BPYC-133

Block 3

CATEGORICAL SYLLOGISM

Block 3 "Categorical Syllogism", comprised of 4 units, discusses categorical syllogism, non-categorical syllogism, and propositional logic. Study of this block will enable learners to understand the distinction between traditional and modern logic and the need for modern or symbolic logic as well.

Unit 11 "Nature of Categorical Syllogism" aims to introduce and discuss the definitions of syllogism and categorical syllogism; basic terms of categorical syllogism; the form of a categorical syllogism, the distinction between the standard form categorical syllogism and non-standard form categorical syllogism; the notion of validity and invalidity of standard form categorical syllogism.

Unit 12 "Methods for Testing Categorical Syllogism" introduces mainly two methods for testing categorical syllogism. The unit discusses the Classical technique (developed by Aristotle) and Venn diagram technique to test the validity of a categorical syllogism.

Unit 13 "Non-categorical Syllogism" discusses the two major kinds of non-categorical syllogisms, namely: disjunctive syllogism and hypothetical syllogism. This unit also elaborates another kind of non-categorical argument called dilemma; the different types of dilemmas and the ways to tackled/resolved dilemmas.

Unit 14 "Propositional Logic" introduces learners to Propositional Logic. Understanding Propositional Logic is crucial to understand the development of Modern Logic or Symbolic Logic. This unit also helps learners to understand the techniques used by Propositional Logic to test the truth and validity in Logical expressions. This unit also briefly mentions the parallels of Propositional logic in Indian Logic.

UNIT 11 NATURE OF CATEGORICAL SYLLOGISM*

Structure

11.0 OBJECTIVES

This unit entitled Nature of Categorical Syllogism aims to:

- Introduce and discuss the definitions of syllogism and categorical syllogism. The unit also introduces the basic terms of categorical syllogism.
- Discuss the form of a categorical syllogism, distinction between the standard form categorical syllogism and non-standard form categorical syllogism
- Discuss moods and figures that make the form and their importance in evaluating a categorical syllogism.
- Discuss the notion of validity and invalidity of standard form categorical syllogism.

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11.1 INTRODUCTION

Propositions are the building blocks of arguments. Arguments in turn are considered the basic unit of reasoning. We use different kinds of arguments in our day to day lives for different purposes ranging from normal conversations to strict logical reasoning and professional decision-making. In all these activities, some arguments are able to convince others sooner than some other arguments. For example, in a court room, the advocate who is able to produce more robust arguments from the available facts is most likely to win the case. In logic, the argument is evaluated in terms of validity and soundness, and not on the basis of its convincing ability. An argument can be very convincing yet could turn out to be invalid. In this unit, we will discuss a kind of argument which is distinct from other kinds of arguments because of its formal nature, namely categorical syllogism. We will look at the key features of a categorical syllogism, the terms involved, the moods of categorical syllogism and the figures in which these moods can be found. Mood and figure taken together constitute the form of a categorical syllogism. As for any argument, the notion of validity and invalidity is important, so it will also be discussed in the context of our topic of categorical syllogism.

11.2 DEFINITION

To begin with, a *syllogism* may be defined as any deductive argument which has three propositions out of which the first two are called premises and third is conclusion and the conclusion is entailed by those two premises.

For example,

> All Men are Stupid.
> Mohan is a Man.
> Therefore, Mohan is Stupid.

If all the three propositions of a syllogism are categorical propositions (namely, A, E, I, O) then the syllogism is called a *categorical syllogism*. This distinction was maintained by traditional logicians, but for the modern logicians starting from Boole, only a categorical syllogism can be of syllogistic nature. In other words, it will not be called a syllogism, unless it is categorical.

Therefore, modern logicians use the word syllogism and categorical syllogism interchangeably. However, any three propositions even if categorical, grouped together, would not result into a categorical syllogism, as pointed out earlier; the conclusion should be necessitated by the other two premises. The three propositions in categorical syllogism must consist only three terms as their subjects and predicates, each of which occurs twice.

Let us look at the example below to understand a categorical syllogism:

> All Ministers are Members of council of Ministers.
> All Members of council of ministers are Politicians.
> Therefore, All Ministers are Politicians..

In the above example, there are three categorical propositions (all of them are Universal Affirmative, A, in this case), and there are three terms namely, 'Ministers', 'Members of Cabinet of Ministers' and 'Politicians', and each of these terms has occurred twice in the syllogism.

However, in order to find out the validity or the invalidity of a categorical syllogism, the syllogism must be in its *standard form*. A categorical syllogism in its standard form is called standard form categorical syllogism. We will discuss in the next section how a categorical syllogism is changed in its standard form, what is the form of a categorical syllogism and other questions related to standard form of categorical syllogism.

11.3 FORM OF CATEGORICAL SYLLOGISM

We can find out the validity of a categorical syllogism only when the syllogism is in the standard form because the rules of the validity and invalidity of a syllogism are applicable only to the standard form syllogism. In the standard form categorical syllogism, the categorical propositions forming the syllogism must be in a *particular order*: the first proposition in this order is called major premise, the second is called minor premise and the third proposition is called conclusion. If a given categorical syllogism whose validity is to be checked is not given in the standard form, then one must convert it into the standard form before determining its validity. Let's consider the example that was mentioned in the previous section,

> All Ministers are Members of council of Ministers.

All Members of council of Ministers are Politicians.

Therefore, All Ministers are Politicians.

How we can find out whether the given syllogism is in the standard form or not? In order to check that, the first thing that is required is to identify the proposition that is serving as the conclusion in the given syllogism. The conclusion can be located by identifying certain conclusion indicators, such as, 'therefore', 'thus', 'hence', 'subsequently', 'so', 'consequently', so on and so forth. Sometimes, a syllogism may contain no conclusion indicators and in that case one can look for premise indicators. Some premise indicator words are: 'since', 'for', 'because' etc. If a premise is identified (by a premise indicator) then the other proposition in conjunction with this premise would be the other premise. And the remaining proposition would be the conclusion of the syllogism. In the considered example, the conclusion is evident because of the conclusion indicator 'therefore' being present:

So the conclusion here is- 'All Ministers are Politicians.'

Now, once the conclusion is identified, we can look for the method that would help us determine which of the premises is to be called minor and which is to be called major. A major premise is the one which contains the major term and minor premise which contains the minor term. There is another term which is called middle term. The method to identify these terms and their relation with each other is explained in the next subsection.

11.3.1Terms of Categorical Syllogism

As mentioned earlier in the introductory section, a categorical syllogism consists of only three terms, namely - middle term, minor term and major term. Once we have identified the conclusion of a categorical syllogism, it is very easy to identify the three terms of the syllogism.

The subject term of the conclusion is called minor term and the premise containing that term is called minor premise. This occurrence of minor term is other than in the conclusion as each of the terms appears twice in a categorical syllogism. The predicate term of the conclusion is called major term and the premise containing the major term is called major premise. Again, this occurrence of major term is other than in the conclusion as each of the terms appears twice in a categorical syllogism. There is a third term which is present in both the premises but is not present in the conclusion; this third term is called middle term. The commonly accepted denotation for major, minor and middle terms are P, S and M, respectively.

The conclusion of a categorical syllogism asserts the relation between the classes represented by the major and the minor terms and this relation is a product achieved by the relation of minor term and major term with the middle term.

So considering the example mentioned earlier, we can now identify which term is which. The conclusion is - All Ministers are Politicians.

Major term in this case is - Politicians

Minor term in this case is - Ministers

The third term which is not present in the conclusion but is there in other two propositions (premises), middle term, in this case is – Members of council of Ministers.

Now, it becomes clear that the categorical syllogism as given to us is not in its standard form, for the minor premise is placed before the major premise. So let us arrange it to get the standard form categorical syllogism,

Major Premise: All Members of council of Ministers are Politicians.
Minor Premise: All Ministers are Members of council of Ministers.
Conclusion: All Ministers are Politicians.

Now if we replace these terms with their standard denotation, we will get

All M is P
All S is M
Therefore, All S is P

11.3.2 Mood of a Categorical Syllogism

The arrangement of premises and conclusion in a syllogism, essentially gives the mood of a syllogism. Once a syllogism has been arranged into a standard form categorical syllogism, then writing the denotations of the categorical propositions used, would give us the sequence known as mood of the syllogism.

Considering the syllogism discussed in the previous section, we have it in its standard form,

That is:	All M is P	(A)
	All S is M	(A)
Therefore,	All S is P	(A)

Here, all the three categorical propositions are of A form. Hence, the mood of this syllogism is AAA.

Example: Let us consider another categorical syllogism and try to determine its mood,

Some Scholars are Students.
All Students are Brave.
Therefore, Some Scholars are Brave.

To decide the mood of the argument, first we need to see whether the syllogism is in its standard form or not. So the conclusion, 'Some Scholars are Brave' gives us the predicate (major term) and the subject (minor term), i.e., 'Brave' and 'Scholars' respectively. Once we have our major and minor terms, we can see that the third term, 'Students' is the middle term. It also becomes evident that the syllogism is not in its standard form yet. So, we restructure it to follow the standard form- that is, major premise, followed by minor premise, followed by conclusion. We get the standard form categorical syllogism as:

Major premise: All Students are Brave.
Minor Premise: Some Scholars are Students.
Conclusion: Some Scholars are Brave.

Replacing these terms by their standard denotations, we will get:

	All M is P	(A)
	Some S is M	(I)
Therefore,	Some S is P	(I)

Hence the mood of the syllogism here is AII.

Now, one can see that there could be so many syllogisms which will have a common mood, no matter what their content is, yet a logical form cannot be determined solely on the basis of the mood of the argument. The reason why it is not possible to determine the logical form of a syllogism solely on the basis of mood is because the factor of the positioning of these terms (middle term, major term and minor term) is not being taken into account. Middle term can either be the subject or the predicate in both major and minor premises and yet the mood will not change if there is no qualitative change in the propositions. So, there is another factor known as figure of the syllogism which taken together with mood provides us with the logical form of the categorical syllogism

11.3.3 Figures of Categorical Syllogism

Figures of categorical syllogisms are determined by the position of the middle term in the premises of any given categorical syllogism. The middle term (M) can appear in four ways in a standard form categorical syllogism. One must make sure that the order of the propositions is according to the standard form, i.e., major premise, minor premise, and conclusion, before one tries to identify the figure of the syllogism.

i. First Figure: M P
S M
Therefore, S P

In the first figure, the middle term is the subject in the major premise and it is the predicate in the minor premise.

ii. Second Figure: P M
S M
Therefore, S P

In the second figure, the middle term is the predicate in both the premises.

iii. Third Figure: M P
M S
Therefore, S P

In the third figure, the middle term is the subject in both the premises.

iv. Fourth Figure: P M

M S

Therefore, S P

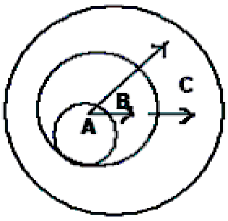
In the fourth figure, the middle term is the predicate in the major premise and it is the subject in the minor premise.

From the scheme of the four figures discussed above, it is clear that neither P nor S determines the figure of syllogism. As per historical records, Aristotle accepted only the first three figures. The origin of the fourth figure remains disputed. While Quine said that Theophrastus, a student of Aristotle, invented the fourth figure, Stebbing said that it was Gallen who invented the fourth figure. This dispute is not very significant. But what Aristotle says on the first figure is rather significant. Aristotle regarded the first figure as most 'scientific' one. It is likely that by 'scientific' he meant 'satisfactory'. One of the reasons, which Aristotle has adduced in defense of his thesis, is what the nature of laws of mathematics and physical sciences suggest.

Aristotle argued that the sciences often establish laws in the form of the first figure. Second reason is that a reasoned conclusion or a reasoned fact is generally found, according to Aristotle, in the first figure. Aristotle believed that only universal affirmative conclusion can provide complete knowledge and universal affirmative conclusion is possible only in the first figure. Aristotle quotes the fundamental principle of syllogism. 'One kind of syllogism serves to prove that A inheres in C by showing that A inheres in B and B in C'. This principle can be expressed in this form:

Minor: A inheres in B
Major: B inheres in C
Therefore, A inheres in C

Evidently, this argument satisfies transitive relation. This can be further clearly illustrated with the help of the following diagram.



Keeping the structure in mind, let us consider an example, corresponding to each of the four figures:

First figure:

All Artists are Poets. (major premise)
All Musicians are Artists. (minor premise)
Therefore, All Musicians are Poets. (conclusion)

Second figure:

All Saints are Pious. (major premise)
No Criminals are Pious. (minor premise)
Therefore, No Criminals are Saints. (conclusion)

Third figure:

All Great Works are Worthy of Study. (major premise)
All Great Works are Epics. (minor premise)
Therefore, Some Epics are Worthy of Study. (conclusion)

Fourth figure:

No Soldiers are Traitors. (major premise)
All Traitors are Sinners. (minor premise)
Therefore, Some Sinners are not Soldiers. (conclusion)

The above discussed examples make the schematic structure of the four types of figures of categorical syllogism clear.

