white light will be diffracted independently. This gives rise to a **white central spot** surrounded by coloured fringes. The outer part of this pattern would tend to be reddish. You can easily observe this diffraction pattern by looking through the tines of a dinner fork at a candle in a dimly illuminated room. On twisting the fork about its handle, you will observe the diffraction pattern as soon as the cross-section area becomes small enough.

Diffraction Pattern of a Rectangular Aperture

So far, we have described Fraunhofer diffraction pattern of a slit aperture. Let us now consider as to what will happen if both dimensions of the slit are made comparable. In other words, we now have a rectangular aperture of width b and height a as shown in Fig. 10.9a. We expect that the emergent diffracted

waves will spread along the length as well as the width of the slit. Can you depict the diffraction pattern? It is shown in Fig. 10.9b. Mathematically, the intensity is given by $I = \frac{I_0 \sin^2 \beta \sin^2 \alpha}{\alpha^2 \beta^2}$ where $\beta = \frac{b \pi \sin \theta}{\lambda}$ and $\alpha = \frac{\pi a \sin \theta}{\lambda}$.

Slit Source

Now, let us consider the nature of the diffraction by a single slit if point source is replaced by a slit source. Refer to Fig. 10.10 which shows the experimental arrangement which is a modified form of the arrangement shown in Fig. 10.1 so that, instead of the point source of light, we have a slit source of light.

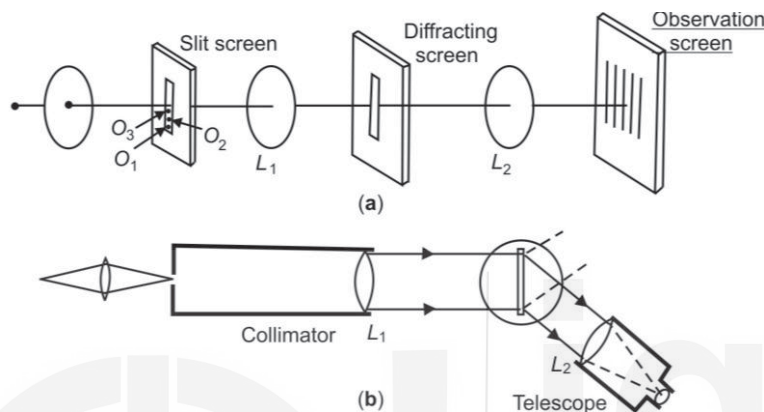


Fig. 10.10: a) Experimental arrangement for diffraction from a vertical single slit illuminated by a slit source; b) corresponding experimental arrangement in a physics laboratory using a spectroscope.

As a matter of fact, the experimental arrangement, which is commonly employed in most experiments, uses a spectrometer (Fig. 10.10b). The slit of the collimator arm is illuminated so that each point of the slit source acts as an independent source. You know that a point source gives a horizontal streak of light as the diffraction pattern of a vertical slit (Fig. 10.1). Now when we use a slit as a source, we can imagine a series of point sources $O_1, O_2, O_3, \dots$ etc. one above the other to form the slit source (Fig. 10.10a). Each point source will give its own diffraction pattern since each point is to be regarded as an independent point source. With the same diffracting slit and the same lenses L_1 and L_2 , the central diffraction maximum due to all point sources will lie above one another and give a **central bright vertical fringe**. Similarly, for secondary maxima and minima points, we will obtain a series of vertical fringes, which will be situated at equal intervals on either side of the central fringe.

Thus, when the point source is replaced by a slit source, the resulting pattern is a series of horizontal diffraction streaks stacked on each other in a vertical direction. The intensity along any line in the diffraction pattern will be the same as in Fig. 10.2. This is because each point such as $O_1, O_2, O_3, \dots$ of the slit source (Fig. 10.10a) acts as an independent source.

Further, it is observed that clear fringes are obtained only when the width of the source slit is small. If the width of the source slit is gradually increased, it leads to an increase in the width of its image on the observation screen. A stage will come when the width of its image becomes comparable with the distances between successive vertical fringes. This will gradually make the

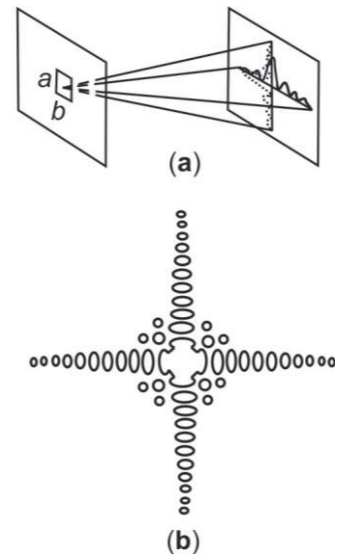


Fig. 10.9: a) Diffraction from a rectangular aperture; both dimensions of the rectangular aperture are small and a two-dimensional diffraction pattern is discernible on the screen; b) diffraction pattern of a single square aperture.

vertical fringes less clear and indistinct. Now, before proceeding further, let us recapitulate the important points of this section.

Recap

- When a single slit is illuminated by light from a point source, we observe a diffraction pattern comprising a horizontal streak of light.
- The intensity of the light in the diffraction pattern at a point located at angle θ is given as

$$I_{\theta} = I_0 \left(\frac{\sin \beta}{\beta} \right)^2$$

where, $\beta = (\pi b \sin \theta) / \lambda$ and I_0 is the intensity of the central spot (principal maximum).

- The position of minima in the pattern is given by

$$\beta = m\lambda$$

- As the width of the single slit decreases, the spread of central maximum increases.

You may like to know the nature of diffraction pattern when single slit is replaced by two slits. The resulting diffraction pattern is qualitatively different from that of the single slit. In particular, we observe both the diffraction and the interference fringes simultaneously when double slit is used. Let us now study the double slit diffraction.

10.4 DIFFRACTION FROM A DOUBLE SLIT

Refer to Fig. 10.11a which shows the experimental arrangement for observing diffraction from two vertical parallel slit (apertures in an opaque screen). Both slits have the same width b and height h .

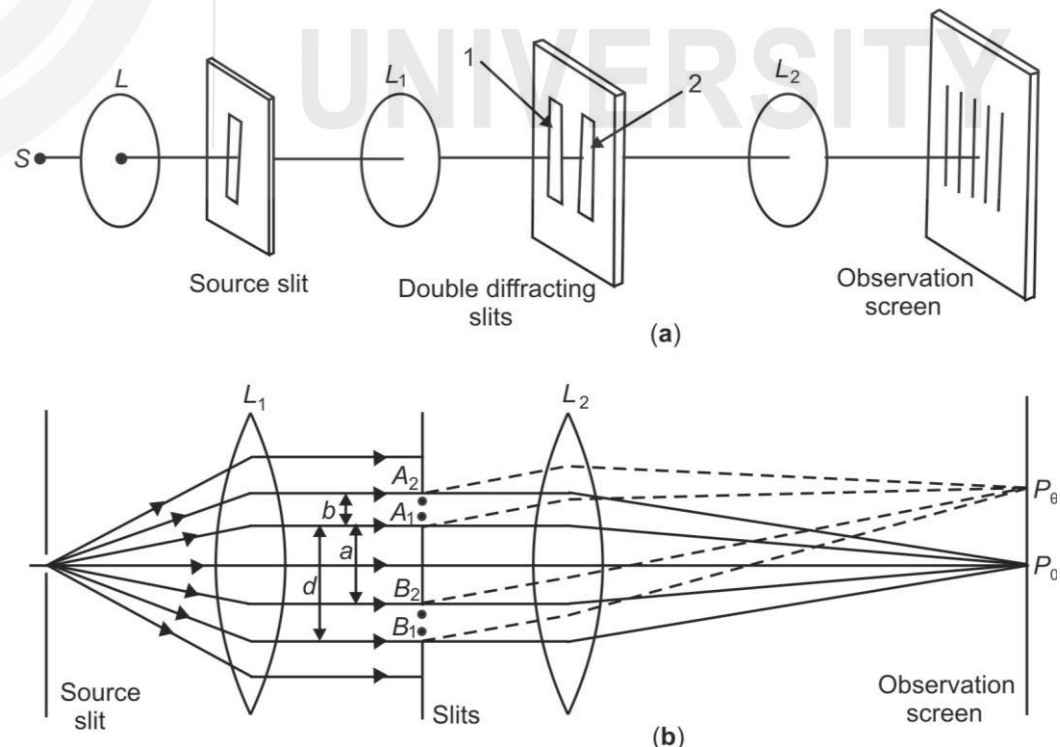


Fig. 10.11: a) Experimental arrangement for observing diffraction from two identical vertical slits; b) corresponding ray diagram.

The width of the intervening opaque space between the two slits is a . Therefore, the distance between two similar points in the slits is $d = a + b$.

Note that the diffracting double slit is illuminated by a slit source rather than a point source of light. We have used this arrangement because this corresponds more nearly to the actual conditions under which an experiment is performed. That is, the diffraction pattern from a slit source is of greater practical importance than from a point source. The ray diagram corresponding to the arrangement shown in Fig. 10.11a is shown in Fig. 10.11b. The length of the source slit in the arrangement should be adjusted to be parallel to the lengths of the diffracting slits.

10.4.1 Observed Pattern

Suppose we block one of the diffracting slits, say slit 1 (Fig. 10.11a) and observe the diffraction pattern on the screen. Obviously, you should expect the single slit diffraction pattern (due to slit number 2 which has not been blocked). Next, we uncover slit 1 and block the other. You should again expect single slit diffraction pattern with exactly the same intensity distribution. But, what may surprise you at the first glance is that both diffraction patterns are not only identical, but they are also located at the same position. Were you not expecting these diffraction patterns to be laterally displaced? **These patterns are not laterally shifted with respect to one another because of the lens L_2 .** This is true even if we have large number of identical vertical slits.

The diffracted wavefronts originating from any slit and travelling along the axis of lens L_2 are focussed at P_0 , which forms the peak of the central spot. And, the diffracted wavelets originating from either slit and moving at an angle θ are focussed at P_θ .

Consider parallel beams of light travelling in a direction inclined to the axis of lens L_2 and falling on its different parts. All these rays are brought to focus on the back focal plane at a point which is located by the beam passing through the optical centre of the lens.



Fig. 10.12: Observed double slit diffraction pattern.

Now uncover both the slits so that each slit gives its own diffraction pattern. The salient features of the observed diffraction pattern, shown in Fig. 10.12, are summarised below:

- i) The double slit diffraction pattern consists of a number of equally spaced fringes similar to what is observed in interference experiments.
- ii) The intensities of all fringes are not equal. The fringes are the brightest in the central part of the pattern.
- iii) As we move away on either side of the central fringe, the intensity gradually falls off to zero.

- iv) The fringes reappear with reduced intensity three or four times and become too faint to be observable thereafter.
- v) The intensity at the maximum of double slit pattern is greater than the intensity of principal maximum in single slit pattern.
- vi) The single-slit diffraction pattern acts as an envelope to the double slit pattern.

You would now like to know: **How do we explain these features of the double slit diffraction pattern?** How bright are double slit fringes compared to those in the single slit pattern? You will discover answers to these and other related questions in the following section.

10.4.2 Intensity Distribution

We can explain the salient features of the double slit diffraction pattern by obtaining an expression for intensity of light at different positions in it. For calculating the intensity distribution for the arrangement shown in Fig. 10.11, it is sufficient to consider a point source. This is because a point source gives the intensity distribution along a section perpendicular to the vertical fringes formed due to a slit source.

For obtaining an expression for intensity of double slit pattern, we extend the procedure used for the single slit discussed in Sec. 10.3. Slit 1 in Fig. 10.11 acts as a source of diffracted plane wavefronts originating from points $A_1, A_2, A_3, \dots$ in it. We represent these by $a_0 \cos \omega t$, $a_0 \cos(\omega t - \phi)$, $a_0 \cos(\omega t - 2\phi), \dots$, where ϕ is the constant phase difference. The amplitude E_1 of the electric field produced by this slit at the point P_θ can be written as [Eq. (10.7)]:

$$E_1 = A \left(\frac{\sin \beta}{\beta} \right) \cos(\omega t - \beta) \quad (10.14)$$

where, $\beta = \frac{\pi b \sin \theta}{\lambda}$.

