UNIT 7 CONSUMPTION AND ASSET PRICES

Structure

7.0 OBJECTIVES

After going through this Unit you should be able to explain

- the Fisherian idea of consumption as an outcome of households' intertemporal choices;
- the Life-cycle hypothesis of Modigliani and Brumberg and the Permanent Income hypothesis of Friedman;
- the Random Walk Hypothesis of Hall; and
- the asset price determination through the Consumption Asset Pricing Model (CAPM).

7.1 INTRODUCTION

In Economics, there exist close links between the current economic variables and their past and future values. As you have seen in Block 2 (on Economic Growth), past state often determines the current state, just as the current state determines the future. But there is also a link from future to the present. Future of course is often unknown. However, expectations about future variables sometimes influence the current economic decisions. In other words, the decision making process of an economic agent is often 'inter-temporal' in nature — involving different periods of time. Consumption and savings are two prime examples of such inter-temporal decision making.

How does a household decide upon how much to consume today and how much to save? Keynes identified current income as the prime determinant of current consumption. In its simplest form, a Keynesian consumption function can be represented by the following linear equation:

$$C_t = \bar{C} + cY_t, \bar{C} > 0, \ 0 < c < 1, \qquad ...(7.1)$$

where C_t is current consumption and Y_t is current income. Notice that one important feature of this consumption function is that the *average propensity to consume* (C_t / Y_t) falls as income rises.

Early empirical studies using cross-sectional household data found evidence in support of decreasing average propensity to consume: it was found that richer households (with higher Y_t) indeed consumed a lower fraction of their income. However, in 1940s Simon Kuznets using long run time series data on aggregate consumption and income found that the average propensity to consume remained more or less constant, even though aggregate income increased substantially over the period under consideration. This apparent puzzle led to the development of a number of theories of households' consumption behaviour, all of which focus on the intertemporal nature of consumption expenditure. The following section discusses some of the important theories from this literature.

7.2 CONSUMPTION AS INTERTEMPORAL CHOICE

One of the earliest works that depicts households' consumption expenditure as an outcome of households' intertemporal optimization exercise is that of Irving Fisher (1930). The Fisherian idea can be easily explained in terms of a two period model of consumer optimization. Let us assume that each member of a household lives for exactly two periods[1] and let C_1 and C_2 denote his consumption during the first and the second period of his life respectively. The person derives utility from both period's consumption and his preferences are represented by the utility function $U(C_1, C_2)$. Let Y_1 and Y_2 be the income of the individual in the two periods respectively. Out of his first period income, the person consumes C_1 and saves the rest ($S_1 = Y_1 - C_1$) to earn an interest income $(1+r)S_1$ in the next period. In Fisher's model, the only purpose of savings is future consumption. Accordingly, consumption during second period is equal to $C_2 = Y_2 + (1+r)S_1$.

By re-arranging the terms in $C_2 = Y_2 + (1+r)S_1$, we find that $\frac{C_2}{1+r} = \frac{Y_2}{1+r} + S_1$.

Since $S_1 = Y_1 - C_1$, we have $\frac{C_2}{1+r} = \frac{Y_2}{1+r} + (Y_1 - C_1)$ or $C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$.

Thus the intertemporal budget constrain is given by

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r} \equiv \hat{Y}(\text{say}) \qquad \text{... (7.2)}$$

The left hand side of the intertemporal budget constraint denotes the present discounted value of the total consumption expenditure of the person, while the right hand side measures the present discounted value of his total life-time income ($\hat{Y}$). The person decides on the levels of C_1 and C_2 by maximizing his utility $U(C_1, C_2)$ subject to his intertemporal budget constraint. Notice that the optimization exercise of the household defines the current consumption as a function of the present discounted value of life time income ($\hat{Y}$) as well as the rate of interest (r) on saving[2].

[1] **The time periods can be broadly defined so that the first time period covers his entire youth and the second one covers his entire old age.**

[2] **Strictly speaking, the rate of interest r is the 'expected' future rate of interest (which is expected to prevail in period 2). We are assuming here that future is certain and known.**

In Fig. 7.1 we depict the optimal consumption choices. We measure C_2 on x-axis and C_1 on y-axis. The intertemporal budget constraint intersects the x-axis at point $(1+r)Y_1 + Y_2$ because when $C_1 = 0$ in (7.2) we have $C_2 = (1+r)Y_1 + Y_2$. Similarly, we find that $C_1 = Y_1 + \frac{Y_2}{1+r}$ when $C_2 = 0$.