Check Your Progress I

Note: (a) Use the space provided for your answer.

(b) Check your answers with those provided at the end of the unit.

1. Give an example of AAA-1.

11.3.4 Evaluation of the Form of Categorical Syllogism

Mood and figure taken together provide the form of categorical syllogism. Consider the example of a categorical syllogism given in the beginning of this section

All Ministers are members of council of ministers.
All Members of council of Ministers are Politicians.
Therefore, All Ministers are Politicians.

When we convert this categorical syllogism to obtain the standard form categorical syllogism we get:

All Members of council of Ministers are Politicians
All Ministers are Members of council of Ministers
All Ministers are Politicians

This standard form when converted into standard denotations gives us:

All M is P
All S is M
Therefore, All S is P

This gives us the mood of the categorical syllogism as AAA. Now if we notice the position of the middle term, we find that in the major premise, middle term is in subject's position and in the minor premise, it is in predicate's position, which is the condition of figure 1.

Hence the form of the syllogism is AAA-1.

Example 1: Consider another argument:

Some Cricketers are Athletes.
All Cricketers are Runners.
So, Some Runners are Athletes.

This would give us:

Some M is P
All M is S
Some S is P

It is evident that the mood of the syllogism is IAI and the position of middle term gives figure 3. Both of these taken together give us the logical structure as IAI-3.

Example 2: Let us consider another categorical syllogism,

All Phoenix are Immortals.
No One who ever dies is Phoenix.
Therefore, No One who ever dies is Immortal.

In the above example, the mood of the categorical syllogism is AEE. On replacing the terms with their denotations, we get:

All M is P
No S is M
Therefore, No S is P

It is evident from the position of the middle term in the premises that the figure of the categorical syllogism is 1. Hence, the form of the syllogism is AEE-1.

In the context of figures and moods of categorical syllogism, it is important to keep in mind that, there are exactly 256 distinct forms of categorical syllogism: four kinds of major premise (categorical propositions A, E, I, O), multiplied by four kinds of minor premise, multiplied by four kinds of conclusion, multiplied by four relative positions of the middle term (figures). That is to say, there are three categorical propositions in each categorical syllogism and four types (A, E, I, O) or 4^3 (4*4*4) = 64 possible combinations or arrangements (moods). Further, with four figures possible for each of 64 moods there are (64*4) 256 total possible arrangements of mood and figure. Out of these distinct and unique 256 forms that standard form categorical syllogisms may assume, only some possible forms are valid forms. Each of the valid forms also has a unique name associated with it. This will be discussed in further more detail in the upcoming unit on 'Techniques for Testing the Validity of Categorical Syllogism'.

Check Your Progress II

Note: (a) Use the space provided for your answer.

(b) Check your answers with those provided at the end of the unit.

1. Point out the major term, minor term and middle term of the given syllogism. Also give the mood and figure.
   a. All Sharks are Biters. Some Fishes are Sharks. Therefore, Some Fishes are Biters.
   b. Some Dogs are Dangerous. Some Cats are Dangerous. Hence, Some Cats are Dogs.
   c. Some Four-wheeler Vehicles are Cars. No Cars are Solar-powered Vehicles. Therefore, Some Four-wheeler Vehicles are not Solar-powered Vehicles.

11.4 NATURE OF CATEGORICAL SYLLOGISM

It is important to note that one of the primary concerns of logic is to deal with validity of arguments. A categorical syllogism is formal in nature. So, the analysis of the syllogism pertains to its logical structure or form and not it's content per se. In a standard form categorical syllogism, the assumption about the truth value of its constituent proposition is taken to be contingent. Therefore, the validity or the invalidity of syllogisms remains unaffected from the content of the propositions. The validity and the invalidity are tested against the form of a syllogism. A corollary to this character is that two syllogisms which may have completely different content in their propositions, would share the validity or the invalidity if they share a form. Either both of them would be true or both of them would be false, independent of the content of their propositions.

This feature of formal validity enables us to employ the tools of ‘logical analogies’ in our everyday usage of arguments. Logical analogy is very frequently used in debates and discussions to prove the validity of one’s own argument and to prove the invalidity of the argument of one’s opponent. The method of logical analogy is a testimony to the fact that the validity or the invalidity arising out of form (formal structure) is independent of specific content.

11.4.1 Axioms of Categorical Syllogism

There are six axioms that govern the formation of a standard form categorical syllogism. The rules are listed as follows.

11.4.1.1 Axioms of Quantity (Axioms pertaining to distribution of terms):

i. The middle term must be distributed at least one time in any of the premises.
ii. A term which is undistributed in the premise must remain undistributed in the conclusion as well.
iii. A term which is distributed in the conclusion should necessarily be distributed in the premise.

11.4.1.2 Axioms of Quality:

i. Two negative premises do not lead to any conclusion.
ii. Affirmative premises give only affirmative conclusion.
iii. Negative premise (given that there can be only one negative premise) leads to only negative conclusion.

Each of these rules, when broken gives rise to a specific fallacy. Further, the fallacy so occurred - occurs in the *form* of the syllogism and is called a formal fallacy. There are different types of formal fallacies. These rules in turn can be used to test the validity of a given categorical syllogism. This method of testing validity of categorical syllogisms is called the Traditional or Aristotelian Technique. These rules and the associated fallacies will be discussed in detail in the upcoming unit on ‘Techniques for Testing the Validity of Categorical Syllogism’.

11.5 LET US SUM UP

Syllogism is an important form of inference in traditional Aristotelian logic. It is a deductive argument in which a conclusion is inferred from two premises. The structure of a categorical syllogism is determined by moods and figures. Mood is determined by the arrangement of the categorical propositions (premises and conclusion) involved in the syllogism. The position of the middle term determines the figure to which syllogism belongs. Mood and figure of a categorical syllogism together provide a unique way of describing the logical structure of each categorical syllogism. The validity or the invalidity of syllogisms remains unaffected from the content of the propositions. The validity and the invalidity are tested against the form (logical structure) of a syllogism.

Check Your Progress III

Note: (a) Use the space provided for your answer.

(b) Check your answers with those provided at the end of the unit.

1. Point out the conclusion, premises and the middle term in the given argument. (Note: Always change the syllogism into standard form first, if it is not already in standard form.)

a. All Saints are Polite. No Sinners are Saints. Therefore, No Sinners are Polite.
b. All Eagles are Birds. Some Birds are Herbivores. Therefore, Some Herbivores are Eagles.

11.6 KEY WORDS

Categorical syllogism: A categorical syllogism is an argument which consists of exactly three categorical propositions (A, E, I, O - two premises and a conclusion) in which there are exactly three categorical terms, each of which occurs exactly twice.

Major premise: The first categorical proposition in a standard form categorical syllogism is the major premise. It introduces the major term.

Major term: The term occurring in the predicate of the conclusion in a categorical syllogism.

Middle term: The term occurring in both the major and the minor premises of a standard form categorical syllogism.

Minor premise: The second categorical proposition in a standard form categorical syllogism is the minor premise. It introduces the minor term.

Minor term: It is the subject of the conclusion.

Mood: Mood of a categorical syllogism is determined by the arrangement of the three propositions.

Figure: It is determined by the middle term.

Standard form categorical syllogism: Standard form of a categorical syllogism is the form in which a syllogism is said to be when its premises and conclusion are all standard form categorical propositions (that is, A, E, I, or O) and are arranged in standard order that is, major premise, then minor premise, and then conclusion.

Syllogism: A syllogism is any deductive argument in which a conclusion is inferred from two premises.

11.7 FURTHER READINGS AND REFERENCES

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11.8 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress I

All graduates are Poets. (major premise)

All Musicians are graduates. (minor premise)

Therefore, All Musicians are Poets (conclusion)

Check Your Progress II

1.

a. Major term: Biters, Minor term: Fish, Middle term: Sharks, Mood: AII, Figure: First (AII-1).
b. Major term: Dogs, Minor term: Cats, Middle term: Dangerous, Mood: III, Figure: Second (III-2).
c. Major term: Solar-powered Vehicles, Minor term: Four-wheeler Vehicles, Middle term: Cars, Mood: IEO, Figure: Fourth (IEO-4).

Check Your Progress III

1.

a. The syllogism is already given in standard form. Conclusion: Therefore, No Sinners are Polite, Major premise: All Saints are Polite, Minor premise: No Sinners are Saints, Middle term: Saints.
b. The syllogism is already given in standard form. Conclusion: Therefore, Some Herbivores are Eagles, Major premise: All Eagles are Birds, Minor premise: Some Birds are Herbivores, Middle term: Birds.

UNIT 12 METHODS FOR TESTING CATEGORICAL SYLLOGISM: ARISTOTELIAN AND VENN DIAGRAM*

Structure

12.1 OBJECTIVES

The objective of the unit is to introduce the methods for testing categorical syllogism. It must be clear by now that an argument is tested for its validity. This unit will enable learner;

- to use the Classical technique to test the validity of categorical syllogism.
- to use the Venn diagram technique to test the validity of categorical syllogism.

12.1 INTRODUCTION

A syllogism, in the most general terms, can be defined as a deductive argument with three propositions where two of them serve as premises and the third one is the conclusion based on

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the premises. If all the three prepositions are categorical proposition, the syllogism thus obtained is called categorical syllogism. In order to test the validity or the invalidity of a categorical syllogism, the first requirement is that we must restructure the given categorical syllogism into its standard form. We can apply any technique, be it the traditional Aristotelian technique or technique of Venn diagram, only when the syllogism is in its standard form. A standard form categorical syllogism has a certain fixed order in which its constituent proposition should be arranged. The first proposition is major premise; the second is minor premise, followed by conclusion.

Aristotelian method of testing the validity of categorical syllogisms can be seen as a corollary to the six rules or axioms of the formation of categorical syllogism. If those six rules are followed in the construction of a syllogism, the syllogism thus formed would be a valid syllogism in principle. The unit will provide a detailed illustration of the six rules and in addition to that the fallacies which arise when one or more than one of these six rules are broken.

In the Venn diagram method, after restructuring a given categorical syllogism into standard form, we need to draw the Venn diagram of the given standard form categorical syllogism. Since the most important aspect of Venn diagram technique to test the validity of categorical syllogism is to be able to represent the categorical syllogism in Venn diagrams, the unit would provide a recapitulation of how categorical prepositions are represented in Venn diagrams.

12.2METHODS/TECHNIQUES

12.2.1 Aristotelian Technique

A syllogism can fall short of establishing its conclusion from its premises in more than one ways. The rules prescribed in traditional logic work as a prescription for anyone trying to use syllogisms, so that he or she may be able to form a valid argument. These rules are six in number. To test the validity of a given categorical syllogism, one should start with these rules - testing the syllogism against each one of them one by one, and if it is found that one of these rules is not being followed, it can be argued immediately that the given syllogism is invalid, without checking other rules. However, this does not imply that any given syllogism cannot have more than one fallacy. The rules are explained as follows:

12.2.1.1 Rule 1: Avoidance of more than three terms

A categorical syllogism, by definition contains only categorical propositions and the conclusion of the syllogism establishes the relation between the two categories represented by major term (predicate class of the conclusion) and minor term (subject class of the conclusion). This is done with the help of the middle term which is found in both the minor and the major premises but is unavailable in the conclusion. So there should not be any other term/class in a categorical syllogism. If there are more than three terms, then it is logically impossible to establish the conclusion about any two terms out of the given four terms using just three propositions. Hence, it becomes clear that if there are more than three terms, the argument fails to qualify as a categorical syllogism. A categorical syllogism should have at least and at most three terms, each occurring twice, spread across three propositions.

The fallacy that arises when this rule is faltered is called *Fallacy of four terms*.

Example:

All teachers are graduates.

Some graduates are sportsmen.

Therefore some teachers are athletes.

In this example, there are four terms, teachers, graduates, sportsmen and athletes. Hence the argument does not qualify to be a categorical syllogism despite the fact that all propositions used in the argument are in standard form. The information provided by the premises does not give us sufficient reason to assert the conclusion. Hence, the argument suffers from the fallacy of four terms.

12.2.1.2 Rule 2: Distribution of the middle term

A term is said to be distributed when it occurs in such a place in a proposition that the proposition refers to all the members of the class represented by that term, as it should be clear from the section on categorical propositions. This rule requires that the middle term must be distributed in either of its occurrence and it does not matter which. In other words, middle term must be distributed in the major or in the minor premise. As explained earlier, the middle term is the bridge that fills the gap between the major and the minor term and helps us assert a relation between the two in our conclusion. The major premise asserts the relationship between the major term and the middle term and the minor premise on the other hand asserts the relationship between the minor term and the middle term. So if the middle term is not distributed in either of these propositions then it means that no relationship has been established either by major term or by minor term with *all the members* of the class represented by middle term. This leaves out a vast unknown gap for us to be able to establish anything between major and minor term conclusively. Hence the conclusion cannot be said to have derived from the premises.

The fallacy which arises when this rule is faltered is called *Fallacy of undistributed middle*.