Now, note in Fig. 10.11b that for every point A_1 in slit 1, we have a corresponding point B_1 in slit 2 at a distance $d (= a+b)$. And, the phase difference between diffracted wavefronts reaching P_θ from A_1 and B_1 can be written as

$$\delta = \frac{2\pi}{\lambda} (a+b) \sin \theta = \frac{2\pi}{\lambda} d \sin \theta \quad (10.15)$$

Thus, the plane wavefronts emerging from point sources $B_1, B_2, B_3 \dots$ can be represented as $a_0 \cos(\omega t - \delta)$, $a_0 \cos(\omega t - \delta - \phi)$, $a_0 \cos(\omega t - \delta - 2\phi), \dots$. As a result, the amplitude E_2 of the electric field produced by slit 2 at the point P_θ can be written as

$$E_2 = A \left(\frac{\sin \beta}{\beta} \right) \cos(\omega t - \beta - \delta) \quad (10.16)$$

Since the secondary sources $A_1, A_2, A_3, \dots$ and $B_1, B_2, B_3, \dots$ are coherent, the amplitude of resultant field at P_θ due to the double-slit is obtained by the superposition of individual fields:

$$E = E_1 + E_2 = A \frac{\sin \beta}{\beta} [\cos(\omega t - \beta) + \cos(\omega t - \beta - \delta)]$$

Using the trigonometric identity $\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$, we

can rewrite the above expression as

$$\begin{aligned} E &= 2A \left(\frac{\sin \beta}{\beta} \right) \cos \left[(\omega t - \beta) - \frac{\delta}{2} \right] \cos \left(\frac{\delta}{2} \right) \\ &= 2A \left(\frac{\sin \beta}{\beta} \right) \cos(\omega t - \beta - \gamma) \cos \gamma \end{aligned} \quad (10.17)$$

where, using Eq. (10.15), we have

$$\gamma = \frac{\delta}{2} = \frac{\pi}{\lambda} d \sin \theta \quad (10.18)$$

Now, you know that the intensity is given as the square of the amplitude. So, using Eq. (10.17), we can write the intensity at point P_θ as

$$I_\theta = 4A^2 \left(\frac{\sin \beta}{\beta} \right)^2 \cos^2 \gamma \quad (10.19)$$

For $\theta = 0$, we note from Eqs. (10.6) and (10.18) that, both β and γ vanish. So, at the centre (bright fringe), the intensity is

$$I_{\theta=0} = 4A^2 = 4I_0$$

Thus, the expression for intensity [Eq. (10.19)] of double slit diffraction pattern can be written as

$$I_\theta = 4I_0 \left(\frac{\sin \beta}{\beta} \right)^2 \cos^2 \gamma \quad (10.20)$$

Since the maximum value of I_θ is $4I_0$, we see that the double slit provides **four times** as much intensity in the central maximum as the single slit. This is exactly what you should have expected since the diffracted rays reaching the central point P_0 are in phase and amplitudes superpose constructively.

If you closely examine Eq. (10.19) you will recognise that the term $(\sin^2 \beta)/\beta^2$ represents the diffraction pattern produced by a single slit of width b [refer to Eq. (10.9)]. The $\cos^2 \gamma$ term represents the interference pattern produced by two diffracted beams (of equal intensity) having phase difference δ . **That is, the intensity of double slit diffraction pattern is product of the irradiances observed for the double-slit interference and single slit diffraction.** For $a > b$, the $\cos^2 \gamma$ factor will vary more rapidly than the $(\sin^2 \beta)/\beta^2$ factor. Then, **we obtain Young's interference pattern for slits of very small widths.** In general, the product of sine and cosine factors may be considered as a modulation of the interference pattern by a single slit diffraction envelope. We shall discuss it in detail a little later.

Before we investigate the positions of maxima and minima in the double slit diffraction pattern, let us understand the physical phenomenon that takes place. Diffracted light emerging from these two slits constitute two coherent beams. These interfere leading to the formation of fringes on the screen. But the intensity of a fringe depends upon the intensities of interfering beams and the phase difference between them when they reach the point under observation. We know that the intensities of diffracted beams are controlled by

Recall from Sec. 10.3 that when $\theta=0$, both β and $\sin \beta$ equals zero but

$$\lim_{\beta \rightarrow 0} \frac{\sin \beta}{\beta} = 1$$

the diffraction conditions and the directions of observation. **Consequently, the intensities of interference fringes are not the same at different points of the screen.** In particular, in those directions in which the intensities of diffracted beams are large, the constructive interference will lead to brighter fringes whereas in directions where the two diffracted beams themselves have lower intensities even their constructive interference will lead to faint fringes.

You should note that we have described the phenomenon as **interference between two diffracted beams**. So, you may ask: How do we distinguish between the two words interference and diffraction which we have used? When secondary wavelets originating from different parts of the same wavefront are made to superimpose, we call it diffraction. Such a case arises when we consider all the wavelets arising from the various points situated in the aperture between the two jaws of a slit. But when two separate beams coming from two different slits are superimposed, we call it interference. It should be clear that in all cases where we apply the principle of superposition, the wavelets have to be coherent in nature to produce an observable pattern.

Before you proceed further, you may like to answer an SAQ.

SAQ 3 - Double slit pattern for non-monochromatic source

If instead of a monochromatic slit source, we use a source emitting two wavelengths, λ_1 and λ_2 ($< \lambda_1$), how will the double slit diffraction pattern get influenced?

Positions of Minima and Maxima

To study the position of minima and maxima in the double slit pattern, we use Eq. (10.20):

$$I_\theta = 4I_0 \left(\frac{\sin \beta}{\beta} \right)^2 \cos^2 \gamma$$

Note that the intensity I_θ will be zero when either $(\sin \beta / \beta)^2$ or $\cos^2 \gamma$ is zero. From Sec. 10.3, recall that the diffraction factor $(\sin \beta / \beta)^2$ will be zero for

$$\beta = \frac{\pi b \sin \theta}{\lambda} = \pi, 2\pi, 3\pi, \dots, m\pi; \quad (m \neq 0)$$

$$\text{or} \quad b \sin \theta = \lambda, 2\lambda, 3\lambda, \dots, m\lambda \quad (10.21)$$

Eq. (10.21) specifies the directions along which the available intensity of either beam is zero by virtue of diffraction taking place at each slit.

The second factor $(\cos^2 \gamma)$ is an interference term and will be zero [Eq. (10.18)] when

$$\gamma = \frac{\pi d \sin \theta}{\lambda} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots, \left(n + \frac{1}{2}\right)\pi$$

$$\text{or} \quad d \sin \theta = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \dots, \left(n + \frac{1}{2}\right)\lambda \quad (10.22)$$

Eq. (10.22) gives the values of angle θ for the intensity to be zero by virtue of destructive interference between two beams. You may recall from Unit 6 of Block 2 that this is the same as the condition for the minimum of the

interference pattern between two point sources. So, Eqs. (10.21) and (10.22) specify the direction where the intensity is zero.

We cannot obtain the exact positions of the maxima by any simple relation. This is because we have to find the maximum of a function which is product of two terms. But we can find their approximate positions if we assume that $(\sin\beta/\beta)$ does not vary appreciably over a given region. We are quite justified in making this approximation if the slits are very narrow (see Fig. 10.8). Note that we observe the maxima near the centre of the pattern. Under these conditions, the positions of maxima are solely determined by the $\cos^2 \gamma$ factor. You know that this factor defines maxima for

$$\gamma = 0, \pi, 2\pi, \dots, n\pi$$

$$\text{or} \quad d \sin\theta = 0, \lambda, 2\lambda, \dots, n\lambda \quad (10.23)$$

We know that $d \sin\theta$ represents the path difference between the corresponding points in the two slits. When this path difference is a whole number of wavelengths, constructive interference occurs between the two beams. Then, we get a maximum which leads to the formation of a series of bright fringes. The central fringe corresponds to $d \sin\theta = 0$. The n th fringe (on either side of the central fringe) occurs when $d \sin\theta = n\lambda$. We, therefore, say that n represents the order of interference.

Missing Orders

From Eq. (10.20), you know that the expression for intensity

$$I_\theta = 4I_0 \left(\frac{\sin\beta}{\beta} \right)^2 \cos^2 \gamma, \text{ we have } \beta = \frac{\pi b \sin\theta}{\lambda} \text{ and } \gamma = \frac{\pi d \sin\theta}{\lambda}. \text{ Thus, we see}$$

that β and γ are not independent. These are connected to each other through the relation

$$\frac{\gamma}{\beta} = \frac{\pi d \sin\theta}{\pi b \sin\theta} = \frac{d}{b} = \frac{a+b}{b} \quad (10.24)$$

Cases of special interest arise when d is an integral multiple of b , say it is an integer p so that $d = pb$. This will happen when the opaque portion a is an integral multiple of the transparent part b . The possibilities are: $a = b$, $a = 2b$ or $a = 3b$ etc, so that $d/b = p = 2, 3, 4, \dots$ in these cases. Under these conditions, the directions of diffraction minimum and interference maximum will necessarily coincide. To show this, let us assume that a direction of diffraction minimum is given by

$$b \sin\theta = m\lambda$$

We will automatically have the interference maximum in this direction when $d = pb$ since

$$\begin{aligned} d \sin\theta &= (pb) \sin\theta = p (b \sin\theta) \\ &= pm \lambda = n \lambda \end{aligned}$$

where $n = pm$. The possible values of p are 2, 3, 4, ... and those of m are 1, 2, 3, ... Thus the n th order interference fringes for which $n = pm$ will have zero intensity since the intensity of both beams is zero by virtue of diffraction

condition. As a result their constructive interference also leads to net zero intensity. These are usually known as **missing orders**. For example, when $p = 2$, we will have 2, 4, 6, 8, ... orders missing for m values of 1, 2, 3, ... etc. Similarly, when $p = 3$, we will have 3, 6, 9, ... order missing and so on.

The special case when $d/b = 1$ means that the opaque part $a = 0$ and the two slits exactly join one another. Then we find that all the interference orders are missing. Actually this means that we now have a single slit of double width and consequently we get a single slit diffraction pattern (with no interference fringes).

These ideas are illustrated in the following example.

EXAMPLE 10.4 : MINIMA AND FRINGE WIDTH OF DOUBLE SLIT DIFFRACTION

Consider a double slit diffraction arrangement with $b = 7.0 \times 10^{-3} \text{ cm}$, $d = 3.5 \times 10^{-2} \text{ cm}$ and $\lambda = 6300 \text{ \AA}$. How many interference minima will occur between the diffraction minima on either side of the central maximum? If a screen is placed at a distance of 5 m from the diffracting aperture, what is the fringe width?

SOLUTION ■ The first diffraction minima on either side of central maximum (i.e. $\theta = 0$) will occur when $b \sin \theta = \pm \lambda$. That is, for $\sin \theta = \pm \lambda / b = 9 \times 10^{-3}$. The interference minima will occur when Eq. (10.22) is satisfied, i.e. when

$$d \sin \theta = \left(n + \frac{1}{2} \right) \lambda$$

On substituting the given values, we find that

$$\sin \theta = \left(n + \frac{1}{2} \right) \frac{\lambda}{d} = \left(n + \frac{1}{2} \right) \times (1.8 \times 10^{-3}); \quad n = 0, 1, 2, \dots$$

Thus, the possible values of $\sin \theta$ which are less than 9×10^{-3} , the value for first diffraction minima, are obtained by taking $n = 1, 2, 3, \dots$ in the above expression for $\sin \theta$. That is,

$$\sin \theta = 0.9 \times 10^{-3}, 2.7 \times 10^{-3}, 4.5 \times 10^{-3}, 6.3 \times 10^{-3} \text{ and } 8.1 \times 10^{-3}$$

Thus, we find that there are five interference minima between central maximum and first diffraction minima. This implies that there will be a total of ten interference minima between the two first order diffraction minima. If θ is small, we have $\sin \theta \cong \theta$ and we may write

$$\theta_1 = 0.9 \times 10^{-3} \text{ rad}, \theta_2 = 2.7 \times 10^{-3} \text{ rad},$$

$$\theta_3 = 4.5 \times 10^{-3} \text{ rad}, \theta_4 = 6.3 \times 10^{-3} \text{ rad}, \theta_5 = 8.1 \times 10^{-3} \text{ rad}$$

and the angle between successive interference minima is $1.8 \times 10^{-3} \text{ rad}$.