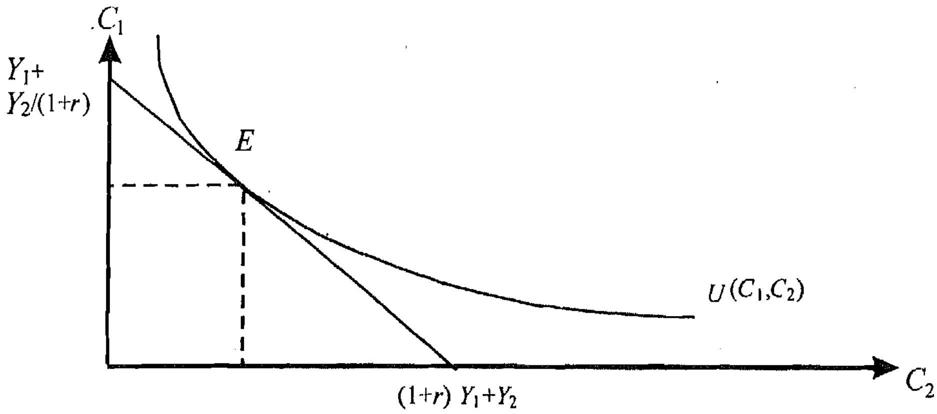

Fig. 7.1: Intertermporal Utility Maximisation

It is now easy to analyze the impact of a change in current income (Y_1) on current consumption (C_1). Under the assumption that consumption in both periods are normal goods (i.e., associated with positive income effects), an increase in current income, ceterisparibus, increases the life-time income and hence both C_1 and C_2 will increase.

The impact of a change in the interest rate (r) on current consumption is more ambiguous. Note that an increase in *r* implies a decline in the relative price of future consumption in terms of current consumption.[3] Such a change in the relative price is typically associated with two effects: (i) an income *effect*, which in this case will lead to an increase in consumption in both the periods (since a decline in the price level implies the choice set of the consumer becomes broader), and (ii) a substitution *effect*, which in this case will lead to a fall in current consumption (since a decline in the relative price of future consumption implies that people will substitute current consumption by future consumption). The overall effect on an increase in r on current consumption depends on the relative strength of the two effects. In ceterisparibus an increase in *r* leads to an increase in C_1 if the income effect dominates the substitution effect; on the other hand, an increase in *r* leads to a decrease in C_1 if the substitution effect dominates the income effect. In this context also note that in so far as $S_1 = Y_1 - C_1$, an increase in the rate of interest would also imply higher or lower savings depending on the relative strength of the income and the substitution effect.

This Fisherian view of looking at consumption as an outcome of an intertemporal optimization exercise on the part of the households came to play a key role in the

[3] **This is because for the same amount of current consumption foregone (in the form of savings), one will now get higher amount of future consumption.**

development of the subsequent influential theories of consumption, which are: (a) the *life-cycle* hypothesis of Modigliani and Brumberg (1954), and (b) the permanent income hypothesis of Friedman (1957). Both these theories attempt to explain the empirically observed discrepancy between the cross-sectional and time series evidence on the relationship between the average propensity to consume and the income level.

7.2.1 Life Cycle Hypothesis

To explain the life-cycle and permanent income hypotheses, let us extend the two period model that we have just discussed to a T-period model where each person lives for *T periods* (where $T \geq 2$). The utility function of the representative member of the household is again given by $U(C_1, C_2, C_3, \ldots\ldots, C_T)$. For expositional simplicity let us assume an additive utility function of the form:

$$U(C_1, C_2, C_3, \ldots\ldots, C_T) = u(C_1) + u(C_2) + \ldots\ldots + u(C_T) = \sum_{t=1} u(C_t). \qquad \ldots (7.3)$$

The utility function (7.3) is well behaved in the sense that marginal utility is positive (in symbols $u' > 0$) and increases at a decreasing rate (implies that the second derivative is negative, $u'' < 0$). For simplicity let us also assume that the rate of interest is zero[4] so that the intertemporal budget constraint of the household now becomes:

$$\sum_{t=1}^{T} C_t = A_0 + \sum_{t=1}^{T} Y_t, \qquad \ldots (7.4)$$

where $Y_1, Y_2, \ldots\ldots, Y_T$ are the incomes of the household in periods 1,2,.....,T respectively and A_0 is the amount of initial level of wealth stock of this household. Maximization of utility subject to the intertemporal budget constraint will yield a Lagrangian function of the form:

$$L = \sum_{t=1}^{T} u(C_t) + \lambda\left(A_0 + \sum_{t=1}^{T} Y_t - \sum_{t=1}^{T} C_t\right), \qquad \ldots (7.5)$$

where λ is the associated Lagrangian multiplier. First order conditions for optimization yield:

$u'(C_t) = \lambda$ for $t = 1,2,\ldots, T$, where $u'(C_t)$ is the marginal utility in period t.