Example:

All horses are mammals.
All whales are mammals,
Therefore, all whales are horses

In the above example, the argument is a standard form categorical syllogism of form AAA-2. Since the middle term (mammals) is undistributed in both minor and major premises, the argument is invalid.

12.2.1.3 Rule 3: Any term distributed in the conclusion must be distributed in the premise

By now this would have become clear that the concept of distribution is important for applying these rules to test the validity of categorical syllogism by the Traditional method. As explained earlier, the conclusion establishes the relation between the classes represented by its subject and predicate terms, which are called in the discussion of syllogism, minor and major terms respectively. So if either of these terms is distributed in the conclusion, it means that the conclusion is referring to all the members of this class. And if this conclusion is to be followed from the premises, then that term must be distributed in the premise in which it has occurred before the conclusion. If the major term (predicate) of the conclusion is distributed, then the major term must be distributed in the major premise and the same applies in the case of minor term. However, this rule does not apply vice versa, that is to say if a term is distributed in the premises then it may not be a necessary condition for that term to be distributed in the conclusion. If one imagines the class, and if the relationship is known for all the members of that class, then it is logically possible for someone to assert something for some members of that class but it is not possible vice versa.

The fallacy that arises when this rule is not observed is called *Fallacy of Illicit Process*. There are two subclasses within the fallacy of illicit process, *illicit minor* and *illicit major* referring to the cases of minor term being distributed in the conclusion and not in the premise and the case of major term not being distributed in the conclusion and not in the premise, respectively.

Example:

All Sofas are Chairs
No Recliners are Sofas
Therefore, No Recliners are Chairs.

The argument is a standard form categorical syllogism (AEE-1). The predicate term of the conclusion, which is the major term (Chairs) is distributed in the conclusion, but is not distributed in the major premise. Hence the argument is invalid as it suffers from the fallacy of illicit major. Had it happened for the minor term, the fallacy would have been illicit minor.

12.2.1.4 Rule 4: Avoidance of two negative premises

A negative proposition asserts the exclusion or non-inclusion of two classes or some parts of two classes. In other words, it establishes that some part of a class or the whole class is excluded from the other class and if both the premises of a categorical syllogism are negative in quality - that denies the inclusion of middle term with both minor and major term, and hence no relation can be established in the conclusion between major and the minor term, based on these premises. In other words, if both the premises assert exclusion, then they cannot base a conclusion asserting any kind of inclusion. Such a rule is very useful when it comes to test the validity of a given syllogism, as it becomes evident at the first instant itself if the given syllogism has two negative premises, and the given syllogism can be classified as invalid if it happens to have two negative premises and no further scrutiny is required there.

The fallacy that arises when this rule is faltered is called *Fallacy of exclusive premises*.

Example:

No cows are carnivores.

Some animals are not carnivores.

Therefore, some animals are not cows.

In this example, despite syllogism being in its standard form, we need not look any further for the validity or the invalidity of the argument. The argument suffers from the fallacy of exclusive premises and is invalid *prima facie* as it contains two negative premises.

12.2.1.5 Rule 5: An affirmative conclusion is not possible if one of the premises is negative

An affirmative conclusion asserts that minor and major terms are contained in each other, wholly or partially. However, such a conclusion requires the assertion from the major and the minor premises of the similar relationship between major and minor terms with middle term respectively. If either of the given premises does not provide such an inclusion, then the conclusion must not provide the inclusion either. That is to say, the conclusion cannot be affirmative if either of the premises is negative. This rule is another very easy rule to apply. As one can figure out in a given categorical syllogism prima facie whether the conclusion drawn from the premises is negative or affirmative, and if affirmative, is one of the given premise is negative.

The fallacy that arises when a syllogism does not follow this rule is called *Fallacy of drawing an affirmative conclusion from a negative premise*.

Example:

All cricketers are athletes

No athletes are drinkers

Therefore all drinkers are cricketers.

This argument is invalid in the very first observation as an affirmative conclusion is being drawn from a negative premise.

12.2.1.6 Rule 6: No particular conclusion may be drawn from two universal premises

As it would have been gathered from the section of categorical propositions that the universal propositions viz., A, E do not have existential import, but the particular propositions, viz., I, O have existential import. Applying Boolean analysis of categorical propositions, no conclusion that does not have existential import can be drawn from the premises that do have existential import. This rule is sometimes not included in the traditional account of categorical syllogism for the reason that traditional or Aristotelian logic does not concern itself with the problem of existential import. However, this rule is another useful tool to test the validity of a given categorical syllogism.

That fallacy that arises when this rule is flouted is called the *Existential fallacy*.

Example:

All vegetables are green
All spices are green
Therefore, some spices are vegetables.

In this argument, a particular conclusion is being drawn from universal premises hence the argument suffers from existential fallacy, and therefore, it is invalid.

Check Your Progress I

Note: (a) Use the space provided for your answer.

(b) Check your answers with those provided at the end of the unit.

1. Test the validity of the given syllogistic moods using the traditional six rules technique.

EAE-2, AII-2, AII-3, IAI-2, OAO-4, AOO-1

12.2.2Venn Diagram Technique

Before we go into the method, let us recall how we represent categorical propositions in Venn Diagrams.

A: All S is P

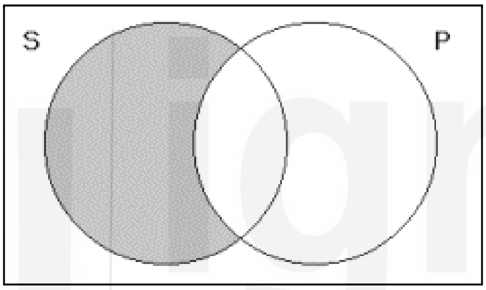

Figure 2.1

The shaded portion of S is an empty class, which is to say that every member of S is a member of P. Or in other words, there is no member of S which is not a member of P.

E: No S is P

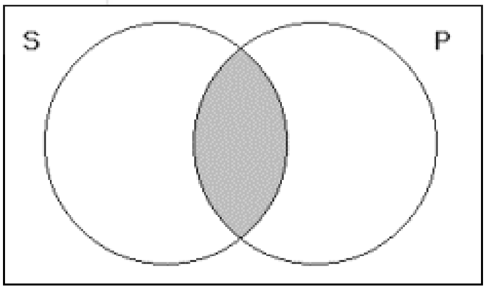

Figure 2.2

The shaded portion in between S and P again shows that there is no such member of either class which is also a member of the other class.

I: Some S is P

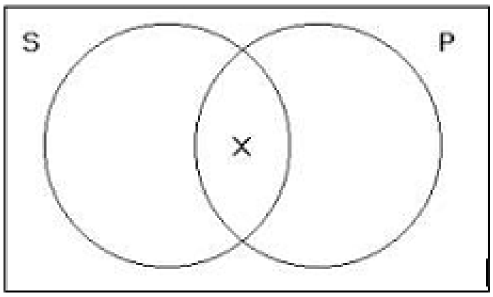

Figure 2.3

The portion where 'x' is written represents that there is at least one member of either class which is the member of the other class.

O: Some S is not P

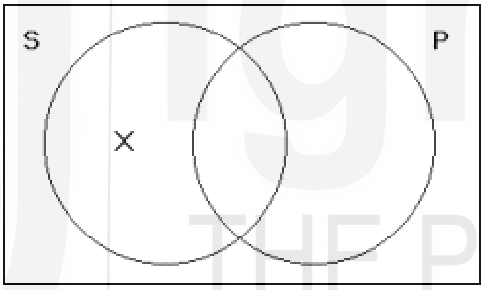

Figure 2.4

The position of 'x' represents that there is at least one member of the class S which is not a member of P.

One thing that should be noted here is that there are other ways to draw some of these propositions and that would not require us to shade some or the other portion, however, to maintain uniformity, it is advisable to draw two overlapping circles and then represent the given proposition by shading out some portion or inserting an X. For example, A can be drawn also as following (Figure 2.5)

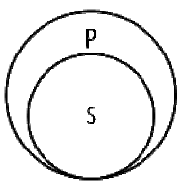

Figure 2.5

This Venn diagram represents the same fact which Figure-2.1 does, A, but we follow a uniform representation so that it's easier for us to visualise the differences between the different categorical syllogisms.

Now as we are clear about the Venn diagram for categorical propositions, we can move forward to testing the validity and invalidity of the categorical syllogism using Venn diagrams. The method involves following steps:

I. As it was mentioned in the introduction, it is required to convert a given categorical syllogism into standard form categorical syllogism, no matter whether the Aristotelian or Venn diagram technique is being followed so the first step would involve obtaining a standard form categorical syllogism.

II. The next step would require us to draw both the premises in a Venn diagram. The important thing to note here is that the two premises taken together contain three classes, viz., the classes represented by Major term (P), Minor term (S) and the Middle term (M). These three overlapping circles give rise to eight classes and the figure would look like the following (Figure 2.6):

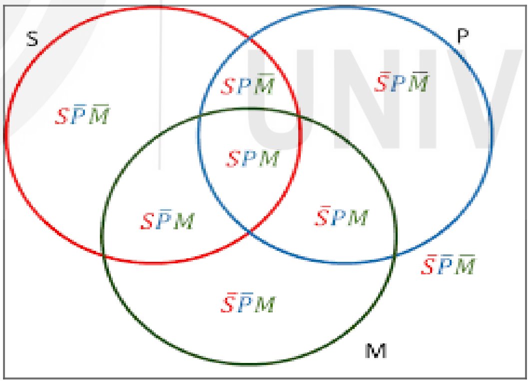

Figure 2.6

The classes being SP̄M̄, SPM̄, S̄PM̄, S̄PM, S̄P̄M, SP̄M, SPM and S̄P̄M̄.

III. After obtaining the Venn diagram representing both the premises, we are required to draw a Venn diagram representing the conclusion.

IV. If both the diagrams (diagram obtained in step II and Step III) look exactly alike, or corroborate each other, then the given syllogism is a valid syllogism, else invalid.

The method will become clearer with an illustration:

Let’s consider the example of syllogism of form AAA-1

So the syllogism will pan out as: All M is P

<u>All S is M</u>

Therefore, All S is P.

So to draw the first premise, we need to focus on the circles labelled as M and P, and shade out all the area of M which is not P. This shaded area will include both the regions $S\bar{P}M$ and $\bar{S}\bar{P}M$. In the next step, we will represent the second or Minor premise. To draw the minor premise, our focus should be on the circles named S and M, and we need to shade out all of S which is not M, and this shade will include both the area $SP\bar{M}$ and $S\bar{P}\bar{M}$. Once we have drawn both the premises, our diagram would look like the following one (Figure 2.7):

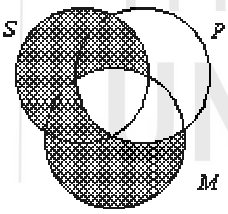

Figure 2.7

Now, a syllogism is considered valid if and only if the premises entail the conclusion and in the Venn diagram technique, this entailment is checked by testing whether after drawing the premises, the diagram obtained represents the conclusion or not. Now in the considered example, if one wishes to draw the conclusion, one need not shade out any portion or need not insert any X in the diagram obtained by the premises in order to represent the conclusion. In other words, if we were to draw the conclusion “All S is P”, that would require us to shade out the portions labelled $S\bar{P}\bar{M}$ and $S\bar{P}M$, and as it is evident in the diagram that these two portions are already shade. Therefore, the considered syllogism is valid.

Let us consider another syllogism which we know to be invalid and see how we arrive at its invalidity using the Venn diagram technique. Let's consider the syllogism of the form AAA-2.

So the syllogism will pan out as: All P is M

All S is M

Therefore, All S is P.

To draw the first or major premise, we will have to focus on the circles named P and M and we will shade out all of P which is not M, this shade will include both the portions $\bar{S}P\bar{M}$ and $SP\bar{M}$. similarly, when we draw the minor premise, we will shade out all the S which is not M. this shade will include $SP\bar{M}$ and $S\bar{P}\bar{M}$. The diagram that we will obtain after representing both the major and the minor premises would look like the following one (Figure 2.8):

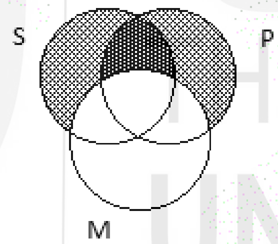

Figure 2.8

Now, does this diagram represent the conclusion? No. As it is evident that if we draw the conclusion "All S is P", we will have to shade out $S\bar{P}M$ which is left unshaded in the diagram obtained by representing the premises. In other words, the conclusion asserts more than what has been asserted by two premises taken together and therefore the conclusion cannot be logically entailed by the premises, hence the considered categorical syllogism is invalid. Since, what we have proved is the invalidity of the form therefore any syllogism of the form AAA-1 will be valid and any syllogism of the form AAA-2 will be invalid.

Both of the examples that we have considered above had only universal propositions as their premises, so it would not have mattered which premise is being drawn first, however, the sequence becomes very crucial if in a given syllogism one of the premises is of Universal quantity and the other is of Particular quantity. In that case, it is crucial to draw the diagram of Universal premise first before we draw the particular one. It is not pertinent to the matter which of these is minor or major. This sequence is followed so that we can become sure about inserting an element 'x' to represent the particular proposition. Let's consider an example of a syllogism which has one universal and one particular proposition as its premises, AII-3

So the syllogism will pan out as: All M is P

Some M is S

Therefore, Some S is P.