Since the screen is placed at a distance of 5 m from the aperture, we can write the fringe width as

$$f \Delta \theta = (500 \text{ cm}) \times 1.8 \times 10^{-3} = 0.9 \text{ cm}.$$

Graphical Representation

To understand the nature of the double slit diffraction pattern, we can plot $\cos^2 \gamma$, $(\sin^2 \beta / \beta^2)$, and their product separately. But, before doing that, we must decide on the relative scale of the abscissas γ and β since the shape of the pattern will depend upon this choice. We have already shown [Eq. (10.24)] that γ/β is equal to d/b . Let us say that in a particular case, $\gamma/\beta = d/b = 3$. We must then plot the proposed curves for $\gamma = 3\beta$.

In Fig. 10.13, the curves (a) and (b) are plotted to the same scale of θ .

Fig. 10.13a depicts the curve for $\cos^2 \gamma$ which gives a set of equidistance

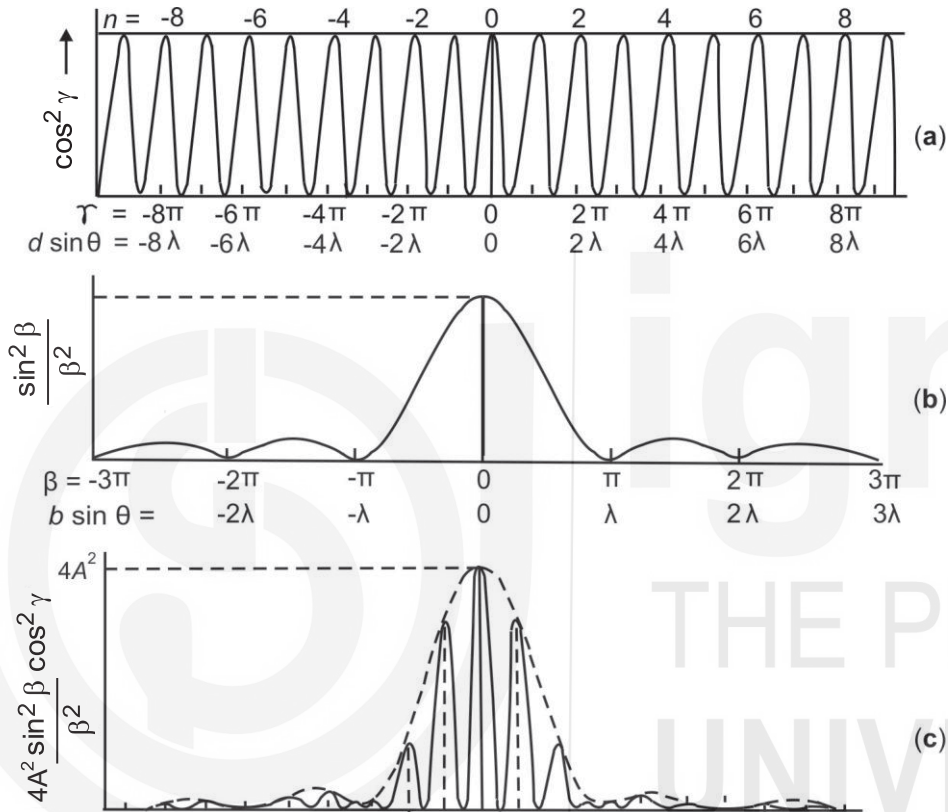


Fig. 10.13: Intensity curves for double slit diffraction pattern for $\gamma = 3\beta$: a) plot of $\cos^2 \gamma$, b) plot of $(\sin^2 \beta) / \beta^2$; c) plot of (a) and (b) together.

maxima of equal intensity located at $\beta = 0, \pm \pi, \pm 2\pi, \pm 3\pi, \dots$. In Fig. 10.13b, we have plotted $(\sin \beta / \beta)^2$ which gives a maximum at $\beta = 0$ and minima at $\beta = \pm \pi, \pm 2\pi, \dots$. In Fig. 10.13c we have plotted their product. What do you observe? The intensity of the fringes in the resultant pattern is not the same as it was in Fig. 10.13a. It is modulated (reduced) by the factor $\sin^2 \beta / \beta^2$. This means that the central fringe or the zeroth order fringe is the brightest and the successive fringes are of decreasing intensity until we reach the point $\beta = \pi$ where the intensity is zero. Thus, the third fringe corresponding to $\cos^2 \gamma = \pm 3\pi$ falls at $\beta = \pi$ or $-\pi$ and their product is zero. Therefore, the third fringe on either side of the central maxima has zero intensity and its location at the angle θ satisfies simultaneously

$$\beta = \pm \pi \quad \text{and} \quad \gamma = \pm 3\pi$$

$$\text{or} \quad b \sin \theta = \pm \lambda \quad \text{and} \quad d \sin \theta = \pm 3\lambda$$

This third fringe will therefore be missing. We will observe the 4th and 5th fringes. We can argue in a similar manner that for 6th fringe

$$\beta = \pm 2\pi \text{ and } \gamma = \pm 6\pi$$

which will therefore have zero intensity and thus be missing.

You may now like to answer the following SAQ.

SAQ 4 - Missing order in double slit pattern

Write down the general condition for missing orders in terms of the ratio d/b .

Let us now summarise what you have learnt in this unit.

10.5 SUMMARY

Concept	Description
Fraunhofer diffraction	To observe Fraunhofer diffraction pattern, the distance of the diffracting object from the source and observation screen should be infinite. This condition is achieved in experimental set up by using converging lenses.
Single slit pattern	The Fraunhofer diffraction pattern of a single slit illuminated by a point source consists of a horizontal streak of light. This horizontal diffraction pattern may be regarded as a spread out image of the point source and consists of a series of diffraction spots symmetrically situated with respect to central point. The central spot (principal maximum) in the pattern has maximum intensity and its width is twice compared to other spots (secondary maxima) which are of equal width. The intensity of secondary maxima decrease rapidly. In fact, most of the light is concentrated in the principal maximum. The plane wavefront incident on the slit gives rise to a system of vertical plane diffracted wavefronts which originate from each point of the diffracting aperture.
Intensity of the single slit pattern	The intensity in the pattern due to single slit at any point P_θ on the screen is computed by taking into account the phase difference between the successive diffracted waves. The intensity at a point P_θ is given by $I_\theta = I_0 \left(\frac{\sin \beta}{\beta} \right)^2$ where $\beta = \pi \frac{b \sin \theta}{\lambda}$, b is width of the slit and I_0 is the intensity of the principal maximum.
Minima in single slit pattern	If the path difference $b \sin \theta$ between waves diffracted by extreme ends of the slit is an integral multiple of λ, we obtain minima (zero intensity) in the pattern.
Slit source for single slit diffraction	The Fraunhofer diffraction pattern of a single slit (as aperture) illuminated by a slit source of light consists of a series of vertical fringes. In this pattern, the central vertical fringe is the brightest and the intensity of other fringes decreases rapidly. The width of central fringe is double of that for other fringes.

Double slit diffraction

- The double slit diffraction pattern consists of a number of equally spaced fringes similar to what is observed in interference experiments. These fringes are the brightest in the central part of the pattern.

The central maximum in double slit pattern is four times brighter than that in single slit pattern.

Intensity of the double slit pattern

- The intensity of double slit diffraction pattern at an angle θ is given by

$$I_{\theta} = 4I_0 \frac{\sin^2 \beta}{\beta^2} \cos^2 \gamma$$

Here $I_0 = A^2$, $\beta = \frac{\pi b \sin \theta}{\lambda}$ and $\gamma = \frac{\pi}{\lambda} d \sin \theta$, where b is slit width and d is the distance between two similar points in the two slits. It is equal to $a + b$, where a is the width of the intervening opaque space between the two slits.

The intensity of double slit diffraction pattern is product of the irradiances observed for the double slit interference and single slit diffraction. Physically, it arises due to interference between two diffracted beams.

Maxima and minima in double slit pattern

- The conditions of maxima and minima in double slit interference pattern are:

$$d \sin \theta = n\lambda \quad (\text{maxima})$$

$$d \sin \theta = \left(n + \frac{1}{2}\right)\lambda \quad (\text{minima})$$

and the condition for diffraction minima is

$$b \sin \theta = m\lambda \quad (\text{minima})$$

10.6 TERMINAL QUESTIONS

1. A single slit has a width of 0.03 mm. A parallel beam of light of wavelength 5500 Å is incident normally on it. A lens is mounted behind the slit and focussed on a screen located in its focal plane, 100 cm away. Calculate the distance of the third minimum from the centre of the diffraction pattern of the slit.
2. For observing Fraunhofer diffraction of a single slit, the screen is placed at a distance of 1.5 m from the lens to obtain the pattern in its focal plane. The first minima of the pattern is at a distance of 0.5 cm on either side of the central maximum when plane wavefronts of the light of wavelength 6×10^{-5} cm are incident on the slit. Calculate the width of the slit.
3. The Fraunhofer diffraction pattern of a double slit is obtained on a screen placed at a distance of 1.5 m from the slits. The width of each slit is 0.03 mm and the wavelength of the light used is 5×10^{-5} cm. Calculate the distance between the central maximum and the first diffraction minimum of the pattern.
4. In the experimental set up to observe Fraunhofer diffraction of a double slit, the monochromatic source of light is replaced by white light. How will it affect the fringe pattern?

5. In a double slit diffraction set up, the wavelength of light used is $6 \times 10^{-5} \text{ cm}$, slit width is 0.02 mm and the spacing between the slits is 0.15 mm . If the distance of the screen from the slits is 1.0 m , calculate the distance between any two consecutive dark fringes.

10.7 SOLUTIONS AND ANSWERS

Self-Assessment Questions

1. We know that the condition for minima in the diffraction pattern of single slit is given as [Eq. (10.11)]:

$$b \sin \theta = m \lambda; \quad m = \pm 1, \pm 2, \dots$$

Thus, the position of first minima on the either side will be given by above condition for $m = \pm 1$. These minima will decide the angular spread of the central maximum. Thus, angular spread of the central maximum is from

$$\theta = \sin^{-1}(\lambda / b) \text{ to } \theta = -\sin^{-1}(\lambda / b).$$

For small θ , we have $\sin \theta = \theta$ and we find that principal maximum is spread from $\theta = (\lambda / b)$ to $\theta = -(\lambda / b)$. So, the angular width of the principal maximum is $(\lambda / b) + (\lambda / b) = (2\lambda / b)$.

Similarly, you can show that the first secondary maximum ($m = \pm 2$) on the positive side extends from $\theta = (\lambda / b)$ to $\theta = (2\lambda / b)$ and on the negative side from $\theta = -(\lambda / b)$ to $\theta = -(2\lambda / b)$. Thus, the angular width of the first secondary maximum on either side of the principal maximum is $(2\lambda / b) - (\lambda / b) = (\lambda / b)$.

Thus we see that the principal (central) maximum is twice as wide as a secondary maxima.

2. We know that the condition for minima in the diffraction pattern of single slit is

$$b \sin \theta = m \lambda; \quad m = \pm 1, \pm 2, \dots$$

So, the angular position of first minima ($m = 1$) is

$$b \sin \theta_1 = \lambda$$

$$\therefore (0.3 \times 10^{-1} \text{ cm}) \sin \theta_1 = 4358 \times 10^{-8} \text{ cm}$$

$$\text{or} \quad \sin \theta_1 = 1.45 \times 10^{-3}$$

In the small angle approximation $\sin \theta \approx \theta$ we can write

$$\theta_1 = 1.45 \times 10^{-3} \text{ rad}$$

$$\text{and} \quad \theta_2 = 2.90 \times 10^{-3} \text{ rad}$$

On comparing these values of the angular positions of the first and second minima with those calculated in Example 10.1 for light of wavelength $6000 \text{ \AA}$, we find that the values are lower for violet light. Since the position of minima determines the width of maxima, we can conclude that the diffraction pattern has shrunk for violet light. The violet light is diffracted about 27% less.