The above condition implies that $u'(C_1) = u'(C_2) = \cdots\cdots = u'(C_T) = \lambda$, and therefore $C_1 = C_2 = \ldots\ldots = C_T$. Hence from the intertemporal budget constraint (7.4) we obtain

$$C_t = \frac{A_0}{T} + \frac{1}{T}\left(\sum_{t=1}^{T} Y_t\right) \text{ for all } t. \qquad \ldots (7.6)$$

4 Having a positive rate of interest will not have any effect on the subsequent analysis.

The term within the parenthesis in the right hand side of (7.6) is the average value of the total life-time income of the individual. The optimization exercise indicates that consumption in any period is determined the individual's total life-time income. Let us denote the latter by $\hat{Y}$. Then consumption at any point of time t is equal to $(A_0 + \hat{Y})/T$. This phenomenon, whereby the individual divides his total life-time resources equally among each period and consumption at different points of time are spread evenly over the entire time horizon is called ***consumption smoothing.***

At any particular point of time t, the actual income of that period Y_t may exceed or fall short of the average life-time income $(\hat{Y}/T)$. Given initial wealth, since current consumption depends only on the average life-time income, therefore any change in current income will have an effect on current consumption ***only to the extent that it impacts upon the average life-time income.*** To be more precise, suppose at some time period, say $t=\hat{t}$, current income rises by an amount Z for some reason. The corresponding increase in average life-time income is equal to Z/T and as a result, consumption at period $t=\hat{t}$ increases only by the amount Z/T. By the same token, if the current income at some period rises by an amount Z and the income at some subsequent period falls by the same amount, so that the average life-time income remains unchanged, then current consumption does not change in any period.

Notice that while changes in current income have limited impact on the consumption of that period, current income plays a crucial role in determination of current savings. Recall that savings at any time period t is defined as $S_t = Y_t - C_t$.

By using the result obtained at (7.6) we get $$S_t = Y_t - \frac{1}{T}\left(\sum_{t=1}^{T} Y_t\right) - \frac{A_0}{T}. \quad \ldots (7.7)$$

An interpretation of (7.7) is that savings is high when current income is high relative to the average life-time income. It is this idea which forms the basis of Modigliani-Brumberg's life-cycle hypothesis.

According to Modigliani and Brumberg, income varies systematically over the life-time of an individual and savings allow people to smoothen consumption over their life-time, even when incomes in different periods are not equal. In other words, people maintain an even stream of consumption over their life-time by saving more during high income periods and saving less during the low income periods.

The life-cycle hypothesis postulates that an individual typically has an income stream which is low during the beginning and towards the end of one's life, and high during the middle years of one's life. This is because productivity of a person is typically low during the early and the late years of his life, and productivity is at the peak during the middle years. On the other hand, consumption at every period remains the same at

$$C_t = \frac{A_0}{T} + \frac{1}{T}\left(\sum_{t=1}^{T} Y_t\right).$$ The typical income and consumption stream over the life-time of the individual is represented by Fig. 7.2. We observe from Fig.7.2, that during the early periods of one's life, an individual dissaves (by running down his initial wealth, and/or by borrowing); he saves during the middle periods of his life and dissaves again during the later years by running down his accumulated wealth.

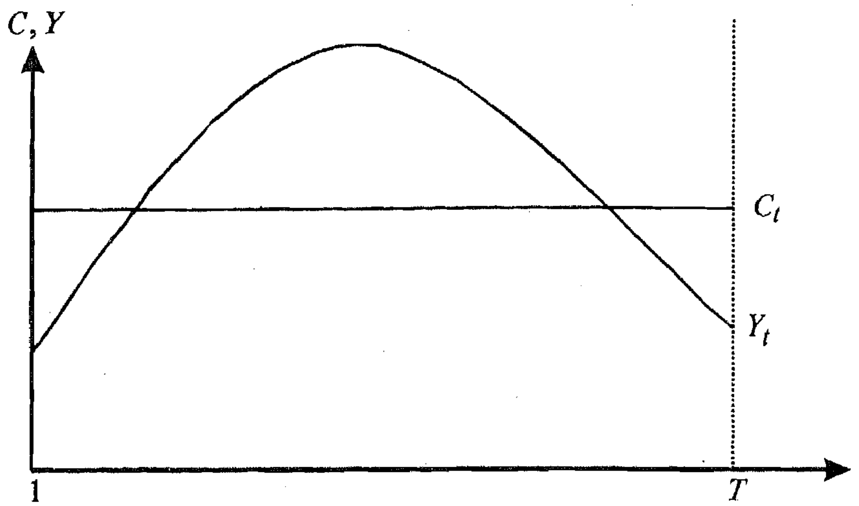

Fig. 7.2: Consumption Stream over Lifetime

Given that consumption and savings are postulated to behave in this manner over the life-cycle of a person, it is now easy to see how this hypothesis can explain the apparent disparity between cross-sectional and aggregate time series data on consumption behaviour. When one is looking at the cross-sectional data for a particular point of time, it is likely that in a randomly selected sample of households (which are classified according to income), the high income category will contain a higher-than-average proportion of people belonging to middle phase of their life cycle, and the low income category will contain a higher-than-average proportion of people belonging to either the early or the late phases of their life. Since during the middle phase, people have a higher average propensity to save (i.e., a higher S_t / Y_t ratio) and correspondingly a lower average propensity to consume (i.e., a lower C_t / Y_t ratio), the cross-sectional data will reflect a lower propensity to consume for the high income category compared to the low income category.