So, the first preposition that we will draw would be "All M is P" by shading out all of M that is not P. This shade will include $\bar{S}\bar{P}M$ and $S\bar{P}M$. Then when we draw the next proposition "Some M is S", we will have to insert an "x" in the intersection area of S and M, but that intersection is divided into two parts, $S\bar{P}M$ and SPM, and since we have already drawn the universal premise, and the class $S\bar{P}M$ is already shaded out, the "x" should come inside the unshaded part of the intersection, that is SPM. The Venn diagram thus obtained from the premises would look as following (Figure 2.9):

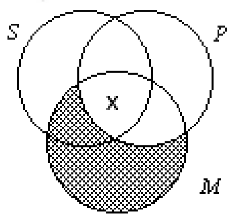

Figure 2.9

Now, once we have obtained the Venn diagram representing both the premises, we can analyse it by corroborating to the Venn diagram of the conclusion to see whether the syllogism is valid or invalid. If the conclusion asserting that "Some S is P" has already been drawn in this Venn diagram then there must be an 'x' somewhere in the intersection of S and P. this intersection part is consist of two sub parts $SP\bar{M}$ and SPM and an 'x' is there in the SPM region, therefore the given syllogism is valid.

Let's consider one more example of the kind of syllogism in which the placing of 'x' to draw a particular proposition does not follow the kind of placement we did in the last example, AII-2.

The syllogism will pan out as: All P is M

Some S is M

Therefore, Some S is P.

As discussed in the last example, in order to draw the premises, we will start with the universal premise, "All P is M" in this case. We will shade out all of P which is not M; this shade will cover the classes $SP\bar{M}$ and $\bar{S}P\bar{M}$. When it comes to placing an 'x' to draw the particular proposition "Some S is M" in the Venn diagram, we still have both the regions SPM and $S\bar{P}M$ unshaded. So where do we place 'x'? - on the line separating the two regions either of which can have 'x' but it is not clear in the syllogism. The reason why 'x' is placed on the line and not in either SPM or $S\bar{P}M$, is because the premises do not provide enough information for us to decide the class where 'x' should be placed. If we place 'x' arbitrarily in either of the two classes, the diagram would represent more information than that which is being provided by the premise. Therefore, it is very important to be cautious about what and how much is being asserted by the premises, if we do not want our Venn diagram technique to fail. So the Venn diagram obtained would look like the following figure (Figure 2.10):

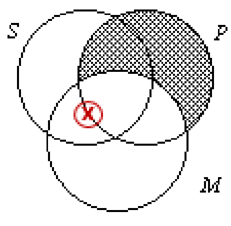

Figure 2.10

Now, once the Venn diagram representing the assertion of both the premises have been obtained, we would go on to the next step to see whether the conclusion of the syllogism has already been drawn here, the conclusion “Some S is P” requires that the ‘x’ be placed neatly in the intersection of S and P, either in SPM or in SP$\bar{M}$. However, as it is clear that SP$\bar{M}$ has already been shaded out and contains no ‘x’ and the other region SPM may or may not have ‘x’ and we do not have enough information in the premises to place an ‘x’ in SPM, hence the syllogism is invalid. Another point that should be noted here is that the premises here do not give us enough information to claim that the conclusion *is* false. It may be false or may be true. All we know that the conclusion is not being *entailed* by the premises.

Check Your Progress II

Note: (a) Use the space provided for your answer.

(b) Check your answers with those provided at the end of the unit.

1. Test the validity of the given syllogistic moods using the Venn diagram technique.

EAE-2, AII-2, AII-3, IAI-2, OAO-4, AOO-1

12.3 FURTHER ON THE CATEGORICAL SYLLOGISM

There are four categorical propositions which can be used to give rise to any syllogism. Since a categorical syllogism involves three propositions, then there could be total 64 possibilities of different combination of categorical propositions (4^3=64, 4 being the number of categorical propositions and 3 being the number of propositions required for a syllogism). In other words, there can be 64 moods of categorical syllogism and since each of these mood can be found in any of the four figures, the total number of forms (mood and figure taken together) of categorical syllogisms possible are 256 (64*4=256). Out of these 256 forms of categorical syllogisms, only fifteen forms are Valid. Since the number of forms that are valid is relatively less, each of these fifteen forms has been given unique names which are as follows:

i.	AAA-1	Barbara
ii.	EAE-1	Celarent
iii.	AII-1	Darii
iv.	EIO-1	Ferio
v.	AEE-2	Camestres
vi.	EAE-2	Cesare
vii.	AOO-2	Baroko
viii.	EIO-2	Festino
ix.	AII-3	Datisi
x.	IAI-3	Disamis
xi.	EIO-3	Ferison
xii.	OAO-3	Bokardo
xiii.	AEE-4	Camenes
xiv.	IAI-4	Dimaris
xv.	EIO-4	Fresison

It is neither important nor required to remember the names of these valid forms. As long as one remembers the basic six rules of categorical syllogism, making a distinction between valid and invalid syllogisms can be carried out easily.

12.4 LET US SUM UP

One of the many purposes of logic itself is to distinguish valid arguments from invalid ones. Categorical Syllogisms are different from other kinds of arguments because of their formal nature. The formal nature of the syllogisms enables us to analyse these solely on the basis of their form. Since, the analysis at the formal level is completely devoid of the content; we saw in this unit that this provides us an opportunity to use abstract mathematical method of Venn diagrams to test the validity of syllogisms. The use of Venn diagrams brings out the relation between the set theory and logic. The unit provided the Aristotelian (the Classical) technique of testing categorical syllogism and Venn diagram technique.

12.5 FURTHER READINGS AND REFERENCES

- Copi, I.M. *Introduction to Logic*. New Delhi: Prentice Hall India, 9th Ed., 1995.
- Copi, I. M., and Carl Cohen. *Essentials of Logic*. 2nd ed. Upper Saddle River, N.J.: Pearson Prentice Hall, 2007.

12.6 ANSWERS FOR CHECK YOUR PROGRESS

Check Your Progress I and II (Please use the traditional and Venn diagram techniques to find the validity and invalidity)

EAE-2	valid
AII-2	invalid
AII-3	valid
IAI-2	invalid
OAO-4	invalid
AOO-1	invalid

UNIT 13 NON-CATEGORICAL SYLLOGISM*

Structure

13.0 OBJECTIVES

The objectives of this unit are,

- to introduce the non-categorical syllogisms.
- to elaborate two major kinds of non-categorical syllogisms, namely: disjunctive syllogism and hypothetical syllogism.
- to discuss another kind of non categorical argument called dilemma, which is very commonly used in everyday life and the different types of dilemmas and how they can be tackled.
- to enable the learners to distinguish logically sound dilemmas from those which are not sound or acceptable.

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13.1 INTRODUCTION

A proposition is classified as a categorical proposition when it establishes the relation between the two categories or classes. The relation can be of exclusion (negative proposition) or of inclusion (affirmative propositions). A syllogism is called categorical syllogism when it is constituted of categorical propositions. However, there are propositions which are not categorical and the syllogisms formed by these propositions are named as per the propositions they are made of. As discussed in the earlier unit, an argument is classified as a syllogism when it has three propositions of which two serve as premises and the third one as conclusion[*]. Since the discussion is about non-categorical syllogisms (syllogism being the key word here), we will still be dealing with the arguments with three propositions. The important distinction between the categorical and non-categorical is that in the latter case, at least one of the propositions must not be a categorical proposition or all of them may not be categorical.

Example:

> Either the global warming is correct or earth's temperature would not rise.
>
> Earth's temperature is rising.
>
> Therefore, Global warming is correct.

The syllogism given above is of compound nature. If we follow the modern logicians' nomenclature, the term non-categorical syllogism becomes contradictory. For them a syllogism has to be categorical, and if it is not categorical then it is not syllogistic in nature. But for traditional logicians, any argument with two premises and a conclusion is syllogistic in nature and then they are classified into categorical and non-categorical depending upon the kind of propositions they contain. There are two most prominent non-categorical syllogisms. They are also called compound syllogisms.

1- Disjunctive syllogism
2- Hypothetical Syllogism

13.2 DISJUNCTIVE SYLLOGISM

[*]This distinction was maintained by the classical or Aristotelian logicians. For modern logicians starting from Boole, an argument is syllogistic only when it is a categorical syllogism. This is the reason that in the modern logic, terms 'syllogism' and 'categorical syllogism' are used interchangeably.

As mentioned earlier, the syllogisms are named in accordance to the kind of propositions they contain. A categorical syllogism is named so for the reason that it contains propositions which are in standard form or categorical. A disjunctive syllogism contains a disjunctive proposition. Disjunctive propositions are also called Alternative propositions. An example of disjunctive proposition is “Rama is either a very intelligent person or a very lazy one”. The two parts of this disjunctive proposition are simple propositions, namely, “Rama is a very intelligent person” and “Rama is a very lazy person”. The components of a disjunctive proposition are called *disjuncts*. A disjunctive proposition does not assert the truth or falsity of any of its disjunct conclusively. The implication that one can draw from a disjunctive proposition is that either one of the two components is true. This leaves the possibility open for both the disjuncts to be true.

If in a given syllogism, one of the premises is a disjunctive proposition and the other proposition of the syllogism is a denial of either of the disjunct of the first premise, then we can conclusively assert the truth of the second disjunct in the conclusion. Such disjunctive syllogisms are valid disjunctive syllogisms.

Example: Either Virat was injured or he was not selected for the team.
Virat was selected for the team.
Therefore, Virat was injured.

In the considered disjunctive syllogism, the two disjuncts of the first premise are, “Virat was injured.” and “Virat was not selected for the team”. The second premise of the given syllogism, “Virat was selected for the team” denies the second disjunct, and the conclusion asserts the other disjunct, hence the given disjunctive syllogism is valid.

A disjunctive syllogism can be stated symbolically in either of the two forms given as follows

i. Either *p* or *q*
~*p*
Therefore, *q*

ii. Either *p* or *q*
~*q*
Therefore, *p*

So in other words, a disjunctive proposition is valid when it follows the following three conditions:

i. The first of the two premises is a disjunctive proposition. (In fact, either of the two premises could be a disjunctive proposition, the order of first and second is primarily to visualise the assertion of argument.)
ii. The next premise is a negation of or is in contradiction to, either of the two disjuncts of the first disjunctive proposition.
iii. The conclusion asserts the remaining disjunct of the disjunctive proposition considered in the first step.

In the example that we had considered before laying out these rules, all the three conditions were being fulfilled. Therefore, the given disjunctive syllogism was classified as a valid disjunctive syllogism.

If we are classifying a category of valid categorical syllogisms, then *ipso facto*, there should be another category which needs some discussion, the category of invalid disjunctive syllogisms. Let’s consider the following disjunctive syllogism,

Either X institute is very expensive or it is very good.
X institute is very good
Therefore, X institute is not very expensive.

If we write the above argument in the symbolic form, we will get the following,

Either p or q
p
Therefore $\sim q$

If we look closely at the argument given above, we find that the conclusion does not necessarily follow from the premises. Even if we assume that both the premises of these disjunctive syllogisms are true, there is no implication of the conclusion. It is possible that the institute X is very good and at the same time it is very expensive. In other words, the truth of either of the two disjuncts of the given disjunctive proposition doesn’t imply the falsity of the other disjunct. Both of the disjuncts can be true together. If we can recall the relationships which were discussed in traditional square of opposition, we can claim that the two disjuncts share the relationship of the contrariety and not of contradiction. The truth of one does not entail the falsity of the other. Both can be true together but cannot be false together. So the disjunctive syllogism of the form mentioned above is invalid. The conditions of the invalidity of the disjunctive syllogisms can be summarised as follows:

i. The first premise is a disjunctive syllogism.
ii. The second premise affirms one of the two disjuncts of the first premise.
iii. The conclusion denies the other disjunct of the disjunctive proposition given in the first step.

In symbolic form, an invalid disjunctive syllogism can have either of the two following forms:

i. Either *p* or *q*
   p
   Therefore, ~*q*

ii. Either *p* or *q*
   q
   Therefore, ~*p*

It is clear from the discussion above that a disjunctive syllogism is valid only when the second premise *contradicts* one of the two disjuncts contained in the disjunctive proposition, and the conclusion asserts the other disjunct. Cases where the categorical premise is simply asserting one of the disjuncts of the disjunctive premise, and conclusion is denying the other disjunct, the disjunctive syllogisms in those cases would be invalid.

13.3 HYPOTHETICAL SYLLOGISM

Another non categorical syllogism that is often used and discussed in reasoning is Hypothetical syllogism. Any syllogism which has one or more than one hypothetical propositions is classified as hypothetical syllogism. Hypothetical proposition is another compound proposition. It is also known as conditional proposition. An example of hypothetical proposition could be, "If it rains then the match will be cancelled". This hypothetical proposition contains two propositions, one which is followed by 'if' and preceded by 'then', the other followed by 'then'. The first component proposition is called *antecedent* and the second component which is followed by 'then' is called *consequent*. The two components of a hypothetical proposition are always related by "If-then" relationship.