3. From Eq. (10.11), we note that the position of minima in the diffraction pattern is dependent on the wavelength of the light used. Therefore, λ_1 and λ_2 both will give their own diffraction patterns within which we will get the interference fringes. The pattern for λ_1 will be smaller (less wider) if $\lambda_2 < \lambda_1$. However, both these patterns will be superimposed on one another coinciding at $\theta = 0$.
4. 'Missing orders' refers to the order of the interference maxima which are absent in the double slit diffraction pattern. It is caused when the directions of diffraction minimum and interference maximum in the double slit pattern coincide. So, to obtain the general condition for the missing order fringes, we note from Eqs. (10.6) and (10.18) that

$$\frac{\gamma}{\beta} = \frac{\pi d \sin \theta}{\pi b \sin \theta} = \frac{d}{b} = \frac{a+b}{b}$$

So, the general conditions for missing orders in terms of γ and β are:

$$\gamma = \pm n\pi \text{ or } d \sin \theta = \pm n\pi \text{ and } \beta = \pm m\pi \text{ or } b \sin \theta = \pm m\pi.$$

Therefore, when $n = pm$ (n, m, p are all integers), we get

$$\frac{d}{b} = \frac{n}{m} = p$$

The missing orders occur when d/b is an integer. So, for $(d/b) = 2$, second, fourth, sixth, ... orders will be missing. What do you say about $(d/b) = 3$?

The case $d/b = 1$ is interesting. This condition implies that the two slits exactly join. In this case, therefore, all the interference orders will be missing! Can you guess what does it signify physically? It means that we have a single slit of double width ($2b$) and consequently no interference pattern at all.

Terminal Questions

1. From Eq. (10.11) we know that the conditions for minima are given by

$$b \sin \theta = m\lambda; \quad m = \pm 1, \pm 2, \dots$$

As per the problem, $b = 0.03 \text{ mm} = 3 \times 10^{-3} \text{ cm}$, $m = 3$ and $\lambda = 5500 \text{ \AA}$.

Thus, we have

$$\sin \theta = \frac{m\lambda}{b} = \frac{3 \times (5500 \times 10^{-8} \text{ cm})}{3 \times 10^{-3} \text{ cm}} = 5.5 \times 10^{-2}$$

In the small angle approximation, $\sin \theta \cong \theta \cong \tan \theta$. So, the distance x of the third minimum from the central maximum is

$$x = 5.5 \times 10^{-2} \times (100 \text{ cm}) = 5.5 \text{ cm}$$

2. The conditions for minima in single slit pattern is

$$b \sin \theta = m\lambda; \quad m = \pm 1, \pm 2, \dots$$

As per the problem, $m = 1$, $\lambda = 6 \times 10^{-5} \text{ cm}$, and distance of the first minimum, say, $x = 0.5 \text{ cm}$, and distance of screen from the lens is 1.5 m . So, under small angle approximation, we can write,

$$\sin \theta \cong \theta \approx \tan \theta$$

$$\text{and } \tan \theta = \frac{0.5 \text{ cm}}{150 \text{ cm}} \approx \sin \theta$$

So,

$$b = \frac{m\lambda}{\sin \theta} = \frac{1 \times (6 \times 10^{-5} \text{ cm}) \times (150 \text{ cm})}{0.5 \text{ cm}} = 0.18 \text{ mm}$$

3. The conditions for diffraction minima in the double slit pattern is

$$b \sin \theta = m\lambda; \quad m = \pm 1, \pm 2, \dots$$

For the first diffraction minimum, $m = 1$. So,

$$\sin \theta = (\lambda / b).$$

In small angle approximation,

$$\sin \theta \approx \theta \approx \tan \theta = (\lambda / b) \quad (i)$$

Since the screen is 150 cm from the slits, the distance x of the first minimum from central maximum ($\theta = 0$) is

$$\tan \theta = \frac{x}{150 \text{ cm}} \quad (ii)$$

So, from Eqs. (i) and (ii), we have

$$\frac{\lambda}{b} = \frac{x}{150 \text{ cm}} \Rightarrow x = \frac{(150 \text{ cm}) \times (5 \times 10^{-5} \text{ cm})}{0.003 \text{ cm}} = 2.5 \text{ cm} \quad (iii)$$

4. Since the direction (and hence, position) of interference maxima and minima depends on the wavelength of light, each wavelength will produce its own interference fringe pattern. However, the central fringe for all wavelengths will coincide and hence the central fringe will be white. Fringes of order $n = 1, 2, 3, \dots$ located on either side of the central fringe, at different θ values given by $d \sin \theta = n\lambda$ for different wavelengths will be coloured.
5. In the double slit diffraction pattern, the conditions for dark fringes are given as [Eq. (10.22)]:

$$d \sin \theta = \left(n + \frac{1}{2}\right)\lambda; \quad n = 0, 1, 2, \dots$$

Let θ_1 and θ_2 be the angles corresponding to the first and second dark fringes, respectively. Then, we have (under small angle approximation),

$$d \sin \theta_1 = \frac{\lambda}{2} \Rightarrow \theta_1 = \frac{\lambda}{2d} \quad (i)$$

$$\text{and } d \sin \theta_2 = \frac{3\lambda}{2} \Rightarrow \theta_2 = \frac{3\lambda}{2d} \quad (ii)$$

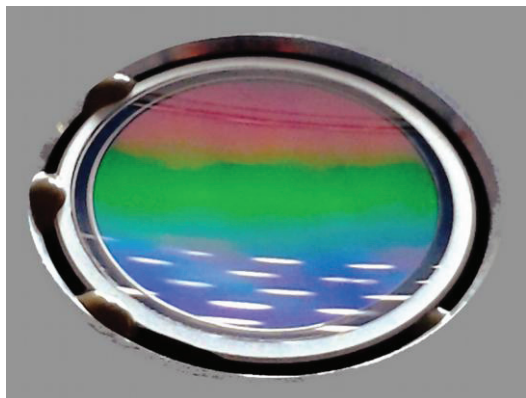
If x_1 and x_2 be the distances of first and second dark fringes from the central fringe, respectively, we can write,

$$\theta_1 = \frac{x_1}{100 \text{ cm}} \Rightarrow x_1 = \frac{(100 \text{ cm}) \times \lambda}{2d} \quad [\text{from Eq. (i)}]$$

$$\text{and } \theta_2 = \frac{x_2}{100 \text{ cm}} \Rightarrow x_2 = \frac{(100 \text{ cm}) \times 3\lambda}{2d} \quad [\text{from Eq. (ii)}]$$

So, the distance x between two consecutive (first and second) minima is

$$x = x_2 - x_1 = \frac{(100 \text{ cm}) \lambda}{2d} [3 - 1] = \frac{2 \times (100 \text{ cm}) \times (6 \times 10^{-5} \text{ cm})}{2 \times (0.015 + 0.002) \text{ cm}} = 0.35 \text{ cm}.$$



Diffraction grating is an optical device used in monochromator, which select a narrow band of wavelength of light from a polychromatic beam. The grating separates the incident beam into its constituent colours at different wavelengths.

(Source: commons.wikimedia.org)

DIFFRACTION GRATING

Structure

- | | | | |
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| | Angular Half-Width of Principal Maxima | | |

STUDY GUIDE

In this unit, we have discussed a very useful optical device called grating which works on diffraction of light. A grating is an array of a large number of identical vertical slits very close to each other. So, diffraction of light from grating can be analysed theoretically by considering the diffraction of light from N slits, where N is the number of slits. While discussing diffraction from a grating, we shall repeatedly refer to the concepts and results obtained in Unit 10 for diffraction from a single slit and a double slit. So, you should read Unit 10 once again before reading this unit. You should focus on the physical significance of the mathematical expressions. Also, you should note and understand the difference between terms like principal maxima (central maximum), minima and secondary maxima. The N slit diffraction (or diffraction by a grating) pattern comprises interference pattern as well as diffraction pattern. You will be able to appreciate this feature better if you revise Unit 6 of the course where we have discussed interference of light. You should also focus on the characteristics of N slit diffraction pattern which enables us to use diffraction grating for spectral analysis. The mathematical treatment of this unit is similar to the one followed in Unit 10. So, we expect that you should not face any difficulty if you have studied Unit 10 carefully.

“Concentrate all your thoughts upon the work at hand. The Sun’s rays do not burn until brought to a focus.”

***Alexander
Graham Bell***

11.1 INTRODUCTION

You have learnt in Unit 10 about Fraunhofer diffraction produced by a single slit aperture. When a narrow vertical slit is illuminated by a distant point source, the Fraunhofer diffraction pattern is produced which consists of a series of spots situated symmetrically about a central spot, along a horizontal line. The intensity of the central spot (called central or principal maximum) is maximum and it decreases rapidly as we move away from the central spot. You have also learnt the qualitative features of the observed double slit diffraction pattern and compared these with those of a single slit pattern. A distinctive feature of the double slit pattern is that it consists of bright and dark fringes similar to those observed in interference experiments. You might have learnt in your school physics that diffraction phenomenon limit the ability of optical devices to form sharp and distinct images of objects. This restriction at one time hampered the spectroscopic work particularly for substances whose spectrum consisted of doublets. The problem could be overcome by an optical device called diffraction grating. A diffraction grating comprises a large number of equally spaced slits and this structure diffracts light in such a way that the individual wavelength making up the incident light are separated. Grating is a powerful scientific tool which enables scientists to determine structure of atoms or helping astronomers determine the chemical composition of stars and galaxies. Fraunhofer first used diffraction grating in 1819 to observe the spectrum of the Sun. In this unit, you will learn about diffraction grating.

We begin this unit with a discussion, in Sec. 11.2, on the intensity distribution in the Fraunhofer diffraction pattern when the number of slits is large. This discussion will be an extension of the intensity distribution for double slit you learned in the previous unit. You will observe that as the number N of slits increases, interference maxima get narrower (sharper). For sufficiently large value of N , interference maxima become narrow lines. In Sec. 11.3, you will learn how the results for N slit diffraction provided a basis for the development of a useful optical device called diffraction grating. You will also discover why diffraction gratings are an excellent tool in spectral analysis. The occurrence of diffraction grating effects in nature is surprisingly common. Do you know that the green on the neck of a male mallard duck and the beautiful colours of the 'eye' of the peacock's feathers are also due to diffraction grating effects? The layered structure in cat's retina acts as reflection grating and is responsible for metallic green reflection at night.

Expected Learning Outcomes

After studying this unit, you should be able to:

- ❖ derive expression for the intensity distribution in the diffraction pattern for N equally spaced slits;
- ❖ obtain conditions for the position of principal maxima, minima and secondary maxima in the N slit pattern;
- ❖ show that the principal maxima of N slit pattern becomes sharper as N increases; and
- ❖ describe the use of a diffraction grating in spectral analysis.

11.2 FRAUNHOFER DIFFRACTION FROM N IDENTICAL SLITS

In Unit 10, you have learnt that interference of waves diffracted by individual slits determines the intensity distribution in the double slit pattern. Let us now consider how will the Fraunhofer diffraction pattern be affected if we replace the double slit by, say, N (> 2) vertical slits. Refer to Fig. 11.1 which depicts the experimental arrangement for obtaining the diffraction pattern for N slits. Note that the experimental set up is same as for double slit with the modification that the double slit has been replaced by N slits.

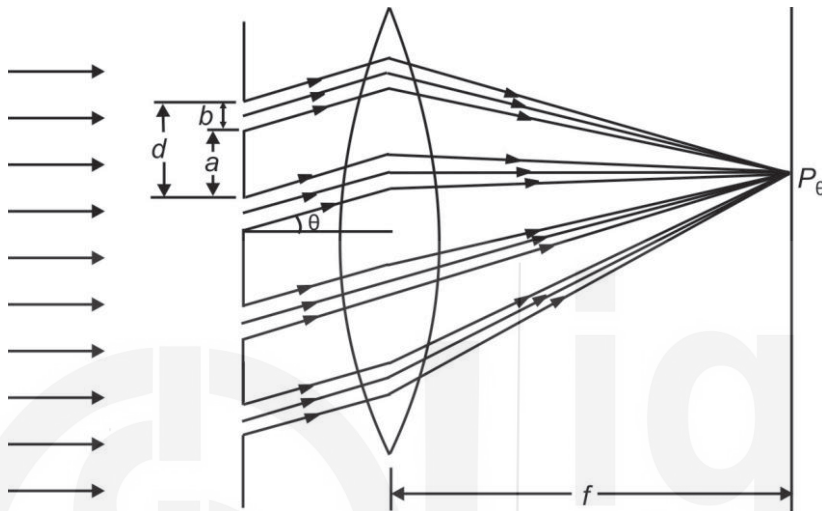


Fig. 11.1: Cross-sectional view of the experimental set up for Fraunhofer diffraction of a plane wave incident normally on a multiple (N) slit aperture.