At this point it is important to note here that current consumption $C_t = \frac{A_0}{T} + \frac{1}{T}\left(\sum_{t=1}^{T} Y_t\right)$ depends not only on the life-time income of the individual, but also on the initial stock

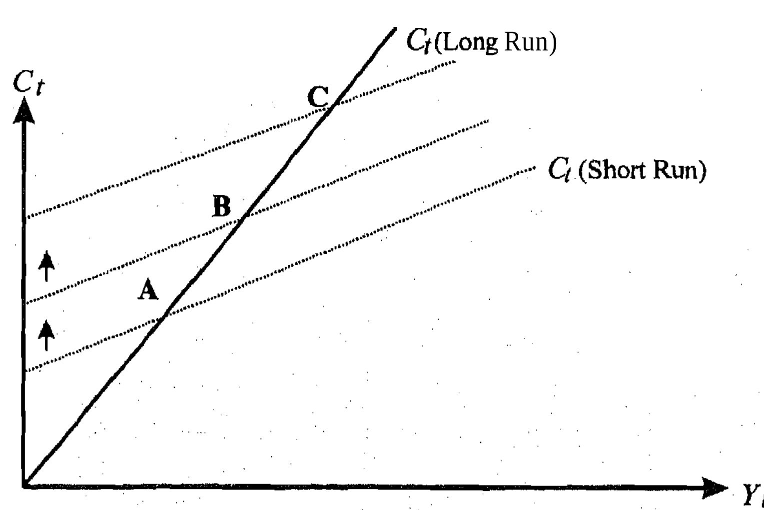

Fig. 73: Short run and Long run Consumption

of asset A_t. In the short run (or at any particular point of time) the asset stock remains constant, In Fig. **7.3** we diagrammatically depict the above relationship between C_t and Y_t. We measure C_t along the vertical axis and Y_t along the horizontal axis. We get an upward sloping line (slope less than unity) with a positive intercept, as shown in Fig. **7.3.** However, when we are looking at the time series data where income is increasing over a long period of time, then the stock of asset (which is positively correlated to the level of income) is also increasing. Thus the intercept term on the line will keep on increasing. In other words, the short run C_t line will keep shifting upward over time (see the dotted line in Fig. **7.3).** Hence if we are examining the long run consumption data to determine the long run relationship between C_t and Y_t, we will observe points like **A,** B, C along successive short run consumption lines. In other words, the long run consumption function will be steeper, where C_t is proportional to Y_t, as shown in Fig. 7.3. Note that along the long run consumption line, the average propensity to consume (C_t / Y_t) remains constant.

7.2.2 Permanent Income Hypothesis

Friedman's permanent income hypothesis provides an alternative explanation to the apparent discrepancy between cross-sectional and time series data on consumption. The peimanent income hypothesis is also founded on the Fisherian theory of consumption as an intertemporal choice. However, unlike the life-cycle hypothesis, Friedman does not postulate that income follow a regular pattem over the life cycle of an individual; he instead argues that individuals experience random and temporary changes in their income from time to time. Accordingly, Friedman views the current income in any period (Y_t) as consisting of two components: *permanent income* (Y_t^P) and *transitory income* (Y_t^T). Permanent income is that part of the income which we expect to prevail over the long run. Friedman interprets this as the long run average income of the individual, i.e., $Y_t^P = \frac{1}{T}\left(\sum_{t=1}^{T} Y_t\right)$. Transitory income is any random deviation from this average, i.e., $Y_t^T = Y_t - \frac{1}{T}\left(\sum_{t=1}^{T} Y_t\right)$. A positive transitory income implies that the current income exceeds the permanent income; a negative transitory income implies that the current income is less than the permanent income. Since $C_t = \frac{A_0}{T} + \frac{1}{T}\left(\sum_{t=1}^{T} Y_t\right) = \frac{A_0}{T} + Y_t^P$, i.e., current consumption of the household depends only on the permanent income, any increase in the transitory part of the current income, which leaves the permanent income unchanged, will have no impact on the level of current consumption.

Let us now see how Friedman's permanent income hypothesis solves the apparent puzzle in the consumption data. According to Friedman's hypothesis, the average propensity to consume (C_t / Y_t) depends on the ratio of permanent to current income Y_t^P / Y_t. Thus when current income temporarily rises above the permanent income the average propensity to consume falls; the opposite happens when current income temporarily falls below the permanent income. Now when we are looking at cross-sectional data of different households at any point of time, typically the high income group will contain some people with a high transitory income, who will have a lower propensity to consume than the average. Similarly the low income group will contain some people with a low transitory income, who will have a higher propensity to consume than the average. As a result, we will observe a falling average propensity to consume as we move from the lower to the higher income group. On the other hand, when we are considering long run time series data, the random fluctuations tend to even out so that any increase in income in the log run reflects a permanent increase in the average income level. Hence in the log run time series data we are likely to observe a constant average propensity to consume.