Hypothetical syllogisms are divided into two kinds, depending on the kinds of propositions they contain. If all the propositions in a given hypothetical syllogism are hypothetical, then the syllogism will be classified as pure hypothetical syllogism. If there is at least one of the propositions in a hypothetical syllogism is categorical, then the given syllogism will be classified as mixed hypothetical syllogism.

13.3.1 Pure Hypothetical Syllogism

let's start by considering an example-

If Shyam is regular in his studies, then he gets good grades.
If he gets good grades, then he has a better job prospect.
Therefore, if Shyam is regular in his studies, then he has a better job prospect.

In the above example, all the three propositions involved are hypothetical or conditional in nature; therefore it is a case of pure hypothetical syllogism. On closer observation, we can see that the antecedent of the first premise is same as that of the conclusion and the consequent of the second premise is same as that of the conclusion. This setup of component parts makes this hypothetical syllogism a valid syllogism. In general terms, only that pure hypothetical syllogism is valid in which the antecedent of the conclusion is shared by one of the premises and the consequent of the conclusion is shared by the other remaining premise. In addition to this, consequent of the premise (which shares its antecedent with the conclusion) must be shared by the antecedent of the other premise (which shares its consequent with the conclusion). In symbolic representation, a valid pure hypothetical syllogism can be represented in either of the two following ways:

i. If p, then q
   If q, then r
   Therefore, if p, then r

ii. If q, then r
   If p, then q
   Therefore, if p, then r

These are the only two ways in which valid pure hypothetical syllogisms can be expressed. Following are the ways in which pure hypothetical syllogisms will be invalid-

i. If p, then q
   If q, then r
   Therefore, if r, then p

ii. If q, then r
   If p, then q
   Therefore, if r, then p

iii. If p, then q
   If r, then q
   Therefore, if p, then r

iv. If p, then q
   If p, then r
   Therefore, if q, then r

13.3.2 Mixed Hypothetical Syllogism

In a mixed hypothetical syllogism, there is only one hypothetical proposition as premise, and the other premise and the conclusion is categorical in nature. Let's consider an example

If it snows, then the roads will be blocked.

It is snowing.

Therefore, the roads will be blocked.

In the above example, the first premise is a hypothetical proposition and that is the only hypothetical proposition here, hence the considered syllogism is hypothetical syllogism. The second premise is same as the antecedent of the first premise and the conclusion is same as the consequent of the first premise. This is a valid hypothetical syllogism. There is another form in which mixed hypothetical syllogism can be expressed validly. Let's take another example,

If it snows, then the roads will be blocked.

Roads are not blocked.

Therefore, it did not snow.

In the above example, the first proposition is the same disjunctive proposition, however, the second premise here is the negation of the consequent of the first premise and the conclusion in this case is the negation of the antecedent of the first hypothetical premise. This is another valid form for mixed hypothetical syllogism. Symbolically, these two valid forms can be expressed as follows,

i. If p, then q
   p
   Therefore, q

ii. If p, then q
   ~q
   Therefore, ~p

The rules of inferences involved in these valid mixed hypothetical syllogisms are *Modus Ponens* and *Modus Tollens*, respectively. The literal meaning of the phrase "Modus Ponens" is "the mode that affirms by affirming". In the first form, we can see that the conclusion is being affirmed by the assertion of the antecedent of the hypothetical proposition. "Modus Tollens", on the other hand, means "the mode that denies by denying". It is again evident in the second case that the antecedent of the hypothetical proposition is being denied in the conclusion based on the denial of the consequent of the hypothetical proposition.

It is important to understand here, that these two valid forms can often be confused with the invalid ones, as they appear similar. The invalid forms of mixed hypothetical syllogism are as follows,

i. If p, then q
   ~p
   Therefore, ~q (fallacy of denying the antecedent)

ii. If p, then q
   q

Therefore, p (fallacy of affirming the consequent)

The first one looks like Modus Tollens but differs from it because of the fact that the antecedent is being denied in the second premise, instead of the consequent and the negation of consequent is being affirmed in the conclusion. This fallacy is called the fallacy of denying the antecedent.

The second one above looks like Modus Ponens but differs from it for the reason that the consequent is being affirmed in the second premise instead of the antecedent. This fallacy is called fallacy of affirming the consequent.

Check Your Progress I

Note: (a) Use the space provided for your answer.

(b) Check your answers with those provided at the end of the unit.

Identify the kind of non categorical syllogism in the following arguments (1-3) and check their validity.

1. Either India does not have enough economic resources or Indian citizens are not willing to change their standards of living.

India does have enough economic resources.
Therefore, Indian citizens are not willing to change their standard of living.

----

1. Either Shyam goes for shopping very frequently or he receives a lot of gifts from his friends.
   Shyam does not go for shopping very frequently.
   Therefore, he does not receive a lot of gifts from his friends.

----

2. If the weather is nice today then we shall go for outing.

   The weather is nice.

   Therefore, we shall go for outing.

Choose the right conclusion for the given Hypothetical Syllogism among the two alternatives so that the syllogism remains valid:

4. If Indian team plays well, then it will win against other team.

If Indian team wins against other teams then it will win the world cup.

i. Therefore, if Indian team wins the world cup, then it plays well.

ii. Therefore, if Indian team plays well then it wins the world cup.

13.4 THE DILEMMA

The dilemma is altogether a different kind of argument as compared to the arguments we have discussed so far. Dilemma is commonly used in ordinary language during debates and discussions. Almost all of us must have heard people using the word dilemma to describe their day to day confusion to decide things. People are often heard saying that they are in a serious dilemma when they are indecisive about choosing one among two equally unpleasant alternatives. Hence, dilemma can be better understood as a rhetoric device employed by debaters to counter or ridicule their opponents' positions. This rhetoric device makes use of different syllogisms and syllogisms used are often ordinary syllogism from the perspective of logic. Because of the use of these ordinary syllogisms, dilemma does not have much importance if evaluated from a strict logical point of you. Dilemma, nonetheless, is considered one of the very lethal and effective techniques of persuasion. If one uses the technique correctly, one can force one's opponent in a corner to make him choose from the given alternatives, both of which would be contradictory or often at least unpleasant to him. It happens more than often, yet dilemma need not always have unpleasant alternatives to chose from. From the view point of logic, and the reason why dilemma is often categorised as one of the non-categorical syllogisms, is that a dilemma can be presented in a standard form to have three propositions, out of which two serve as premises for the third one which will serve as the conclusion. All the three propositions used in a dilemma are compound propositions. One of the two premises is often a conjunction of two hypothetical propositions; the other premise is often a disjunctive proposition. The conclusion drawn from these will often be a disjunctive proposition, presenting the two alternatives to chose from. The two alternatives provided in the conclusion of a dilemma are known as two horns of the dilemma.

There could be two valid forms of inference in which a dilemma can be put, Constructive Dilemma and Destructive Dilemma. The symbolic representation of the two is given as follows-

- **Constructive Dilemma**

  (If p then q) and (if r then s)

  Either p or r

  Therefore, q or s

  In propositional logic, the same can be presented as:

  $(p \supset q) * (r \supset s)$

  $p \vee r$

  $\therefore q \vee s$

- **Destructive Dilemma**

(If p then q) and (if r then s)

Either not q or not s

Therefore, either not p or not r

In propositional logic, the same can be presented as:

$$(p \supset q) * (r \supset s)$$

$$\sim q \vee \sim s$$

$$\therefore \sim p \vee \sim r$$

There are two ways one can deal with a given dilemma, one is logical approach and the other one is informal one. The two approaches are as follows,

13.4.1 Refutation of Dilemma

This is a logical approach to deal with a dilemma. In this approach the attempt is made to prove the dilemma to be invalid or unsound. The *modus operandi* of this approach is to attack either of the premises. If it can be shown that, in the premise containing the disjunctive propositions, both of the given disjuncts are false, and then the proposition itself will be false. (Recall, false ∨ false = false). This would make the conclusion undetermined and the argument, invalid. Because the conclusion is asserted on the assumption that either of the disjuncts presented in this premise is true.

Refutation of a dilemma is difficult than it seems to be, and since as mentioned earlier, the device of dilemma is often used in rhetoric, the informal methods take precedence over the formal one.

13.4.2 Rebuttal of Dilemma

This approach is an informal approach and the purpose here is not to prove dilemma invalid but to find the ways in which *one can try to avoid the conclusion of the dilemma* presented to him, without challenging the formal validity of the dilemma. Rebuttal is often done in three ways, which are discussed as follows:

13.4.2.1 Rebuttal by posing a counter dilemma:

This is the most lethal of the ways to defeat one's opponent in the debates. When presented by a dilemma, one could posit a counter dilemma (often with at least one same premise) without even discussing the alternatives presented, and arriving at a different alternative. A very famous example that highlights the rebuttal by positing a counter dilemma comes from a classical anecdote of Athens, where a mother is trying to persuade her son to not enter into politics. The dilemma presented by mother goes as,

"If you say what is just, men will hate you; and if you say what is unjust, the gods will hate you; but you must either say the one or the other; therefore you will be hated."

The son countered the dilemma presented to him by his mother, by presenting a counter dilemma using one of the premises used by his mother. The counter dilemma goes as,

"If I say what is just, the gods will love me; and if I say what is unjust, men will love me. I must say either the one or the other. Therefore I shall be loved!"

Such rebuttals are considered to be come from those who have a great rhetorical skill.

13.4.2.2 Escaping between the horns

As the name suggests, this technique works by trying to avoid both the horns of a dilemma. The strength of dilemma lies in the fact that the two alternative options presented in the conclusion are taken to be the *only* two possible alternatives that can be derived from the given premises. So if one tries to negate this possibility of there being only two possible alternative, the approach is called escaping between the horns.

Let's consider the following example that will make it clear,

If Ram is a hard working employee, then he needs no incentives and if he is lazy then no incentive will motivate him.

An employee is either hard working or lazy.

Therefore, incentives are either needless or useless.

The disjunctive proposition claims that there are only two alternatives possible, that every employee is either lazy or hardworking, but this is a wrong assumption. There could be many employees who are neither exceptionally hard working nor lazy, rather who do as much asked or required. These employees can be motivated if they are given incentives. Once this false dichotomy is exposed, it will show that the alternatives presented in the conclusion are not the only alternatives. This has been achieved by showing the minor premise to be false. This is an instance of escaping between the horns.

13.4.2.3 Taking the Dilemma by the horns (grasped by the horns)

In the last case, we saw that if the minor premise was proved to be false, it made the dilemma false. However, if the given minor premise is a tautology then it is not possible to escape the dilemma 'between the horns' as there are no means available to us to expose the false dichotomy. In such cases, we need a different strategy to refute the given dilemma. We start by observing the major premise, and as explained earlier the major premise is a conjunction of two hypothetical propositions. Major premise is of the form $(p \supset q) * (r \supset s)$ that means p implies q and r implies s, and if we could show either that q does not follow consequently from p or s does not follow from r, then it will prove the dilemma to be unsound. This approach for the rebuttal of dilemma is called taking the dilemma by the horns or grasping the dilemma by the horns. Consider the following example:

If pregnant women have the freedom of choice, then pregnancy awareness programs should not be conducted and if pregnant women do not have the freedom of choice then pregnancy awareness programs are useless.

Either pregnant women have freedom of choice or they do not have freedom of choice.

Therefore, either the awareness programs should not be conducted or they are useless.

Now in the given example, the minor premise is a tautology (recall that a disjunctive proposition of the form $p \vee {\sim}p$ will always be true) so it is not possible to escape between horns. However, if we look closely at the major premise, the consequents, "pregnancy awareness programs should not be conducted" and "pregnancy awareness programs are useless" are not being followed from their consequents, "pregnant women have the freedom of choice" and "pregnant women do not have the freedom of choice". In the first hypothetical proposition, if the pregnant women have the freedom of choice then it becomes more important to conduct pregnancy awareness programs so that they can make informed choice, hence the consequent is not being followed from the antecedent. In the second hypothetical statement, if the women do not have the freedom of choice does not imply that they should not be made aware about pregnancy. Hence the dilemma becomes unsound.

Check Your Progress II

Note: (a) Use the space provided for your answer.

(b) Check your answers with those provided at the end of the unit.

1. Write the conclusion for the incomplete Dilemma given and then state which technique can be applied for its rebuttal:

   If I go abroad for post doctoral research immediately after my PhD, then I may lose job opportunity here in the country and if I stay here in the country, then I may not get good Post Doc position later.

   Either I go abroad for post doctoral research immediately after my PhD or I stay here in the country.

13.5 LET US SUM UP

In this unit we have discussed two major kinds of non-categorical syllogisms, namely: disjunctive syllogism and hypothetical syllogism, dilemma and its different types. We have also discussed how dilemma can be resolved. Dilemma can be tackled either by refute it or by rebut it. In our everyday life we face so many dilemmas, if we do not pay attention to these dilemmas; we may find ourselves in an unwanted situation. That is why the study of dilemma and the techniques to resolve them or tackle them is very important.