To keep the discussion simple, we assume that (i) each slit is of width b and has the same length (ii) all slits are parallel to each other and (iii) the intervening opaque space between any two successive slits is the same, equal to a . An arrangement consisting of a large number of parallel and equidistant narrow rectangular slits of the same width is called **diffraction grating**. Note that the distance between any two equivalent points in two consecutive slits is $a + b$. Let us denote it by d which is called **grating element**. We take the source of light to be in the form of a slit and adjust the length of this source slit to be vertical and parallel to the length of N slits. As discussed in the double slit pattern, the diffraction pattern obtained on the screen consists of vertical fringes parallel to the slit source and these fringes are similar to that for double slit (refer to Sec. 10.3 of Unit 10). We now wish to study the theoretical basis for the intensity distribution in the N slit diffraction pattern.

11.2.1 Intensity Distribution

To obtain an expression for the intensity distribution, we will follow the procedure and arguments similar to those used for the intensity distribution in the double slit pattern (Sec. 10.3, Unit 10). Consider a point source of light which sends out plane waves. That is, a plane wavefront is incident on the N slit arrangement (grating) shown in Fig. 11.1. (In terms of ray-optics, we may say that light rays fall normally on the grating.) You may recall from

Sec. 10.2 of Unit 10 that **the intensity distribution along any section perpendicular to the vertical fringes formed due to a slit source will be the same as obtained from a point source**. Physically, light emerging from N slits results in N diffracted beams due to diffraction at each slit. Since these beams are coherent, interference takes place between them resulting in the formation of fringes. However, you must note that diffraction controls the intensity of beam from each slit in a given direction.

As in the case of double slit, we consider that the diffracted rays proceed towards point P_θ , where θ is the angle between the diffracted rays and the normal to the plane containing the slits. Let $E_1, E_2, E_3, \dots, E_N$, respectively, denote the magnitude of electric fields associated with the diffracted rays produced by the first, the second, the third ... and the N th slit at the point P_θ .

Then we have [refer to Eq. (10.10)]:

$$\begin{aligned} E_1 &= A \frac{\sin \beta}{\beta} \cos(\omega t - \beta) \\ E_2 &= A \frac{\sin \beta}{\beta} \cos(\omega t - \beta - \delta) \\ E_3 &= A \frac{\sin \beta}{\beta} \cos(\omega t - \beta - 2\delta) \\ &\vdots \\ E_N &= A \frac{\sin \beta}{\beta} \cos(\omega t - \beta - (N-1)\delta) \end{aligned}$$

where,

$\beta = (\pi b \sin \theta / \lambda)$ and $\delta = (2\pi / \lambda) d \sin \theta$ is the phase difference between the fields from two consecutive slits (refer to Sec. 10.3 of Unit 10). Also, we have assumed that the phase changes by equal amount δ from one slit to the next. So, we obtain the resultant field E at P_θ by summing these N terms:

$$\begin{aligned} E &= A \frac{\sin \beta}{\beta} \cos(\omega t - \beta) + A \frac{\sin \beta}{\beta} \cos(\omega t - \beta - \delta) \\ &\quad + A \frac{\sin \beta}{\beta} \cos(\omega t - \beta - 2\delta) + \dots \\ &\quad + A \frac{\sin \beta}{\beta} \cos[\omega t - \beta - (N-1)\delta] \end{aligned} \quad (11.1)$$

We can write it as

$$E = A \frac{\sin \beta}{\beta} [\cos(\omega t - \beta) + \cos(\omega t - \beta - \delta) + \dots + \cos[\omega t - \beta - (N-1)\delta]]$$

When we sum the above series (see the margin remark), we get

$$E = A \left(\frac{\sin \beta}{\beta} \right) \frac{\sin N\gamma}{\sin \gamma} \cos \left(\omega t - \beta - \frac{1}{2}(N-1)\delta \right) \quad (11.2)$$

where

$$\gamma = \frac{\delta}{2} = \frac{\pi}{\lambda} d \sin \theta \quad (11.3)$$

So, the intensity I_θ of the resultant pattern is obtained by squaring the amplitude of the resultant field E [Eq. (11.2)] :

$$I_\theta = A^2 \frac{\sin^2 \beta}{\beta^2} \frac{\sin^2 N\gamma}{\sin^2 \gamma} \quad (11.4)$$

You know that in complex number notation,

$$\exp i(\theta) = \cos \theta + i \sin \theta \quad (i)$$

so that

$$\operatorname{Re} [\exp i(\theta)] = \cos \theta \quad (ii)$$

Thus, we can write

$$\cos(\omega t - \beta) = \operatorname{Re}[e^{i(\omega t - \beta)}]$$

$$\cos(\omega t - \beta - \delta)$$

$$= \operatorname{Re}[e^{i(\omega t - \beta - \delta)}]$$

and

$$\cos(\omega t - \beta - (N-1)\delta)$$

$$= \operatorname{Re}[e^{i(\omega t - \beta - (N-1)\delta)}]$$

Therefore, we can write

$$\begin{aligned} &\cos(\omega t - \beta) + \\ &\quad \cos(\omega t - \beta - \delta) + \dots \\ &= \operatorname{Re}[e^{i(\omega t - \beta)} + e^{i(\omega t - \beta - \delta)} \\ &\quad + \dots + e^{i(\omega t - \beta - (N-1)\delta)}] \quad (iii) \end{aligned}$$

(Contd...)

Eq. (11.4) gives the resultant intensity in the diffraction pattern due to N -slits. We expect it to be true for any number of slits. Note that, for a single slit, $N = 1$ and Eq. (11.4) reduces to

$$I_{\theta} = A^2 \frac{\sin^2 \beta}{\beta^2}$$

which is the same as Eq. (10.9). Now, you should answer a simple SAQ before proceeding further.

SAQ 1 - Intensity of a multiple slit pattern

Show that for $N = 2$, Eq. (11.4) reduces to Eq. (10.11) for the double slit.

After having obtained the expression for intensity distribution for N slit diffraction pattern, let us now examine this expression and determine the conditions for positions of principal maxima, minima and secondary maxima in this pattern.

11.2.2 Positions of Principal Maxima

For obtaining the positions of maxima, let us re-examine Eq. (11.4). We note that the intensity distribution is a product of two terms; the first term ($\sin^2 \beta / \beta^2$) represents the diffraction term produced by a single slit whereas the second term ($\sin^2 N\gamma / \sin^2 \gamma$) represents the interference term (for N slits). **The interference term controls the width of interference fringes, while the diffraction term governs their intensities.**

As in case of the double slit, we cannot locate the exact positions of maxima; their approximate positions can, however, be obtained by neglected the variation of diffraction term $\sin^2 \beta / \beta^2$. This is quite justified for very narrow slits. **Therefore, for obtaining the positions of maxima we consider only the interference term.**

We will now show that the maximum value of $\frac{\sin^2 N\gamma}{\sin^2 \gamma}$ is N^2 which occurs for

$\gamma = 0, \pi, 2\pi, \dots, n\pi$. At the first glance, you will note that the quotient of the interference term becomes indeterminate at these values. To overcome such a situation, we compute the first derivative of the numerator as well as the denominator separately before inserting the value of argument and taking the limit $\gamma = n\pi$. So, following this procedure, you will readily obtain

$$\lim_{\gamma \rightarrow n\pi} \frac{\sin N\gamma}{\sin \gamma} = \lim_{\gamma \rightarrow n\pi} \frac{N \cos N\gamma}{\cos \gamma} = \pm N$$

so that for $\gamma = 0, \pi, 2\pi, \dots, n\pi$ we have

$$\left(\frac{\sin N\gamma}{\sin \gamma} \right)^2 = N^2$$

Thus, the expression for intensity [Eq. (11.4)] for $\gamma = n\pi$ takes the form

$$I_{\theta} = A^2 \frac{\sin^2 \beta}{\beta^2} N^2 = N^2 A^2 \frac{\sin^2 \beta}{\beta^2} \quad (11.5)$$

(Contd...)

The RHS can be written as

$$\begin{aligned} \text{RHS} &= e^{i(\omega t - \beta)} \\ &\quad [1 + e^{-i\delta} + e^{-2i\delta} + \dots \\ &\quad + e^{-i(N-1)\delta}] \quad (\text{iv}) \end{aligned}$$

This is a geometric series with common factor $e^{-i\delta}$ and can be summed up easily using the formula

$$S = \frac{1-r^N}{1-r}$$

$$\therefore \text{RHS} = e^{i(\omega t - \beta)} \times \frac{1 - e^{-iN\delta}}{1 - e^{-i\delta}}$$

$$\begin{aligned} &= e^{i(\omega t - \beta)} \times \\ &\quad \frac{e^{-iN\delta/2} \left(\frac{e^{iN\delta/2} - e^{-iN\delta/2}}{e^{i\delta/2} - e^{-i\delta/2}} \right)}{e^{-i\delta/2} \left(\frac{e^{i\delta/2} - e^{-i\delta/2}}{e^{i\delta/2} - e^{-i\delta/2}} \right)} \\ &= e^{i(\omega t - \beta - (N-1)\delta/2)} \\ &\quad \frac{\sin(N\delta/2)}{\sin(\delta/2)} \end{aligned}$$

Hence, LHS of Eq. (iii) is recovered by the real part of Eq. (iv) which is Eq. (11.2).

$$\text{where, } \beta = \frac{\pi b \sin \theta}{\lambda} \quad (11.6)$$

We, therefore, conclude that the positions of maxima are obtained when

$$\gamma = 0, \pi, 2\pi, \dots, n\pi \quad \text{or} \quad N\gamma = 0, N\pi, 2N\pi, \dots, nN\pi \quad (11.7)$$

Physically, at these maxima, the fields produced by each of the slits are in phase and the resultant field is N times the field due to each of the slits.

When N is large, the intensity, being proportional to N^2 , is very large and we obtain intense maxima, if only $\sin^2 \beta / \beta^2$ is not too small. Such maxima are, therefore, known as **principal maxima**.

Thus, using Eq. (11.3), the condition of **principal maxima** ($\gamma = n\pi$) can be rewritten as

$$d \sin \theta_{\max} = n\lambda \quad (11.8)$$

Let us now pause for a moment and think what does Eq. (11.8) signify physically. We can draw the following conclusions:

1. If you compare Eq. (11.8) with the corresponding equation [Eq. (10.10)] for double slit, you will note that the principal maxima in N -slit pattern correspond in position to those of the double slit.
2. The relative intensities of different orders (that is, $n = 1, 2, \dots$) are modulated by the single slit diffraction envelope [Eq. (11.5)].
3. n cannot be greater than d/λ since $|\sin \theta| \leq 1$. Can you imagine the implications of this condition? If you ponder for a while, you will realise that this condition suggests existence of only a finite number of principal maxima, which are designated as the 1st, 2nd, 3rd, ... order of diffraction. **However, there will be as many first order principal maxima as the number of wavelengths in the incident wave.**
4. The relation between β and γ obtained for double slit in terms of slit width and slit separation does not change. That is, Eq. (10.10) holds for N -slits.