Check Your Progress 1

1) Explain the Fisherian idea of consumption as an intertemporal choice. What are the impact of a change in income and a change in the interest rate on current consumption?

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2) What is the puzzle in the consumption data that has been identified by Kuznets? How do the Life-cycle hypothesis and permanent income hypothesis solve this puzzle?

7.3 CONSUMPTION UNDER UNCERTAINTY: RANDOM WALK HYPOTHESIS

In our discussion so far we have assumed that people know their average life-time income (or the permanent part of the income) with certainty. What happens if there is some degree of uncertainty with the permanent income as well? Obviously. in the presence of uncertainty, people's expectation about the future becomes important. We can extend the logic of the permanent income hypothesis to argue that in the presence of uncertainty, individuals maximize their *expected* utility subject to the constraint that the sum of total *expected* consumption cannot exceed the total value of the *expected* life-time income. In other words, the objective function of a consumer is now given by

$$E[U] = E[u(C_1)] + E[u(C_2)] + ... + E[u(C_T)] = \sum_{t=1}^{T} E[u(C_t)] \quad ... (7.8)$$

which he maximizes subject to ihe constraint

$$\sum_{t=1}^{T} E(C_t) = A_0 + \sum_{t=1}^{T} E(Y_t). \quad . (7.9)$$

Note that since Future is uncertain, the expectation of people about future comes to play an important role, and therefore how people form their expectation also becomes important for the decision making process. You have earlier been introduced to the rational expectation hypothesis in Block-3. According to the rational expectations theory the outcomes do not differ systematically from what people expected them to be. The extension of the permanent income hypothesis in the presence of uncertainty combined with the assumption of rational expectation led to the theory of 'random walk consumption", as expounded by Robert Hall (1978).

Hall argued that if individual's consumption indeed depended on their expected average life-time income, and if people had rational expectations, then changes in consumption over time will be unpredictable, i.e., consumption will follow a 'random walk'. The intuition behind this result is simple: if the life-time income is expected to change at some future point of time and people have this information, then they will use this information optimally (under rational expectation) and will therefore immediately adjust their consumption over different time periods (consumption smoothing) *so* that current consumption would not change at the time when the actual income changes. Current consumption can change only if there are surprises in the life-time income, which were not anticipated. To put it differently, current consumption *can* only change due to events which are unpredictable and as a result changes in consumption would also be unpredictable.

Theoretically it is easy to see how this hypothesis follows from the above formulation of individual's maximization problem, As we saw in (7.6) and (7.7) the individual maximizes $\sum_{t=1}^{T} E[u(C_t)]$ subject to $\sum_{t=1}^{T} E(C_t) = A_0 + \sum_{t=1}^{T} E(Y_t)$. Since the individual is taking his decision at time $t=1$, under rational expectations, all his expectations are based on the information available at period 1. **An** optimizing agent will equate his ***expected*** (as of period 1) marginal utilities in different periods. Now in period 1, consumption in period 1 is a certain event; hence $E_1[u'(C_1)] = u'(C_1)$. On the other hand, *expected* (as of period 1) marginal utility at any subsequent period is given by: $E_1[u'(C_t)]$; $t = 2,3,\ldots, T$. Thus the optimality condition implies:

$$u'(C_1) = E_1[u'(C_2)] = E_1[u'(C_3)] = \cdots = E_1[u'(C_T)] \qquad \ldots (7.10)$$

In order to explain *(7.10)* further let us take a quadratic utility function of the form: $u(C_t) = C_t - \frac{a}{2}C_t^2$ for $t = 1,2,\ldots, T$. Then the marginal utility function is linear in C_t, *i.e.*, $u'(C_t) = 1 - aC_t$ for all t. Therefore,

$$E_1[u'(C_t)] = E_1[1 - aC_t] = 1 - aE_1(C_t).$$

Using this in the optimality condition we get:

$$1 - aC_1 = 1 - aE_1[C_2] = 1 - aE_1[C_3] = \cdots = 1 - aE_1[C_T].$$

On simplification,

$$C_1 = E_1[C_2] = E_1[C_3] = \cdots = E_1[C_T] \qquad \ldots (7.11)$$

The above condition implies that expectation as of period 1 about C_2 equals C_1. In more general terms if expectations are formed at any period t about a future period $t+1$, then optimality condition requires that expectation as of period t about C_{t+1} equals C_t, i.e., $C_t = E_t[C_{t+1}]$. Since under rational expectations the actual value of a variable can differ from its expected value only by a random term, this would imply

$$C_{t+1} = E_t[C_{t+1}] + e_{t+1}$$

$$= C_t + e_{t+1}, \qquad \ldots (7.12)$$

where e_{t+1} is a random term whose expected value is zero. If you look carefully, equation (7.12) says that consumption from period t to period $t+1$ would remain unchanged, except for an unpredictable random term. This is precisely the conclusion of the random walk hypothesis. The hypothesis says that changes is consumption over time is unpredictable, because they can only change due to the presence of unpredictable random events.