13.6 KEY WORDS

Dilemma: Dilemma is a rhetoric device in which the conclusion is a disjunctive proposition containing two equally unpleasant alternatives.

Disjunctive Syllogism: A disjunctive syllogism is an example of non-categorical syllogism which contains at least one and at most one disjunctive proposition, and one simple proposition which denies either of the disjuncts of the disjunctive proposition.

Hypothetical Syllogism: A hypothetical syllogism is another example of non-categorical syllogism which contains at least one hypothetical proposition. All the proposition may be hypothetical in nature.

Non-Categorical Syllogism: A non-categorical syllogism is a syllogism which has at least one non-categorical proposition.

13.7 FURTHER READINGS AND REFERENCES

- Copi, I. M., and Carl Cohen. *Essentials of Logic*. 2nd ed. Upper Saddle River, N.J.: Pearson Prentice Hall, 2007.
- Jain, Krishna. *A Textbook of Logic*. 5th ed. D.K. Printworld New Delhi, 2018.

13.8 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress I

1. Disjunctive syllogism, valid
2. Disjunctive syllogism, invalid
3. Mixed hypothetical syllogism, valid
4. (ii)

Check Your Progress II

1. Conclusion: Either I may lose job opportunity here in the country or I may not get good Post Doc position later.

UNIT 14 PROPOSITIONAL LOGIC*

Structure

14.0 OBJECTIVES

The objective of this unit is to introduce learner to Propositional Logic. Understanding Propositional Logic is crucial for understanding the development of Modern Logic or Symbolic Logic. It also helps learner to understand the techniques used by Propositional Logic to test the truth and validity in Logical expressions. Through the analysis of different aspects of Propositional Logic, learner shall be able to learn:

- the nature of Propositional Logic
- the process of symbolization in Propositional Logic

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- the types of Truth-Functional Proposition
- the use of Truth-Table method
- Indirect Truth Table Method
- Parallels of Propositional logic in Indian Logic

14.1 INTRODUCTION

In the previous Blocks of this Course and in the previous Units of this Block you have learnt that the validity of deductive argument depends on its form. This has been made more clear in the Units on Syllogism. By simply looking at the form you can say whether an argument is valid or invalid. You may have however noticed that the assessment of the form is complicated due to the ordinary use of language. For example, same term can be used in two propositions signifying different meaning, or sometimes the relation between two terms may not be clearly stated. Logicians therefore have been always trying to simplify the process. In Traditional Logic you have seen how the letters S, P, and M are used to represent the terms in Syllogistic Arguments and how a Syllogism was given a Standard Form. This was to some extent able to reduce the obscurity of ordinary language. However, this process was largely limited to two things: using only Categorical Prepositions in Standard Form Syllogism, and symbolization of the terms of propositions. But Modern Logicians went to further extent to make the recognition of logical forms easier by use of special symbols that represent the relation between terms as well. They made efforts to re-classify propositions so that even the form of arguments with complex propositions can also be easily recognized. These efforts led to the development of Propositional Logic.

Unlike Traditional Logic, Propositional Logic, while dealing with arguments, considers the whole proposition and not concerned with the terms of the proposition. Since it deals with Propositions or logical sentences, it is known as Propositional Logic and sometimes it is also called Sentential Logic. Propositional Logic thus does not analyze the internal structure of the proposition (There is however another branch of Modern Logic which analyses the internal structure of propositions. That branch of Logic is known as Predicate Logic. We have no scope to discuss it here) Propositional Logic deals with the whole propositions of an arguments. The propositions are represented by letters and the relationships between propositions are represented by Connectives or operators. Thus Propositional Logic can represent complex arguments in a symbolic form and determining the validity of such arguments become very easy.

14.2 PROPOSITION AND SYMBOLIZATION

Propositional Logic deals with arguments containing propositions and in consideration of the argument for validity and invalidity each proposition is considered as a whole. So it cannot rely on the traditional definition and classification of proposition. Traditional Logic, as you have already seen, was largely confined to categorical propositions. Thus traditional logic was unable to deal with arguments containing complex sentences. Propositional Logic overcomes these limitations by accepting the modern definition of proposition as a statement that is either true or false. So it is not limited to sentences with two terms and a copula.

Modern logicians have broadly categorized propositions into two types- Simple and Compound Propositions. Propositional Logic relies on this categorization. This distinction is significant to understand the symbolization process of argument and testing their validity in propositional logic.

A Simple Proposition is a proposition that does not contain any other proposition as it's component. In other words, a simple proposition can not to be divided into different propositions. For example:

Tansen was a great musician.

Yoga was invented in ancient India.

Tagore was a Nobel laureate.

These propositions do not have other propositions as components. Therefore, these are Simple Propositions. In propositional logic these propositions are symbolically represented using any convenient uppercase letter. Usually the first letter of any significant word may be taken. Here *T*, *Y*, *T* can be used to represent the above three propositions respectively. Now you can see that *T* is used twice taking a clue from the words "Tansen" and "Tagore". Don't you think this would create confusion in certain cases? It will. Especially if these are parts of the same argument. So, for all simple propositions of one argument we need to use unique uppercase letter for each propositions. We shall come back soon on this topic of symbolization. Let us first know about compound proposition now.

A **Compound Proposition** can be defined as a proposition containing one or more simple propositions as component. For example,

It is not the case that Tansen was a great musician.

Yoga was invented in ancient India and Tagore was a Nobel laureate.

In the first case there is one simple proposition as component that is “Tansen was a great musician”. And in the second proposition there are two simple propositions as components - (1) Yoga was invented in ancient India, (2) Tagore was a Nobel laureate.

Logicians broadly categorizes Compound Propositions in to five types: Negation, Conjunction, Disjunction, Conditional and Bi-Conditional

Negation or a Negative Proposition contains the negation of a proposition. In this proposition the word “not” or the phrase “It is not the case that” is used. For instance, “Harish is not an honest person”, “It is not the case that Harish is a honest person.’ Please note that merely using a word with negative connotation does not make a proposition a negative compound proposition. “Harish is a dishonest person” for example, is just a simple proposition, “Harish is not a dishonest person” is a negation, and therefore, a compound proposition.

Conjunction or Conjunctive Proposition is a compound proposition where simple propositions are connected by the word “and” or its equivalent. For example, “Rama is a teacher and Dinesh is a journalist”. The proposition contains two simple propositions- “Rama is a teacher” and “Dinesh is a journalist”. These component propositions are called conjuncts.

Disjunction or Disjunctive Proposition is a compound proposition where simple propositions are connected by the word “and” or its equivalent. For example, “Rama is a teacher or Rama is a journalist”. The proposition contains two simple propositions- “Rama is a teacher” and “Rama is a journalist”. These component propositions are called disjuncts. It may be noted that disjunctive proposition can further be divided into exclusive and inclusive Disjunctions. In exclusive disjunctions only one of the two disjuncts can be true. For

Conditional Proposition is a compound proposition where the relation between the components is expressed by “if” and “then”. For example, if the weather is unsuitable then the cricket match will be cancelled. In this compound proposition the two simple propositions namely “the weather is unsuitable” and “the cricket match will be cancelled” are connected by” if…then..”. The proposition followed by “if” is called antecedent and the proposition followed by “then” iscalled consequent. The relation is called a material implication. Because the material truth of the consequent is implied or conditioned by the material truth of the antecedent. Please note here that implication should not be taken in a sense of derivation like when the proposition of an argument implies the conclusion.

Biconditional proposition, on the other hand, is a compound proposition where the components are connected by the phrase “if and only if”. For example, Seema will go to Cinema if and only if Rihana goes with her. Here the relation is of material equivalence. This proposition is called bi-conditional because it contains a combination of two conditional propositions. If we analyze the present example, we find the following- Seema will go to Cinema if Rihana goes with her and Seema will go to Cinema only if Rihana goes with her. The relation between the components here is a necessary and sufficient condition.
Now let see how these five types of Compound Propositions can be symbolized following the method sated above.

Compound Proposition	Example	Symbolization
Negative Proposition	It is not the case that Harish is a honest person	It is not the case that H
Conjunctive Proposition	Rama is a teacher and Dinesh is a journalist	R and D
Disjunctive Proposition	Rama is a teacher or Rama is a journalist	T or J
Conditional Proposition	If the weather is unsuitable then the cricket match will be cancelled	If W then C
Biconditional Proposition	Seema will go to Cinema if and only if Rihana goes with her	S if and only if R

Table 1: Compound Proposition and Symbolization

Here you can see that the simple propositions from each compound propositions is symbolized using an upper case alphabet. But how to symbolize the words and phrases like "it is not the case that "or", "and", "if ...then…",, "if and only if"? Now before going to symbolize these we need to remember these are unlike simple propositions. These expressions reveal the nature of a compound proposition. In case of Negation, it negates a simple proposition, in case of others there are at least two simple propositions connected in a particular way with a particular meaning. Therefore, while symbolizing the compound prepositions we need to use special logical symbols which are known as Logical Operators or connectives or logical constants. These help us in translating English sentences into symbolic logical expressions. Thus we have five logical operators to express compound propositions. The following list is going to be our guide in the process of symbolization.

Proposition	**Representing Words/Phrase**	**Logical Operator to be used**	**Name**	**Function**
Negative Proposition	Not/it is not the case that/it is false that/	$\sim$	Tilde	Negation
Conjunctive Proposition	And/also/moreover	$\bullet$	Dot	Conjunction
Disjunctive Proposition	Or/unless	V	Wedge	Disjunction
Conditional Proposition	Only if/if..then..	$\supset$	horseshoe	Implication
Biconditional Proposition	If and only if/is a sufficient and necessary condition for	$\equiv$	Triple bar	Equivalence

Table 2

In the light of the above list, let us symbolize the previous list of examples using operators.

Compound Proposition	Example	Symbolization	Symbolization Using Operator
Negative Proposition	It is not the case that Harish is a honest person	It is not the case that H	**~H**
Conjunctive Proposition	Rama is a teacher and Dinesh is a journalist	R and D	R •D
Disjunctive Proposition	Rama is a teacher or Rama is a journalist	T or J	T v J
Conditional Proposition	If the weather is unsuitable then the cricket match will be cancelled	If W then C	W ⊃ C
Biconditional Proposition	Seema will go to Cinema if and only if Rihana goes with her	S if and only if R	S ≡ R

Table 3

In general, this is the process of symbolization. This is the way we can "translate" sentences into logical proposition in symbolic form making it easy to decide the logical structure of a compound proposition. But this "translation" may not always capturefull sense of ordinary language, but it is able to do so to a great extent. However, we do not have scope to discuss that issue here. Now let us discuss some more examples to familiarize with the symbolization process.

As we have noted earlier any negated proposition is a compound proposition with negation of a simple proposition. So we can always use the tilde sign to represent such proposition. The negation can be expressed in various ways in ordinary language but the use of logical operator remains similar in each cases.

All humans are not scientist	~ S
It is not the case that all humans are scientist	~S
It is false that all humans are scientist	~S

So it appears that tilde needs to be placed before a proposition. It cannot be placed as a connector of two propositions. Thus the expression S~ is not proper. Similarly, S~P is also not correct. However, S • ~P is correct because here tilde is placed before P. Here you can see that tilde can be placed after another operator. We can use tilde before a compound proposition including multiple simple propositions also. But it must stand to mean as a negation of what followed by the tilde. For example

~(PvQ)

~(P•.(Q≡R))

~[(P•S) v (Q⊃R)]

All these propositions are negative propositions, because in these expression *tilde* is the main operator. But what is a **main operator**? The main operator is that operator under the scope of which everything else of a compound proposition is included. As you know in a compound proposition there could be one or more simple proposition. So when there are more component propositions in a compound proposition then there may be many operators as well. But the statement can be expressed properly only when we know the main operator that will signify the main proposition. Take for example: A •B **v** C. Here we are not sure what type of compound proposition this is. Because the main operator is not identified. Let us restate the proposition with a main operator: A • (B **v** C). Here dot (•)is the main operator because everything is within its scope, while the scope of wedge (v) is limited to B and C only. One easy way to identify the main operator is that if there are parentheses, brackets or braces, whatever is outside those is the main operator. In such cases if there are two operators then whatever is not tilde is the main operator. For example: (H v H) • ~ (P v Z). Here dot is the main operator.

Now let us check Conjunction. The dot symbol (•) is used to translate such conjunctions as "and," "also," "but", "however," "yet," "still," etc. While translating such propositions we need to make sure that the logical statement is equivalent to the ordinary sentence. For example- Rama and Laxmana are philosopher. This can be translated as R • L because the sentence stands for Rama is a philosopher and Laxmana is a philosopher. But Rama and Laxmana are friends cannot be translated as R • L because here R • L stands for Rama is Friend and Laxmana is a friend and that is not equivalent to the ordinary sentence.

All the following propositions are Conjunctions. Because the main operator is a dot.