11.2.3 Minima and Secondary Maxima

As in the case of locating the position of maxima, we neglect the variation of diffraction term ($\sin^2 \beta / \beta^2$) to locate the position of minima in the diffraction pattern of N slits. That is, we locate the position of the minima of the interference term. Note that the numerator in the interference term $\sin^2 N\gamma / \sin^2 \gamma$ will become zero more often than the denominator because of the factor N in the numerator. The numerator becomes zero for

$$N\gamma = 0, \pi, 2\pi, \dots, p\pi \quad \text{or} \quad \gamma = \frac{p\pi}{N}. \quad \text{Therefore, } \sin \gamma = \sin \frac{p\pi}{N} \text{ will not become zero}$$

for all integral values of p . It will become zero only for special cases when $p = 0, N, 2N, \dots$ so that γ assumes values $0, \pi, 2\pi, \dots$

But, you will recall from the previous section [Eq. (11.7)] that for these special values of γ , both $\sin N\gamma$ and $\sin \gamma$ vanish and the interference term defines the positions of principal maxima. However, for all other integral values of p , the numerator vanishes but not the denominator. That is, intensity vanishes when p , though an integer, is not an integral multiple of N . Hence, the condition for

minimum is $\gamma = p\pi/N$ except when $p = nN$; n being the order of diffraction.

These values correspond to

$$N\gamma = [\pi, 2\pi, \dots, (N-1)\pi], [(N+1)\pi, (N+2)\pi, \dots, (2N-1)\pi], [(2N+1)\pi, \dots]$$

$$\gamma = \left[\frac{\pi}{N}, \frac{2\pi}{N}, \dots, \frac{(N-1)\pi}{N} \right], \left[\frac{(N+1)\pi}{N}, \frac{(N+2)\pi}{N}, \dots, \frac{(2N-1)\pi}{N} \right],$$

$$\left[\frac{(2N+1)\pi}{N}, \dots \right] \quad (11.9)$$

So, using Eqs. (11.3) and (11.9), we can write the path difference for these values [Eq. (11.9)] of γ as

$$d \sin \theta = \left[\frac{\lambda}{N}, \frac{2\lambda}{N}, \dots, \frac{(N-1)\lambda}{N} \right], \left[\frac{(N+1)\lambda}{N}, \frac{(N+2)\lambda}{N}, \dots, \frac{(2N-1)\lambda}{N} \right],$$

$$\left[\frac{(2N+1)\lambda}{N}, \dots \right] \quad (11.10)$$

Eq. (11.10) can be written in the general form for the direction, $\theta_{\min}$ of the minima as

$$d \sin \theta_{\min} = \frac{q\lambda}{N} \quad (11.11)$$

where, q takes all the integral values except 0, N , $2N$, ...; that is

$q = [1, 2, \dots, (N-1)], [(N+1), (N+2), \dots, (2N-1)], [(2N+1), \dots]$ etc.

Note from Eq. (11.11) that, when $q = 0, N, 2N, \dots, nN$, we see that $d \sin \theta$ becomes equal to 0, λ , 2λ , ..., $n\lambda$ so that $d \sin \theta$ becomes equal to $n\lambda$ which represent the principal maxima and are omitted in the values of minima.

You might have found the discussion in Secs. 11.2.2 and 11.2.3 somewhat difficult to follow. In fact the content in these sections are indeed a bit dense. It will be better if you read these sections once again and convince yourself that you have understood all the physical arguments and mathematical expressions well.

Before proceeding further, let us recapitulate the important points of these sections.

- **The condition for principal maxima in the N slit pattern is**

$$\gamma = 0, \pi, 2\pi, \dots, n\pi$$

$$\text{So, } N\gamma = 0, N\pi, 2N\pi, \dots, nN\pi$$

$$\text{And, } \gamma = \frac{\pi d}{\lambda} \sin \theta_{\max} = n\pi; \quad \text{where } n = 0, 1, 2, \dots$$

$$\text{In terms of path difference, } d \sin \theta_{\max} = n\lambda$$

- **The condition for minima in the N slit pattern is**

$$N\gamma = [\pi, 2\pi, \dots, (N-1)\pi], [(N+1)\pi, (N+2)\pi, \dots, (2N-1)\pi], \dots$$

$$\gamma = \left[\frac{\pi}{N}, \frac{2\pi}{N}, \dots, \frac{(N-1)\pi}{N} \right],$$

$$\left[\frac{(N+1)\pi}{N}, \frac{(N+2)\pi}{N}, \dots, \frac{(2N-1)\pi}{N} \right], \dots, \left[\frac{(2N+1)\pi}{N}, \dots \right]$$

In terms of path difference, the condition is given as

$$d \sin \theta_{\min} = \left[\frac{\lambda}{N}, \frac{2\lambda}{N}, \frac{3\lambda}{N}, \dots, \frac{(N-1)\lambda}{N} \right], \left[\frac{(N+1)\lambda}{N}, \frac{(N+2)\lambda}{N}, \dots \right], \dots$$

Recap

You may, at this stage, ask: **How many minima will be there between two consecutive principal maxima?** To answer this, let us examine Eq. (11.9) once again. Note that in Eq. (11.9), the first square bracket lists the positions of $(N - 1)$ minima between the central ($n = 0$) and first ($n = 1$) principal maximum. Similarly, the second square bracket lists the positions of $(N - 1)$ minima between the first and second principal maxima. So, we find that there are $(N - 1)$ positions of minima between any two successive principal maxima. In other words, we find from Eqs. (11.8) and (11.11) that the first minimum on either side of the n th principal maximum given by $d \sin \theta_{\max} = n\lambda$ will be at

$$d \sin \theta_{\min} = n\lambda \pm \frac{\lambda}{N}.$$

Further, we know that between any two consecutive minima, there has to be a maxima. Such maxima in multiple slit diffraction pattern are called **secondary maxima**. From the above discussion on the number of minima between any two consecutive principal maxima, it is obvious that there will be $(N - 2)$ positions of secondary maxima between two consecutive principal maxima. As in single slit diffraction pattern, the secondary maxima are not symmetrical, and the intensity of secondary maxima is very low. They are therefore of little practical importance.

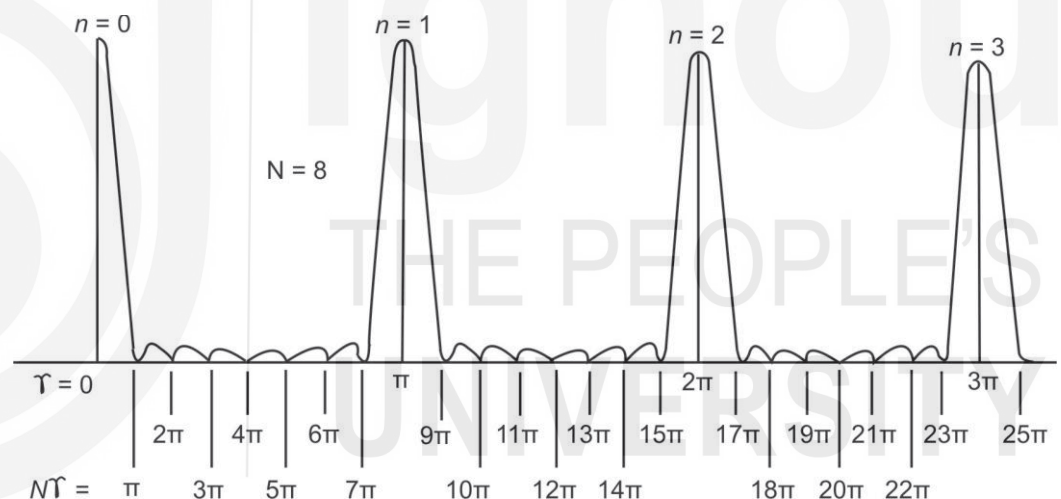


Fig. 11.2: Intensity distribution in the N slit diffraction pattern with $N = 8$ slits.

Fig. 11.2 shows the intensity distribution pattern for N slit diffraction pattern when the number of slits, $N = 8$. Here, we have shown principal maxima corresponding to $n = 0, 1, 2, 3$, and six secondary maxima between adjacent principal maxima. To appreciate the above discussion, go through the following example.

EXAMPLE 11.1: MAXIMUM NUMBER OF PRINCIPAL MAXIMA

Calculate the maximum number of principal maxima that can be formed with a grating having 5000 lines per cm for light of wavelength 5000 Å.

SOLUTION ■ You know that the grating element, d is defined as $d = a + b$, where b is the width of the individual slit and a is the separation

between two consecutive slits. Note that, in a grating, all slits are identical and the value of a is also constant. Now, as per the problem, there are 5000 lines per cm in the grating. So, we can write

$$5000 \times d = 1 \text{ cm}$$

Thus,

$$\text{Grating element } d = \frac{1 \text{ cm}}{5000} = 2 \times 10^{-4} \text{ cm}$$

From Eq. (11.8), we know that the condition for the formation of principal maxima is $d \sin \theta_{\max} = n\lambda$. Since $|\sin \theta| < 1$, we cannot have n greater than $\frac{d}{\lambda}$. So, in this specific case, the number of principal maxima is

$$n = \frac{d}{\lambda} = \frac{2 \times 10^{-4} \text{ cm}}{5000 \times 10^{-8} \text{ cm}} = 4$$

Therefore, the grating will be able to show 1st, 2nd, 3rd and 4th orders of principal maxima.

To concretise the ideas discussed above, you should solve an SAQ.

SAQ 2 - Number of Principal maxima

Calculate the maximum number of principal maxima in the diffraction pattern produced by a grating which has 1500 lines per cm. Take the wavelength of light to be 500 nm.

So, after solving SAQ 2 and on the basis of the results of Example 11.1, you may have realised that the number of principal maxima reduces as the number of lines per cm in the grating increases. Another important feature of the multiple slit diffraction pattern is that it has rather **sharp principal maxima**. The sharpness of maxima is expressed in terms of a parameter called angular half width of principal maxima. Let us learn about it now.

11.2.4 Angular Half-Width of Principal Maxima

From Eqs. (11.7) and (11.9), you now know that for N slits

1. The principal maxima occur when $\gamma = n\pi$ and therefore $N\gamma = nN\pi$.
2. On either side of the principal maxima, we have minimum when

$$N\gamma = Nn\pi \pm \pi \text{ or when } \gamma = n\pi \pm \frac{\pi}{N}.$$

In terms of path difference and angle of diffraction, the above conditions for principal maxima and the adjacent minimum can be written as

$$d \sin \theta_{\max} = n\lambda \quad (11.13)$$

$$\text{and, } d \sin \theta_{\min} = n\lambda \pm \frac{\lambda}{N} \quad (11.14)$$

The angle between the direction (or angle) of the principal maxima, $\theta_{\max}$ and the direction (or angle) of the adjacent minimum, $\theta_{\min}$ is called the **angular**

You may like to know as to why is $\delta\theta$ called **angular half-width**?

You know that the principal maximum extends from minimum on one side to minimum on the other side and $\delta\theta$ is half of it. If we consider the case of 6 slits, the first principal maximum extends from [Eq. (11.9)]:

$$N\gamma = 5\pi \text{ to } N\gamma = 7\pi$$

So, from Eq. (11.3):

$$d \sin \theta_{\max} = (5\lambda/6) \text{ to } (7\lambda/6)$$

Note that the term half width of a spectrum line (or a diffraction curve) has a slightly different meaning. To understand it, refer to Fig. 11.3 which represents the intensity vs. θ curve.

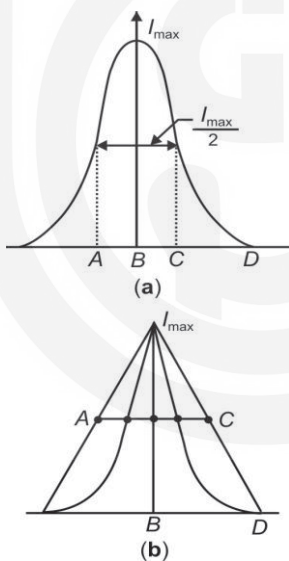


Fig. 11.3: Variation of intensity with angle of diffraction

The half width gives the width of the curve at $(I_{\max}/2)$. It is equal to AC in Fig. 11.3a. The angular half width, on the other hand, is equal to BD . Obviously, AC is not equal to BD . Only in the extreme case when the curve is a triangle (Fig. 11.3b) that we have $AC = BD$.

half-width of principal maxima. Let us denote it by $\delta\theta$. We now proceed to calculate this angle.