The random walk hypothesis has important implications from the point of view of policy effectiveness. It implies that any government policy to influence consumption (for example, a tax cut) can work only to the extent that it is not anticipated. For example, if the government announces a tax cut policy today, which will be implemented from next year, then consumers with rational expectations will adjust their consumption today itself, so that consumption would remain unchanged when the tax policy actually becomes operative next year.

7.4 CONSUMPTION AND RISKY ASSETS: CAPITAL-ASSET PRICING MODEL

While analysing consumption under uncertainty in the previous Section, we had implicitly assumed that the source of uncertainty is the households' income. In other words, we had assumed that there was no uncertainty as far as the asset return is concerned: the return to asset (r) was treated as given. (In fact, for simplicity we had assumed that it has a value equal to zero). If the return to asset is non-zero, but the exact return is uncertain then the optimality condition of the individual's utility maximization will include an expected return term as shown below:

$$u'(C_t) = E_t[(1+r_{t+1})u'(C_{t+1})], \qquad \text{... (7.13)}$$

where t is the time period when expectations are formed.

We know from the theorem for statistical expectation (operator E) for two events A and B that $E(AB) = E(A).E(B) + \mathrm{Cov}(AB)$ (where $\mathrm{Cov}(AB)$ is the co-variance between the two events A and B). We can visualize (7.13) as a case of two joint events and write the optimality condition as:

$$u'(C_t) = E_t[(1+r_{t+1})].E_t[u'(C_{t+1})] + \mathrm{Cov}_t[(1+r_{t+1}, u'(C_{t+1})] \qquad \text{... (7.14)}$$

Now suppose there are two assets: i) one with a certain (risk-free) return given by $\bar{i}_{t+1}$, and ii) another with an uncertain risky return with an expected return value equal to $E_t(r_{t+1})$. For the risky asset application of the condition (7.14) implies that

$$E_t[(1+r_{t+1})] = \frac{u'(C_t) - \mathrm{Cov}_t[(1+r_{t+1}, u'(C_{t+1})]}{E_t[u'(C_{t+1})]},$$

i.e., $$1 + E_t[r_{t+1}] = \frac{u'(C_t) - \mathrm{Cov}_t[(1+r_{t+1}, u'(C_{t+1})]}{E_t[u'(C_{t+1})]} \qquad \text{.. (7.15)}$$

For the risk-free asset, the return is certain and therefore is uncorrelated to C_{t+1}. In other words, for the risk-free asset $\mathrm{Cov}_t[(1+\bar{i}_{t+1}, u'(C_{t+1})] = 0$. Hence for the risk-free asset, the condition (7.14) implies that

$$1 + \bar{i}_{t+1} = \frac{u'(C_t)}{E_t[u'(C_{t+1})]}. \qquad \text{... (7.16)}$$

Comparing (7.15) and (7.16) we get:

$$E_t[r_{t+1}] = \bar{i}_{t+1} - \frac{\mathrm{Cov}_t[(1+r_{t+1}, u'(C_{t+1})]}{E_t[u'(C_{t+1})]}. \qquad \text{.. (7.17)}$$

The condition (7.17) gives us a way of determining the optimal (or equilibrium) expected return of a risky asset. Note that a higher value of C_{t+1} implies a lower value of $u'(C_{t+1})$. Therefore a positive co-variance between r_{t+1} and C_{t+1} implies a negative co-variance between r_{t+1} and $u'(C_{t+1})$. Hence (7.17) implies that the higher is the correlation between r_{t+1} and C_{t+1}, the greater is the required expected return on a risky asset compared to the return from the risk-free asset. To put it differently, the greater is the covariance between r_{t+1} and C_{t+1}, the higher is the premium that an asset must offer relative to the risk-free rate. This model of determination of expected return of risky assets is known as the Consumption Capital Asset Pricing Model (CAPM).

Check Your Progress 2

1) Describe the Random Walk Hypothesis of consumption. What is the implication of this hypothesis from the perspective of government policy?

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2) Explain the Consumption Capital Asset Pricing rule for a risky asset.

7.5 LET US SUM UP

The present unit viewed consumption in a dynamic set up and introduced you to the problem of intertemporal utility maximization. In this context it discussed the issue of consumption and savings when more than one time periods are considered. The Fisherian idea that consumption is an outcome of household's intertemporal optimization exercise has been explained in the unit.