W • L, (W v L) • (D v L), ~ P • [W v (S ⊃ T)]

In disjunction, the wedge symbol (v) is used to translate "or", "unless" or any equivalent expression of "or". What we need to note here is that "or" can be used in inclusive or exclusive senses. When it taken in inclusive sense it means both the propositions connected by "or" can be true. For example- either he is a teacher or he is journalist. Here he actually can be both teacher and journalist. In exclusive sense the components cannot both true at the same time. For example- She went to London yesterday evening or she went to Washington. Here only one proposition can be true. All these types of propositions are translated using the wedge. When the main operator in a compound proposition is a wedge, it is a disjunctive proposition. Check these examples:

P v Q, Z v (G • L), (L• V) v (Q v ~ K)

In case of conditional or implicative propositions the horseshoe symbol is used to translate "if…then…", "only if" or any similar expressions. Please note that in case of Conditional statement the condition can be either sufficient (if indicated by "if") or a necessary (indicated by "only if") condition. Choosing the antecedent and consequent is thus may be little complex. The part that comes with "if" is always identified as antecedent. And what comes with "only if" is always considered as a consequent. For example- D only if M is translated as D ⊃ M, whereas D if M is translated as M ⊃ D.

Any logical expression that has horseshoe as the main operator is a conditional proposition. Take the following examples:

P⊃Q, Z⊃ (G • L), (L• V) ⊃ (Q v ~ K)

In case of Bi-Conditional Proposition the triple bar symbol is used to translate "if and only if" and similar expression. It indicates two conditions- necessary and sufficient. That is why this type of proposition is equivalent to two conditional propositions. For example, Mobile operator x decreases 4G service price if and only Mobile operator y decreases 4G service price. This can be symbolized as x≡y. The proposition can be expressed this way also: Mobile operator x decreases 4G service price if Mobile operator y decreases 4G service price (y⊃x) and Mobile operator x decreases 4G service price only if Mobile operator y decreases 4G service price (x⊃y).

This can be symbolized together as (y ⊃ x)•(x⊃y) and this is equivalent to x≡y.

The following compound propositions are all bi-conditional as the main operator is triple bar

~P ≡ Q, Z ≡(G • L), (L• ~V) ≡ (Q v ~ K)

14.3 WELL-FORMED FORMULAS

The symbolic expressions that we have shown in above examples are called **Well Formed Formulas (WFF)**. They are translations of meaningful and unambiguous sentences. Like a syntactically correct sentence these symbolic expressions are also syntactically correct. We can clarify this with an example- "Mary loves the colors of a rainbow". This is a syntactically correct sentence. The meaning is clear and unambiguous. But "Rainbow the loves a Mary colors of" is not syntactically correct. Similarly, logical expressions can be syntactically correct or incorrect. The correct one is called WFF. Thus "P v Q~", "v P•" these are not WFF.

There are some informal norms that we can follow to distinguish between WFFs and Non-WFFs.

Norms to Follow	Not WFFs	WFFs
Two or more Propositions cannot be combined without an operator occurring between them	AB A(AvB)	A v B A • (A v B)
A tilde cannot be placed immediately after a proposition, but it can be placed immediately before a	A~ (A v B) ~	~ A ~(A v B)

proposition		
A tilde cannot be placed immediately before other operator.	~ • A A ~ v B	~ A• B A v ~ B
A dot, wedge, horseshoe, or triple bar must be placed between propositions	•A BC v ⊃ D F	A•B B v C F ⊃ D
Parentheses, brackets, and braces must be placed properly to prevent ambiguity	A • ~R v Q ≡ T v Z	A •[(~R v Q) ≡ (T v Z)]

Table 4

Check Your Progress I

Note: (a) Use the space provided for your answer.

(b) Check your answers with those provided at the end of the unit.

1. What is propositional Logic?

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2. Symbolize the following propositions using appropriate logical operators

(i) It is not the case that India will abandon the mission to Mars.

(ii) If the Amazon rainforest is destroyed, then global warming will further deteriorate.

14.4 TRUTH FUNCTIONS

A Truth function can be understood in comparison to a mathematical function. Take this for instance: Y = X+2. Here the value of Y can be determined if we know the value of X. Suppose the value of X is 1. So the value of Y will be 3. Similarly, in Propositional Logic the value of a compound proposition is determined by the value of its components. The difference is that in Logic the values are not numbers but truth values. A proposition can be either true or false. The truth value a true proposition is "true" and the truth-value of a false proposition is "false".

A truth function is thus a compound expression whose truth value is completely determined by the value of its components. In logical expression our target is to find the value of the main operator on the basis of the value of all other operators and propositions etc. which are components of the compound proposition we are dealing with. However, in determining the value we need to follow the rules applicable for the five types of compound propositions we have discussed so far.

14.4.1 Statement form or propositional form

Here we need to note here that so far we were concerned with translation of propositions. To represent the propositions, we have used upper case letters of alphabet. But logic as you know is more concerned with logical forms than actual arguments or actual propositions. Therefore, here we are going to introduce what is called **statement form** or **propositional form**. Statement forms can be developed by using the operators and statement variables. Statement variables are lowercase letters. They can represent any proposition. The operators called logical constants, because they always are used in the same way. For example, "p v q" is a statement form. Here if we replace "p" and "q" with "A" and "B" we get a proposition "A v B". So a "statement form is an arrangement of statement variables and operators such that the uniform substitution of statements in place of the variables results in a statement'.

The nature of logical operators can be examined in terms of statement forms. This can be done by formulating Truth-Table for each statement forms. The truth table is an arrangement of truth values that shows in all possible case how the truth value of a particular compound proposition is determined by the truth values of its simple components. The truth value of simple proposition can be true or false. While formulating the table it is customary to use capital letter T for true and capital letter F for false.

Now let us consider tilde of a negation. The truth-table for negations shows how the truth-value of any propositions in negation form (~p) is determined by the truth values of the proposition that is negated (p).

14.4.1.1 Negation

p	**~p**
T	F
F	T

The truth-table shows that ~p is false when p is true and ~p is true when p is false. This we can apply wherever we find any expression with a negation.

Next we shall consider dot (•) of a conjunction. The truth-value of any proposition having the form of a conjunction (p•q) is determined by the value of its conjuncts (p,q):

14.4.1.2 Conjunction

p	**Q**	**p • q**
T	T	T
T	F	F
F	T	F
F	F	F

From this truth-table we can see that a conjunction is true only when its two conjuncts are true and it is false in all other cases.

Next we shall consider wedge (v) of a disjunction. The truth-value of any proposition having the form of a disjunction (pvq) is determined by the value of its disjuncts (p,q):

14.4.1.3 Disjunction

p	**q**	**p v q**
T	T	T

T	F	T
F	T	T
F	F	F

Here we can see that a disjunction is false only when both the disjuncts are false. In all other cases it is true.

Next we shall consider horseshoe (⊃) of a conditional (material implication). The truth-value of any proposition having the form of a conditional (p⊃q) is determined by the value of its antecedent (p) and consequent (q):

14.4.1.4 Conditional

p	**q**	**p ⊃ q**
T	T	T
T	F	F
F	T	T
F	F	T

The truth-table shows that a conditional is false when the antecedent is true and consequent is false. In all other cases it is true.

Finally, we shall consider triple bar (≡) of a Biconditional (material equivalence). The truth-value of any proposition having the form of a Biconditional (p ≡ q) is determined by the value of its components (p, q):

14.4.1.5 Biconditional

p	**q**	**p ≡ q**
T	T	T
T	F	F

F	T	F
F	F	T

This truth-table shows that a Biconditional is true only when both the components have equal value whether it is false or true that is not an issue. But if the value of the components is different, then the conditional is false.

14.5 TRUTH-TABLE FOR PROPOSITIONS

In the above section we have learned the norms to determine the value of different operator. Now we shall see how truth-tables can be used to determine the value of complex propositions.

The truth table is an arrangement of truth values that shows in all possible case how the truth value of a particular compound proposition is determined by the truth values of its components. The method is presented in a table of values. There will be separate columns for each variables and operators. There will be one header row to write the letters and operators and below that there will be other rows showing the possible truth value for each component. In order to calculate the truth value of the compound proposition first we have to determine the number of possible cases for the component simple propositions. But how do we determine the number of all possible cases? There is a simple method. All compound propositions are constituted by simple propositions. Now we already know that a simple proposition has two possible truth values namely "true" and "false". So if we have one simple proposition the possible case will be two. But if we have two simple propositions (for example p and q) then there will be the following possible case:

Possible Case 1: p could be true and q could be true

Possible Case 2: p could be true and q could be false

Possible Case 3: p could be false and q could be true

Possible Case 4: p could be false and q could be false

How did we calculate the possible number if we have more components? The formula is $L=2^n$. L is lines or rows of possible truth value combinations. Here n is the number of unique letter representing simple propositions. So if we have p, q, r as components of a compound proposition the number of possible cases will be $L=2^3$ and hence L=8. We shall have eight rows of values.

Now let us begin a truth-table for compound proposition. Let’s take a very simple example first:

pv(q⊃ ~ p)

Here the number of simple propositions is two. Hence the number of possible cases will be four and the four lines will be placed immediately under the letters. In the next step we need to follow some norms so that all possible cases are actually written so that in every column and row we know where to put T and where to put F. The total number of rows shall be divided 2. Here 4 divided by 2 we get 2. So under two rows of the first letter we put T first then put F in the remaining rows. Next we divide the previous result of 2 by 2 again. This time we get 1. So under the second letter we put one T followed by one F and so on. Let’s check-

p v (q⊃ ~ p)					
T		T			
T		F			
F		T			
F		F			

Here we see that every possible combination of truth and falsity are assigned under p and q. In case of the second p we can just copy the values from the first p.

p v (q⊃ ~ p)					
T		T			T
T		F			T
F		T			F
F		F			F

In the next step we need to find the values of the operators. We need to always find the value tilde first before other type of operators. Then we find the value of the operators which within bracket etc. Finally, we find value of the main operator that lies outside the scope of brackets or parenthesis. We need to follow these rules that we have earlier discussed regarding the operators.

p v (q⊃ ~ p)

T	**T**	T	F	F	T
T	**T**	F	T	F	T
F	**T**	T	T	T	F
F	**F**	F	T	T	F

Here the value of the main operator wedge is computed from the column under p and horseshoe. Let us take another example with three different simple propositions.

p⊃(q v r)

p⊃	(qv		r)	
T	**T**	T	T	T
T	**T**	T	T	F
T	**T**	F	T	T
T	**F**	F	F	F
F	**T**	T	T	T
F	**T**	T	T	F
F	**T**	F	T	T
F	**T**	F	F	F

This compound proposition has three simple propositions and so we have 8 lines to write all possible combinations of truth and falsity. The main operator is a horseshoe. Therefore, following the rules for horseshoe this compound proposition is false in the row where the value of p is true but value wedge is false. We know that in a conditional proposition if the antecedent is true and consequent is false then the proposition is false.

In this way we can construct Truth-tables for any type of compound propositions.

14.6 TAUTOLOGY, SELF-CONTRADICTORY AND CONTINGENT PROPOSITIONS

On the basis of the truth-tables of the compound proposition we can classify propositions into three categories. As we have seen the value of the main operator can be all T or all F or some T, some F. When the main operator has only T under it that means the compound proposition is true even when the component propositions may be or may not be true. In this situation the proposition is called **Logically true** or **tautologous proposition**. On the other hand, if a compound proposition is false independently of the truth of its components, it is called **logically false or self-contradictory proposition**. And if a proposition's truth value is varying depending on the truth value of its components than its called **contingent proposition**. The easy way to identify these three: if all values under the main operator is T than it is a tautology; if all the values under the main operator are **F** than the proposition is Self-Contradictory. If the value under the main operator is at least once **T** and at least once **F** then the proposition is Contingent.

14.7 TRUTH-TABLE FOR ARGUMENTS

So far we have discussed about propositions and how to determine the truth value of propositions in propositional logic. But the main aim of logic is to deal with the validity of arguments. Propositional logic provides truth-table method as a standard technique to test the validity of an argument. To construct a truth table for an argument we need to follow certain steps.

First, symbolize the arguments using letters to represent the simple propositions.

Secondly write the symbolized argument like a symbolized proposition, put the first premise on the left-hand, the second premise next and the conclusion at the end. Place a single slash between the premises and a double slash between the last premise and the conclusion so that they are easily distinguishable.

Thirdly, draw a truth table for the symbolized argument in a similar ways just like a proposition showing the columns representing the premises and conclusion.

Finally, find a line of values in which all of the premises are true and the conclusion is false. If such a line exists, the argument is invalid; otherwise, it is valid.

For example, let us test the following argument for validity:

If Seema goes to Cinema than Mary stays at home

Seema does not go to Cinema

Therefore, Mary does not stay at home

Let us symbolize the argument

S ⊃ M

~S

. ˙ . ~ M

Now we can construct a truth table for this argument. Since the symbolized argument contains two letters so we shall have four lines of values. Next we shall assign the values under each letters. If there are identical letter the values will be same. In the next step we shall assign values under each column that represents the premises and the conclusion as per rules of those operators they contain. Thus we get the following truth-table.