We can calculate $\delta\theta (= |\theta_{\max} - \theta_{\min}|)$ by computing $\theta_{\max}$ and $\theta_{\min}$ from these equations. Alternatively, by choosing $\theta_{\min} > \theta_{\max}$, we substitute

$$\theta_{\min} = \theta_{\max} + \delta\theta \text{ in Eq. (11.14) to obtain}$$

$$d \sin(\theta_{\max} + \delta\theta) = n\lambda + \frac{\lambda}{N}$$

$$\text{or } d \sin \theta_{\max} \cos \delta\theta + d \cos \theta_{\max} \sin \delta\theta = n\lambda + \frac{\lambda}{N}$$

For $\delta\theta \rightarrow 0$, $\cos \delta\theta \rightarrow 1$ and $\sin \delta\theta \rightarrow \delta\theta$. Hence, we can write the above expression

$$d \sin \theta_{\max} + d \cos \theta_{\max} \delta\theta = n\lambda + \frac{\lambda}{N}$$

Using Eq. (11.13), we can write the above equation in a compact form:

$$d \cos \theta_{\max} \delta\theta = \frac{\lambda}{N}$$

$$\delta\theta = \frac{\lambda}{N d \cos \theta_{\max}} \quad (11.15)$$

Eq. (11.15) shows that the principal maximum becomes sharper as N increases. This is an important result and it is used in spectral analysis.

Before proceeding further, answer the following SAQ.

SAQ 3 - Angular half width

Monochromatic light of wavelength 500 nm illuminates a diffraction grating having 1500 lines. If the value of the grating element is $1.8 \mu\text{m}$, calculate the angular half width of the first order principal maxima.

So far, you have learnt about the diffraction pattern produced by a system of N parallel equidistance slits. As we mentioned earlier in this unit, an arrangement of a large number of equidistance narrow vertical slits is known as diffraction grating. Let us now learn about it.

11.3 DIFFRACTION GRATING SPECTRUM

Diffraction grating is a series of equidistant vertical apertures (or obstacles). The theoretical analysis of the action of a grating is done by considering it as a regular array of parallel slit (such as an arrangement of N slits discussed in the previous section). There are two types of gratings: **reflection gratings** and **transmission gratings**. A grating is constructed by forming parallel, evenly spaced grooves or scratches on a flat surface of metal (reflection grating) or glass (transmission grating) plate. The grooves scatter the light and are effectively opaque and the space between grooves behaves like a slit.

The first grating was made by Fraunhofer. He stretched fine silver wire on a frame. His grating had nearly 200 wires to a centimetre. Afterwards, gratings were made by ruling fine lines with a diamond pen on a glass plate. Rowland was among the first to rule gratings on a metallic surface. He produced plane

as well as concave gratings with nearly 5000 lines per centimetre. These gratings are difficult to make and are expensive but celluloid replicas can be made fairly cheaply and are commonly used in the physics laboratory for spectral analysis. You can make a simple coarse grating for demonstration purposes on a plate by drawing equidistant and parallel scratches on the photographic emulsion on a plate.

A typical grating plate is rectangular with dimensions of several centimetres on each side. The spacing between the slits is very small ($\sim 2 \mu\text{m}$) and the number N of slits is usually very large, about 10,000.

Diffraction grating is a widely used optical component in spectroscopy. You know that spectroscopy is the study of emission and absorption of light (and other radiation) by matter. Every element of the periodic table, under suitable conditions, emit light of characteristic wavelength(s) called spectra. The technique of spectroscopy involves separating the light emitted by matter into its constituent wavelengths. This is similar to what we achieve when white light from the Sun is passed through a prism: it is split into its seven constituent colours each having a unique wavelength. The splitting of light emitted by a substance is achieved with better precision by using a diffraction grating and we obtain spectra of the substance. Let us now learn about this technique of formation of spectra by a grating.

11.3.1 Formation of Spectra

We have seen that for a monochromatic light of wavelength λ_1 , the direction θ_1 of principal maxima are given by the grating equation [Eq. (11.8)]:

$$d \sin \theta_1 = n \lambda_1 \quad n = 0, 1, 2, 3, \dots$$

With the experimental arrangement described in Sec. 11.2 for N slits, we will get these principal maxima as one line in each order. Using another source of light which emits a longer wavelength λ_2 , we will get a corresponding line in each order at a larger angle θ_2 :

$$d \sin \theta_2 = n \lambda_2 \quad n = 0, 1, 2, 3, \dots$$

However, if the same source of light emits light of both the colours corresponding to wavelengths λ_1 and λ_2 , we will get two lines simultaneously in each order. These two lines will be seen as two spectrum lines separated from each other. This is so because, except the central maximum (zeroth order), the angles of diffraction for λ_1 and λ_2 are different in various other orders. In the central maximum, $\theta = 0$ for all wavelengths and therefore different colours are not separated from each other. What do you expect to observe when we have a white light source? The central maximum will be white while maxima in all other orders will show colours and we will see a continuous spectrum.

We note that in the grating equation [Eq. (11.8)], if we know d , θ and n , we can calculate the wavelength of light. Since the grating element (d) is known for a grating and θ can be measured, this arrangement provides a simple and accurate method of determining the value of wavelength λ of light used. This is discussed in the following section.

11.3.2 Observing Grating Spectra

The steps to mount a grating for normal incidence on the turn table of a spectrometer are as given below:

1. Take the reading of the turntable when the telescope arm is in line with the collimator. Let it be ϕ .
2. Rotate the telescope to position $\phi \pm 90^\circ$.
3. Mount the grating on the turn table and rotate the turn table till you see the image of the slit after reflection from the grating. At this position the surface of the grating is inclined at 45° to the parallel beam of light emerging from the collimator.
4. Obviously, turning the grating through 45° in the proper direction will make the light fall normally on the grating surface.

Light emitted by an atom consists of sharp spectrum lines. Light emitted by a molecule consists of a group of lines which when unresolved give a band like appearance and is often called band spectrum, while an incandescent lamp or similar sources will give a continuous spectrum, where various colours merge into one another.

In the experimental set up for observing a grating spectra, a simple spectrometer is used. This arrangement is depicted in Fig. 11.4. The light from the given source is focussed (with the help of a lens) on the slit of the collimator which sends out a parallel beam of light.

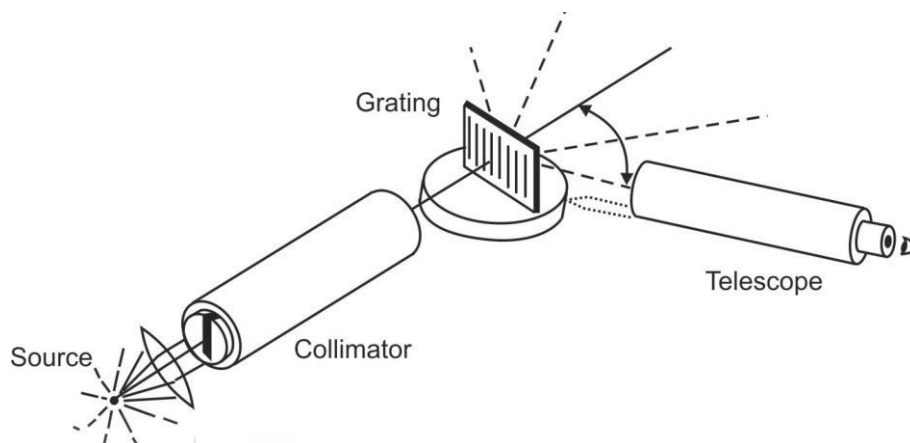


Fig. 11.4: A schematic diagram of experimental set up for observing grating spectra.

The telescope arm is rotated and brought in line with the collimator. This ensures that the parallel beam of light falling on the objective of the telescope is focussed at the cross wires, which is in the focal plane of the eye piece. The position of the source of light should be adjusted to get the brightest image. We mount the diffraction grating on the turntable and adjust it so that the light is incident normally on the grating (see the margin remarks).

Next, we rotate the telescope arm to the left or right to get the first order spectrum in the field of view. If the source of light is a discharge tube containing sodium, mercury or argon, the spectrum will consist of a series of spectrum lines. **Each spectrum line is a diffracted image of the slit, formed by different wavelengths present in the source.** To get sharp line images, we adjust the grating so that the diffracting slits are parallel to the collimator slit. This can be done by rotating the grating in its own plane.

To measure the wavelength of each line, we set the vertical crosswires at the centre of each spectrum line and note the position of the telescope in each case. The difference between this position of the telescope and the direct position gives the angle of diffraction for each of the lines. To reduce error, the position of the telescope is noted on both sides of the direct position and half of this angle gives the angle of diffraction.

The salient features of the observed spectra are

1. The spectrum exists on both sides of the direct beam.
2. Apart from the first order, the second or even third order spectrum is also present depending on the grating element.
3. Different spectrum lines are not equally bright or sharp. This depends on the energy levels and the transitions of the atom giving the spectrum. These concepts are further illustrated in the following example.

EXAMPLE 11.2: ANGLE OF DIFFRACTION

Rowland ruled 14438 lines per inch in his grating. i) Calculate the angles of diffraction for violet ($\lambda = 4000 \text{ \AA}$) and red ($\lambda = 8000 \text{ \AA}$) colours in the first order of spectrum. ii) What is the largest wavelength which can be seen with this grating in the third order?

SOLUTION ■ i) The grating element $d = \frac{2.54 \text{ cm}}{14438} = 0.0001759 \text{ cm}$
 $= 1.759 \times 10^{-4} \text{ cm}$

Suppose that the violet colour ($\lambda = 4000 \text{ \AA}$) is diffracted through angle θ_v . Then, from Eq. (11.8), we can write

$$d \sin \theta_v = n\lambda$$

For first order ($n = 1$), on substituting the values of d , and λ , we get

$$\sin \theta_v = \frac{4 \times 10^{-5} \text{ cm}}{1.759 \times 10^{-4} \text{ cm}} = 0.2274 \Rightarrow \theta_v \cong 13^\circ.$$

Similarly, for red colour ($\lambda = 8000 \text{ \AA}$), we have

$$\sin \theta_r = \frac{8 \times 10^{-5} \text{ cm}}{1.759 \times 10^{-4} \text{ cm}} = 0.4548 \Rightarrow \theta_r = 27^\circ$$

This means that the entire visible spectrum in the first order extends from nearly $\theta = 13^\circ$ to $\theta = 27^\circ$, i.e. covers an angle of about 14° .

ii) For third order ($n = 3$) spectrum, the condition for principal maxima will be

$$d \sin \theta = 3\lambda_{\max}$$

According to the given condition, the largest value of λ in the third order will correspond to $|\sin \theta| = 1$ which gives $\theta = 90^\circ$. Thus, we have

$$d = 3\lambda_{\max}$$

$$\lambda_{\max} = \frac{d}{3} = \frac{1.759 \times 10^{-4}}{3} \text{ cm} = 5860 \text{ \AA}$$

This calculation suggests that in the third order spectrum, the sodium doublet consisting of $5890 \text{ \AA}$ and $5896 \text{ \AA}$ will not be visible. It will be worthwhile to verify this result.

If you calculate $\sin \theta_v$ and $\sin \theta_r$ for 1st, 2nd and 3rd orders, you will find that for

1 st order	$\sin \theta_v = 0.2274 \Rightarrow \theta_v \sim 13^\circ$
	$\sin \theta_r = 0.4548 \Rightarrow \theta_r \sim 27^\circ$
2 nd order	$\sin \theta_v = 0.4548 \Rightarrow \theta_v \sim 27^\circ$
	$\sin \theta_r = 0.9096 \Rightarrow \theta_r \sim 65^\circ$
3 rd order	$\sin \theta_v = 0.6822 \Rightarrow \theta_v \sim 43^\circ$
	$\sin \theta_{\max} = 1$ for $\lambda_{\max} = 5860 \text{ \AA}$
	and $\theta_{\max} = 90^\circ$

$\sin \theta_r > 1$ cannot be observed.

Note that the spread of the spectrum in 1st, 2nd and 3rd orders are 14°, 38° and 47°, respectively. Also note that the value of largest wavelength which can be observed in the 3rd order is 5860 Å which is less than the wavelength of red colour. Thus, in the 3rd order, the entire visible spectrum cannot be observed; only a part of the spectrum can be observed for the given grating. Schematically, it is shown in Fig. 11.5 below for the spectrum on the left side of the centre. A similar spectrum will be observed on the right side of the central maxima.