Empirical studies involving cross-section data find that richer households consume a smaller fraction of their income compared to poorer households. However, studies based on time series data show that the proportion of consumption in income has remained more or less the same although household income has increased manifold, This apparent discrepancy has been attempted to be resolved through life-cycle hypothesis and permanent income hypothesis. According to the life cycle hypothesis the short run consumption function shows a declining average propensity to consume (APC) while the long run consumption function exhibits constant APC due to increase in asset base. On the other hand, the permanent income hypothesis explains the discrepancy in terms of permanent income and transitory income.

In the presence of uncertainty actual consumption behaviour may be difficult to predict. In this situation consumption may follow a random pattern. This theory of consumption, known as the Random Walk Hypothesis, has been explained here. Finally, in the presence of uncertainty the price of the risky asset may follow a specific pattern which has been explained in the discussion on the CAPM model.

7.6 KEYWORDS

Average Propensity to Consume It is defined as the ratio of total consumption (C) to total income (Y). Thus APC = $\frac{C}{Y}$

Ceteris Paribus It means 'every thing else remaining the same'.

Consumption Smoothing Even though the income of an individual changes across time periods, he divides his life time resources equally among each period and consumption at different points of time are spread evenly.

Expected Utility The total utility expected to be derived from the future income stream.

Intertemporal Decision A decision which involved more than one time period.

Permanent Income That part of the current income that is expected to remain stable over the long run.

Rational Expectations The hypothesis that expectations s f people on the whole is unbiased.

Transitory Income That part of the current income which is a deviation from permanent income.

7.7 SOME USEFUL BOOKS

Branson, William H., 1989, ***Macroeconomic Theory and Policy*** (3rd Edition), Harper and Row, chapter 12.

Romer, David, 2001, ***Advanced Macroeconomics*** (2nd Edition), McGraw-Hill, chapter 7.

7.8 ANSWERS/HINTS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress 1

1) Read Section 7.2 and write your answer.

2) Read Section 7.2 and write your answer.

Check Your Progress 2

1) Read section **7.3** and **write** your answer.

2) Read section 7.4 and write your answer.

UNIT 8 THE RAMSEY MODEL

Structure

8.0 OBJECTIVES

After going through this Unit you should be able to:

- explain the Ramsey problem of optimal growth;
- compare the optimal growth problem of the central planner and that of the decentralized perfectly competitive economy;
- explain the role of government in the optimal growth framework and the corresponding concept of Ricardian Equivalence; and
- if you go through the Mathematical Appendix carefully you should be in a position to solve any standard dynamic optimization exercise.

8.1 INTRODUCTION

In our discussion of household's intertemporal consumption and saving decisions in the previous unit have, we considered optimization problems where the time horizon was finite. While this is a valid assumption for an individual, a household (or for that matter, the society as a whole) is generally infinitely lived. In other words, one can think of a household as consisting of individual members, each of whom are finitely lived, but new members are born in each time period who are an exact replica of the older members – so that the household as a dynasty continues to live forever.

An important economic question that is often asked in the context of an infinitely lived household is the following: how should the household allocate its resources between consumption and savings in every period so that the utility of each of its member are maximized? In other words, what are the optimal consumption path and optimal savings path for this household? To put it more generally, in an economy consisting of *identical* infinitely lived household, whaf should be the optimal savings/accumulation pattern that maximizes the utility of *each* of dynastic household? This latter question was first tackled by Frank Ramsey (1928), which was later generalized by Cass (1965) and Koopmans (1965). The model has subsequently been known as the Ramsey-Cass-Koopmans *optimal growth* model.

The Ramsey-Cass-Koopmans optimal growth model essentially involves a dynamic optimization exercise whereby an agent maximizes its objective function defined over a period of time (finite or infinite) subject to some constraints. The constraints are also dynamic in nature in the sense that they relate to values of the variables for different time periods. The technique for solving such dynamic optimization exercises has been specified in the Mathematical Appendix at the end of the Unit. In the text we shall simply state the conditions for optimization (without getting into the underlying technique) and explain the economic intuition behind each of these conditions.

8.2 PLANNER'S PROBLEM

In the original Ramsey formulation, the optimal growth problem was postulated as the problem of a social planner who seeks to maximize the welfare of each household subject to the economy's resource constraint at each point of time. The households are all assumed to be identical in terms of preferences and their welfare is represented by an infinite horizon utility function of the form $W = \int_0^\infty u(c_t)\exp^{-\rho t}dt$, where c_t is the ***per capita consumption*** in period t, $u(c_t)$ is the associated ***instantaneous utility*** in period t, and ρ is positive constant term which represents the ***subjective discount rate*** of the household, or its ***rate of time preference.*** Before we proceed further, it is important to explain the presence of the discount factor in the household's welfare function. The positive discount factor reflects the household's preference for present over future. In other words, it implies that a household puts more weight on consumption today vis-a-vis consumption tomorrow. Note that when $t = 0$ (i.e., the initial time period) $\exp^{-\rho t} = 1$. Subsequently when $t = 1$, $\exp^{-\rho t} = \exp^{-\rho}$; when $t = 2$, $\exp^{-\rho t} = \exp^{-2\rho}$ and so on, such that $1 > \exp^{-\rho} > \exp^{-2\rho}$ .... Therefore in the infinite-time utility function W, current utility at period zero is associated with the highest weight (unity), and each subsequent utility term is associated with lower and lower weights.