S	⊃	M	/	~	S	//	~	M
T	**T**	T		**F**	T		**F**	T
T	**F**	F		**F**	T		**T**	F
F	**T**	T		**T**	F		**F**	T
F	**T**	F		**T**	F		**T**	F

Here you can see that in the third line both the premises are true but the conclusion is false. You have learned in previous units that in a valid argument if the premises are true then the conclusion must be true. If there is an F in any of the premises, then it does not matter if the value in that line in conclusion is F or T. But if in any line of values all premises column have T but conclusion column has F than by definition the argument is invalid. To understand clearly let us test a valid argument:

G ⊃ ~H

~H ⊃ I

. ˙ . G ⊃ I

G	⊃	~	H	/	~	H	⊃	I	//	G	⊃	I
T	**F**	F	T		F	T	**T**	T		T	**T**	T
T	**F**	F	T		F	T	**T**	F		T	**F**	F
T	**T**	T	F		T	F	**T**	T		T	**T**	T
T	**T**	T	F		T	F	**F**	F		T	**F**	F
F	**T**	F	T		F	T	**T**	T		F	**T**	T
F	**T**	F	T		F	T	**T**	F		F	**T**	F
F	**T**	T	F		T	F	**T**	T		F	**T**	T
F	**T**	T	F		T	F	**F**	F		F	**T**	F

The main column of the premises and conclusion are shown in bold. In the eight lines of possible truth value combination there is no instance where all the premises are true and the conclusion is false. Therefore, this argument is a valid one. In this way we can easily test the validity of any argument.

14.8 INDIRECT TRUTH TABLE METHOD

Indirect Truth table method provides a shorter method of testing arguments using truth-table.

This method is very useful if we have arguments containing large number different simple propositions. In this method we need to work backward in comparison to normal truth-table.

We know that if an argument is valid and its premises are true then it’s impossible for the conclusion to be false. So in the indirect method we assign T under the premises’ column and F under the conclusion column. In the next step on the basis of these values and following the rules of the operators we assign values to the other components and finally the letters representing the simple propositions. If we can do this without any contradiction, then it means that it is possible that there will be at least one line of values where premises are true and conclusion is false. In that case the argument is not valid. But if arrive at contradiction, say at one place we have assign T under letter A and we have assigned F under A in another column then this is self-contradiction and it implies that the assumption that the premises are true and the conclusion false is self-contradictory. Hence indirectly the argument is proved to valid.

Let us test one argument using this method.

(A v B)⊃ (C •D)

(D v E) ⊃ F

.˙. A ⊃ F

(A	V	B)	⊃	(C	•	D)	/	(D	V	E)	⊃	F	//	A	⊃	F
T	T		**T**	T	T	T		F	F	F	**T**	F		T	**F**	F

Here you can see that we have assigned T under both the premises and F under the conclusion. These three are values shown in bold letters. In the next step we assign truth value to A and F of conclusion. The conclusion is a Conditional proposition so here the proposition can be assigned F for one combination only, when antecedent is true and consequent false. So we put value T under proposition A and value F under proposition F. The values can be assigned to all occurrences of A and F. The first premise is a conditional proposition (A v B) ⊃ (C •D) and the value is T and its antecedent is a disjunctive proposition. We have already assigned T under A so the disjunctive proposition will be T in any case because disjunctive proposition is true when at least one of its disjuncts is true. Now since this is the antecedent of the main conditional proposition which needs to be true, therefore, the consequent (C •D) cannot be F. So we must put T under (C •D) which is a conjunctive proposition. A conjunctive proposition is true only when both the conjuncts are true. We must therefore assign T under both C and D.

Now let us check the second premise which is also a conditional proposition (D v E) ⊃ F. Here the value of proposition F (Consequent) is already assigned as an F. So we must assign F to the antecedent (D v E), also because otherwise the premise cannot be true. Now we have assigned F under (D v E), which is disjunctive proposition. A disjunctive is false only when both its disjuncts are false. So we are forced assign F under both D and E.

But we have already assigned T under D in the first premise. This means that if we are assuming this argument as invalid we shall face such self- contradiction in assigning truth values to the components. Hence indirectly the argument is proved to valid.

14.9 ARGUMENT FORMS

So far we have discussed about arguments expressed in logical symbols. We know that an argument is valid when it has a valid form. In Propositional logic many arguments are valid or invalid because of specific forms and these forms are given names. With the help of these we can immediately recognize if an argument is valid of invalid

We have previously discussed about Propositional forms. A propositional form is an arrangement of propositional variables and logical operators such that the uniform substitution of proposition in place of the variables results in a proposition. An example of a propositional form is p v q, where p and q are

If use proposition A and B in place variable p and q, we get the proposition A v B. Analogously, an argument form is an arrangement of propositional variables and operators such that the uniform substitution of propositions in place of the variables produces an argument. This resulting argument is termed as a substitution instance of its related argument form. For example, the following arrangement of statement variables and an operator is an argument form:

p •q

∴ p

If the proposition A and B are uniformly substituted in place of the propositional variables p and q of this argument form, we get the following substitution instance, which is an argument:

A •B

∴ A

Now in the same form we can have substitution instance with more complex propositions. But the form will be same and hence the validity will also be same.

(A v B) • (B⊃ D)

. ˙ . (A v B)

If the above argument form is valid then both these given examples are also valid because of the form. Whether the form is valid that can be tested in truth-table method.

Please note that any substitution instance of a valid argument form is always valid. Suppose there is an argument form from which we can have sometimes valid substitution instance. But if it has even one substitution instance in which the premises are true and the conclusion is false, then we must know that the argument form is invalid.

But how do we know that an argument form can have invalid substitution instances? Take the following example of an argument form:

P ⊃ q

q

. ˙ . p

When we construct a truth-table for this argument form there will be four lines of truth value combination. These lines represent all possible cases and hence all possible categories of substitution instances.

p	⊃	q	/	q	//	P
T	T	T		T		T
T	F	F		F		T
F	T	T		T		F
F	T	F		F		F

Now checking this truth-table we find that in the third line the first premise p⊃q has T, the second premise q has T but the conclusion p has F as truth value. This means there is at least one substitution instance of the given argument form in which the premises can be true yet the conclusion is false. By the definition of validity, the argument form is therefore invalid. Now let us consider some valid argument forms.

14.9.1 Valid Argument Forms

14.9.1.1 Disjunctive Syllogism (DS)

p v q

~ p

. ˙ . q

Let us check this form using a truth-table.

p	v	q	/	~	p	//	q
T	T	T		F	T		T
T	T	F		F	T		F
F	T	T		T	F		T
F	F	F		T	F		F

From this table we can see that there is not a single line where both the premises are true and the conclusion is false. In the third line both premises have T and the conclusion also has T. So this argument form is a valid one. Any argument which is substitution instance of this argument form will be valid. This form is termed as Disjunctive Syllogism. Please note that substitution instance must be exact replacement of each variable. If we replace q with a substitution instance that is supposed to be replacement of p, for then it is not correct substitution instance.

14.9.1.2 Hypothetical Syllogism (HS)

p ⊃ q

q ⊃ r

. ˙ .p ⊃ r

To show the validity of this form the following truth-table is constructed.

P	⊃	q	/	q	⊃	r	//	p	⊃	r
T	T	T		T	T	T		T	T	T
T	T	T		T	F	F		T	F	F
T	F	F		F	T	T		T	T	T
T	F	F		F	T	F		T	F	F
F	T	T		T	T	T		F	T	T
F	T	T		T	F	F		F	T	F
F	T	F		F	T	T		F	T	T
F	T	F		F	T	F		F	T	F

In all the lines above there is not a single instance where both the premises are true and the conclusion false. This argument form is a valid one. In this argument form the propositions are linked like a chain. So in a substitution instance component should be uniformly substituted.

14.9.1.3 Modus Ponens (MP)

p ⊃ q

p

∴ q

Any argument of this form is valid. Let us test it:

p	⊃	q	/	P	//	q
T	T	T		T		T
T	F	F		T		F
F	T	T		F		T
F	T	F		F		F

Here you can see that there is no possible arrangement where both the premises are true and the conclusion is false. Wherever there is F in the conclusion column we can find F in at least of one the premise column also.

14.9.1.4 Modus Tollens (MT)

p ⊃ q

~q

. ˙ .~p

p	⊃	Q	/	~	q	//	~	p
T	T	T		F	T		F	T
T	F	F		T	F		F	T
F	T	T		F	T		T	F
F	T	F		T	F		T	F

There is only one line where both the premises have T and the corresponding conclusion line also have T. So any substation instance of this form will be valid argument. Therefore, the argument form is a valid one.

In this way we can find 9 different forms of argument which are regarded as elementary valid argument forms. Any substitution instance of these forms shall result in valid arguments. These 9 forms are very useful in testing the validity of arguments.

Check Your Progress II

Note: (a) Use the space provided for your answer.

(b) Check your answers with those provided at the end of the unit.

1. In what condition a disjunctive function becomes false?

----

2. Test the following argument using Truth table.

p v (p • q)

P

. ˙ . q

14.10 FORMAL PROOF OF VALIDITY

Truth-table method becomes very difficult when the number of component propositions increases. Suppose there are five different simple propositions in an argument. Then the number of rows in truth-table will be 32. It will be cumbersome to assign and check all the values in a table form. Logicians therefore use a more convenient shorter method. In this method the validity of argument is established by deducing the conclusions from the premises by a sequence of elementary argument forms whose validity is already proved.

An elementary valid argument form is defined as any argument that is a substitution instance of an elementary valid argument form. For example, there is a valid form called *Modus Ponens*. If there is a substitution instance of this form in the argument, then argument is valid. There are nine elementary valid argument forms accepted as Rules of Inference. However, there are many valid truth-functional arguments that cannot be proved valid using simply the nine Rules of Inference. We need additional rules in such cases. The Rule of Replacement is an additional principle of inference. There are ten rules of Replacement which allows us to replace any component of a proposition which is logically equivalent to that part as per these rules. Using these19 rules we can easily prove the validity of complex arguments just in few steps.

14.11 A NOTE ON INDIAN LOGIC

Propositional Logic is a largely a contribution of modern western logician. But as we have already seen the seed of this approach was already contained in Aristotelian logic where standard form syllogism was developed and the terms of the propositions were symbolically represented for easy and fast identification of the syllogistic forms. Can we find similar traces Indian Logic also?

The oldest Indian text which hints at the knowledge of some theorems of propositional calculus is the *Kathāvatthu*. This text discusses heretical theses following a stereotypical schema, which had been expressed as follows: 'if A is B, then C is D; but C is not D; therefore, A is not E'. As one can see, this formula is equivalent to the *Stoic modus Tollens*. According to Ganeri, the elements with which the logic in the *Kathāvatthu* operates are evidently propositional variables. In his commentaries, Buddhaghosha calls the conclusion from the premise $p \supset q$ 'direct' (*anuloma*) and the conclusion from the premise $\sim q \supset \sim p$ 'inverse' (*pratiloma*). So we can say that Indian Logic also carried some traces of propositional logic.

14.12 LET US SUM UP

In this Unit we have learnt about various aspects of propositional logic. We have seen that unlike Traditional Logic, Propositional Logic, while dealing with arguments, considers the whole proposition and not concerned with the terms of the proposition. Since it deals with Propositions or logical sentences, it is known as Propositional Logic and sometimes it is also called Sentential Logic. Propositional Logic thus does not analyze the internal structure of the proposition. Propositional Logic can represent complex arguments in a symbolic form and determining the validity of such arguments become very easy.

14.13 KEY WORDS

Argument Form: An argument form is an arrangement of propositional variables and operators such that the uniform substitution of propositions in place of the variables produces an argument.

Compound Proposition: A Compound Proposition can be defined as a proposition containing one or more simple propositions as component.

Propositional Logic: Propositional Logic, while dealing with arguments, considers the whole proposition and not concerned with the terms of the proposition. Since it deals with Propositions or logical sentences, it is known as Propositional Logic and sometimes it is also called Sentential Logic.

Simple Proposition: Simple Proposition is a proposition that does not contain any other proposition as it’s component.

14.14 FURTHER READINGS AND REFERENCES

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14.15 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress I

1. Propositional Logic deals with Propositions or logical sentences. It is known as Propositional Logic and sometimes it is also called Sentential Logic. Propositional Logic thus does not analyze the internal structure of the proposition h arguments, considers the whole proposition and not concerned with the terms of the proposition.

2. (i) It is not the case that India will abandon the mission to Mars.

It is not the case that M

~M

(ii) If the Amazon rainforest is destroyed, the global warming will further deteriorate

If A then G

A ⊃ G

Check Your Progress II

1. In disjunctive function becomes false when both the disjuncts are false.

p	V	(p	•	q)	/	p	//	q
T	T	T	T	T		T		T
T	T	T	F	F		T		F
F	F	F	F	T		F		T

2.

F	F	F	F	F		F		F

In the 2nd line both premises are true but the conclusion is false. Therefore, the argument form is invalid.