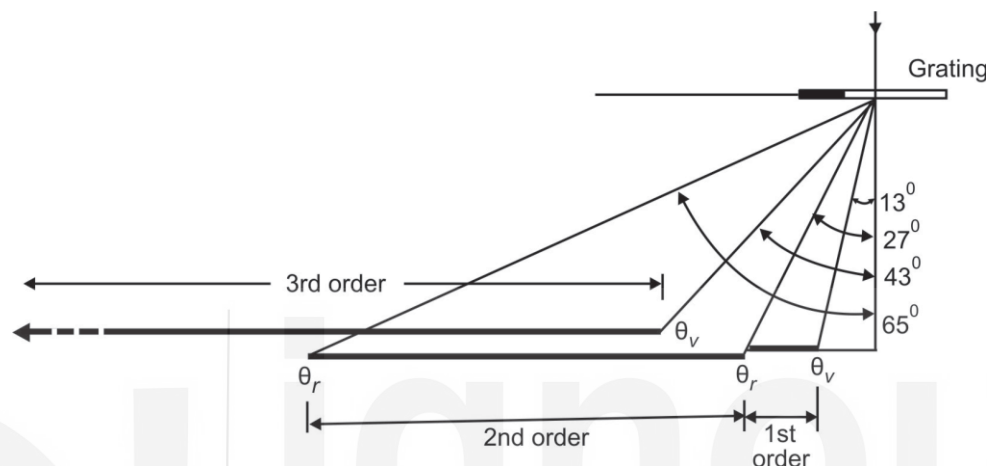


Fig. 11.5: The Angular spread of various orders of spectrum.

Thus, from Fig. 11.5, we find that in the 1st order, red just touches second order violet. (This is because we have selected $\lambda_v = 4000 \text{ Å}$ and $\lambda_r = 8000 \text{ Å}$.) It means that there is essentially no overlapping of first and second order spectra. In third order, λ_v begins at $\theta \cong 43^\circ$. If we calculate wavelength say, λ_x of 2nd order present at $\theta \cong 43^\circ$ we find that

$$d \sin 43^\circ = 3\lambda_v = 2\lambda_x \Rightarrow \lambda_x = \frac{3 \times 4000 \text{ Å}}{2} = 6000 \text{ Å}$$

Therefore $\lambda = 6000 \text{ Å}$ of the 2nd order occurs at the same place as $\lambda = 4000 \text{ Å}$ of third order. Therefore, from 6000 Å to 8000 Å we will observe overlapping colours.

You should now solve an SAQ to concretise your understanding of the diffraction grating.

SAQ 4 - Grating spectra and wavelength of light

Light from a laser is incident on a grating with 5300 lines per cm. The central and the first order principal maxima are separated by a distance of 0.480 m on a wall that is 1.50 m from the grating. Determine the wavelength of the laser light.

We now summarised what you have learnt in this unit.

11.4 SUMMARY

Concept

Description

Intensity in N slit pattern ■ The intensity distribution in N -slit diffraction pattern is given by

$$I_{\theta} = A^2 \frac{\sin^2 \beta}{\beta^2} \frac{\sin^2 N \gamma}{\sin^2 \gamma}$$

where $(\sin^2 \beta / \beta^2)$ and $(\sin^2 N \gamma / \sin^2 \gamma)$, respectively, represent diffraction and interference terms.

The value of parameters β and γ is given as

$$\beta = \frac{\pi b \sin \theta}{\lambda} \quad \text{and} \quad \gamma = \frac{\pi d \sin \theta}{\lambda}$$

The interference term controls the width of interference fringes while diffraction term governs their intensities.

The grating element, $d = a + b$.

Principal maxima

■ The condition for principal maxima in the diffraction pattern due to N slit (or diffraction grating) is

$$d \sin \theta_{\max} = n\lambda; \quad n = 0, 1, 2, \dots$$

The integer n is called order of the maximum.

Minima and secondary maxima

■ The condition for minima in the diffraction pattern due to N slits (or a diffraction grating) is

$$d \sin \theta_{\min} = \frac{q\lambda}{N}$$

where q takes all integral values except $q = 0, N, 2N, \dots$. There are $(N - 1)$ positions of minima between any two consecutive principal maxima.

Between any two consecutive minima, there is a secondary maxima. There are $(N - 2)$ secondary maxima between any two consecutive principal maxima. The intensity of the secondary maxima is very small.

Angular half-width

■ The angular half-width of principal maxima is given as

$$\delta\theta = \frac{\lambda}{Nd \cos \theta_{\max}}$$

So, as the number N of slits in a grating increases, the principal maxima becomes narrower/sharper.

11.5 TERMINAL QUESTIONS

- The first-order principal maximum of light of wavelength 656 nm is at $\theta = 18^\circ$ for a particular grating. a) Calculate the slit spacing of the grating. b) Using the same grating, the first-order principal maximum of another

- source of light is observed at $\theta = 15.7^\circ$. What is the wavelength of this light?
- A beam of monochromatic light with $\lambda = 532 \text{ nm}$ is incident normally on a grating with $d = 2.16 \text{ }\mu\text{m}$. Calculate the angular positions of the first – and second order interference maxima.
 - A grating having 760 slits is illuminated by a light of wavelength 580 nm. If the value of the grating element d is equal to $2.12 \text{ }\mu\text{m}$, calculate the angular half-width of first-order principal maxima whose angular position is 14.2° .
 - Consider a diffraction grating with 15000 lines per inch. Calculate the angular separation of D_1 and D_2 lines of sodium in the second-order. Given that $\lambda_{D_1} - \lambda_{D_2} = 6 \text{ }\text{\AA}$.

11.6 SOLUTIONS AND ANSWERS

Self-Assessment Questions

- For $N = 2$, Eq. (11.4) takes the form

$$\begin{aligned}
 I_\theta &= A^2 \frac{\sin^2 \beta}{\beta^2} \frac{\sin^2 2\gamma}{\sin^2 \gamma} \\
 &= A^2 \frac{\sin^2 \beta}{\beta^2} \frac{(2 \sin \gamma \cos \gamma)^2}{\sin^2 \gamma} \\
 &= 4A^2 \frac{\sin^2 \beta}{\beta^2} \cos^2 \gamma
 \end{aligned}$$

which is the required result [Eq. (10.11)] for the double slit.

- You know from Eq. (11.8) that the condition for principal maxima is

$$d \sin \theta = n\lambda$$

Since $|\sin \theta|$ has the maximum value of 1, we can write

$$\frac{n\lambda}{d} = 1 \Rightarrow n = \frac{d}{\lambda}$$

As per the problem, the grating element

$$d = \frac{1 \text{ cm}}{15000} = 6.6 \times 10^{-5} \text{ cm}; \quad \text{and} \quad \lambda = 500 \text{ nm} = 5 \times 10^{-5} \text{ cm}$$

So, the maximum value of order of diffraction, n is

$$n = \frac{d}{\lambda} = \frac{6.6 \times 10^{-5} \text{ cm}}{5 \times 10^{-5} \text{ cm}} = 1.32$$

So, only first order principal maxima will be obtained.

3. From Eq. (11.15), we note that the angular half-width is given as

$$\delta\theta = \frac{\lambda}{Nd \cos \theta_{\max}} = \frac{\lambda}{Nd \sqrt{1 - \sin^2 \theta}} = \frac{\lambda}{Nd \sqrt{1 - (\lambda^2 / d^2)}}$$

So, substituting the values of N , A and d , we get

$$\begin{aligned} \delta\theta &= \frac{5 \times 10^{-5} \text{ cm}}{1500 \times (1.8 \times 10^{-4} \text{ cm}) \times \sqrt{1 - (5 \times 10^{-5} \text{ cm})^2 / (1.8 \times 10^{-4} \text{ cm})^2}} \\ &= 0.0006 \text{ rad} = 0.04^\circ \end{aligned}$$

So, the angular half-width is very small and hence the principal maximum is sharp and it appears as image of the source slit.

4. We know that the condition for principal maxima in the diffraction pattern due to grating is

$$d \sin \theta = n\lambda$$

For first order principal maxima, $n = 1$, and we have

$$d \sin \theta = \lambda$$

where, θ is the angle between central maximum and the first order principal maxima. As per the problem, the distance between the central maximum and first order maximum is 0.480 m on the screen which is at a distance of 1.50 m from the grating. So, we can write

$$\delta\theta = \frac{0.480 \text{ m}}{\sqrt{(0.480 \text{ m})^2 + (1.50 \text{ m})^2}} = 0.32 \text{ rad.}$$

So,

$$\lambda = d \sin \theta = \frac{(1 \text{ cm}) \times (0.32)}{5300} = 6.04 \times 10^{-5} \text{ cm.}$$

Terminal Questions

1. a) The condition for principal maxima is given by Eq. (11.8):

$$d \sin \theta = n\lambda$$

For first-order, $n = 1$; and $\lambda = 6.56 \times 10^{-5} \text{ cm}$, $\theta = 18^\circ$

$$d = \frac{\lambda}{\sin \theta} = \frac{6.56 \times 10^{-5} \text{ cm}}{\sin(18)} = \frac{6.56 \times 10^{-5} \text{ cm}}{0.309} = 2.12 \mu\text{m}$$

- b) As per the problem, $\theta = 15.7^\circ$, $d = 2.12 \mu\text{m}$ and $n = 1$. So,

$$\lambda = d \sin \theta = (2.12 \times 10^{-6} \text{ m}) \times \sin(15.7) = 573 \text{ nm.}$$

2. As you know, the principal maxima in the diffraction pattern of a grating is due to interference of diffracted light from the slits of the grating. So, these maxima are interference maxima. And, the condition for interference maxima is

$$d \sin \theta = n\lambda$$

For first order, $n = 1$; and we have, $\lambda = 532 \times 10^{-9} \text{m}$, $d = 2.16 \times 10^{-6} \text{m}$

$$\sin \theta = \frac{\lambda}{d} = \frac{532 \times 10^{-9} \text{m}}{2.16 \times 10^{-6} \text{m}} = 0.246$$

So,

$$\theta = \sin^{-1}(0.246) = 14.2^\circ$$

And, for second order, $n = 2$. So,

$$\sin \theta = \frac{2\lambda}{d} = \frac{2 \times (532 \times 10^{-9} \text{m})}{2.16 \times 10^{-6} \text{m}} = 0.492$$

So,

$$\theta = \sin^{-1}(0.492) = 29.4^\circ$$

3. The angular half-width of principal maxima is given by Eq. (11.15):

$$\delta\theta = \frac{\lambda}{Nd \cos \theta}$$

$\lambda = 580 \times 10^{-9} \text{m}$, $N = 760$, $d = 2.12 \times 10^{-6} \text{m}$ and $\theta = 14.2^\circ$. So,

$$\delta\theta = \frac{580 \times 10^{-9} \text{m}}{760 \times (2.12 \times 10^{-6} \text{m}) \times \cos(14.2^\circ)} = 0.37 \times 10^{-3} \text{rad} = 0.02^\circ$$

So, the angular half-width is very low which means the maximum is very sharp.

4. Grating element, $d = \frac{2.54}{15000} = 1.69 \times 10^{-4} \text{cm}$

For second order; $d \sin \theta = 2\lambda$

So,

$$d \sin \theta_1 = 2\lambda_1 \quad \text{and} \quad d \sin \theta_2 = 2\lambda_2$$

Let,

$$\theta_2 = \theta_1 + \delta\theta$$

So,

$$d [\sin \theta_1 - \sin (\theta_1 + \delta\theta)] = 2(\lambda_1 - \lambda_2)$$

$$d [\sin \theta_1 - \sin \theta_1 \cos \delta\theta - \cos \theta_1 \sin \delta\theta] = 2\Delta\lambda$$

or

$$d \cos \theta_1 \delta\theta = 2\Delta\lambda \quad (\text{because, for small } \delta\theta, \cos \delta\theta \approx 1 \text{ and } \sin \delta\theta \approx \delta\theta)$$

or

$$\delta\theta = \frac{2 \times 6 \times 10^{-8} \text{cm}}{d \sqrt{1 - \sin^2 \theta_1}} = \frac{2 \times 6 \times 10^{-8} \text{cm}}{d \sqrt{1 - (2\lambda_1 / d)^2}} \approx 0.001 \text{rad} \approx 0.06^\circ$$