The social planner maximizes the integral W, but he is constrained by the fact that at each point of time, the economy's total consumption and total investment cannot exceed its total output.[1] To put it formally, $C_t + \frac{dK}{dt} = Y_t$, where C_t is ***aggregate consumption*** in period t; $\frac{dK}{dt}$ is the amount of investment in period t which augments the capital stock (K) of the economy and Y_t is the total output produced in period t.

Total population at time t is given by L_t, which is assumed to grow at a positive exogenous rate n: $\frac{1}{L}\frac{dL}{dt} = n$.

Output at any point of time is produced using the existing capital and labour at that point of time. Technology is represented by a neoclassical production function $Y_t = F(K_t, L_t)$, where F is continuous, concave and exhibits constant returns to scale

[1] We rule out international borrowing here.

(CRS). We have explained the neoclassical production function in Unit 3, while discussing Solow growth model. The CRS property of the production function implies that per capita output (y) can be written as the function of the capital-labour ratio (k) in the following way:

$$y \equiv \frac{Y}{L} = \frac{F(K,L)}{L} = \frac{L\,F(K/L,1)}{L} = F\left(\frac{K}{L},1\right) \equiv f(k).$$

Moreover the marginal products of capital and labour can also written as the following functions of the capital-labour ratio:

$$\frac{\partial F}{\partial K} = f'(k)\ ;\ \frac{\partial F}{\partial L} = f(k) - kf'(k).$$

The continuity and concavity properties of $F(L, K)$ ensure that $f(k)$ is also continuous and concave. Additionally we assume that $f(0) = 0$; $f'(0) = \infty$; $f'(\infty) = 0$. The first of these assumptions implies that no production is possible with zero capital-labour ratio. The last two assumptions are known as the *Inada* ***conditions*** which state that when the capital-labour ratio tends to zero, the marginal product of capital tends, to infinity, while an infinitely high capital-labour ratio implies that the corresponding marginal product of capital approaches zero.

The aggregate resource constraint for the economy is, $C_t + \frac{dK}{dt} = Y_t$. Dividing both sides by L, we can write it as $\frac{C_t}{L_t} + \frac{1}{L_t}\frac{dK}{dt} = \frac{Y_t}{L_t}$, i.e., $c_t + \frac{dk}{dt} + nk_t = f(k_t)$ where the last equation denotes the resource constraint faced by the social planner in per capita terms.[2]

The economy starts with a given amount of capital stock and population; hence the initial capital-labour ratio is given, denoted by k_0. At every point of time the existing capital and labour stocks are fully employed. Hence the capital-labour ratio k also denotes the per capita capital stock. While referring to k we shall use these two terms interchangeably.

The complete dynamic optimization problem for the social planner can now be written in terms of the two time dependent variables per capita consumption (c_t) and per capita capital stock (k_t):

$$\text{Maximize } W = \int_0^\infty u(c_t)\exp^{-\rho t}\,dt \text{ subject to } \frac{dk}{dt} = f(k_t) - nk_t - c_t; k_0 \text{ given.}$$

The corresponding Hamiltonian function and the first order conditions in terms of the

[2] Note that $\frac{dk}{dt} = \frac{1}{L}\frac{dK}{dt} - \frac{K}{L}\left(\frac{1}{L}\frac{dL}{dt}\right) = \frac{1}{L}\frac{dK}{dt} - nk$.

[3] See the Mathematical Appendix for the definitions of the Hamiltonian function, control, state and co-state variables in a dynamic optimization problem.

control variable (c_t), state variable (k_t) and co-state variable (λ_t) are given by:[3]

$$H = u(c_t)\exp^{-\rho t} + \lambda_t\left[f(k_t) - nk_t - c_t\right]$$

ia) $u'(c_t)\exp^{-\rho t} = \lambda_t$

iia) $\dfrac{d\lambda}{dt} = -\lambda_t(f'(k_t) - n)$

iiia) $\dfrac{dk}{dt} = f(k_t) - nk_t - c_t$

iva) $\lim\limits_{t\to\infty} \lambda_t k_t = 0$

Notice that λ_t is the shadow price of capital (as explained in the Appendix) or the value of capital in utility terms. Hence condition (ia) states that along the optimal path the present discounted value of marginal utility from consumption would be equal to the shadow price of capital. The economic significance of this condition would become clear if you note that at any point of time one unit of output can be put to two different uses – one can either consume it or invest it which augments the capital stock. Now optimality condition requires that the returns in terms of utility from these two uses should be equal, which is precisely what condition (ia) states. Conditions (iia) and (iiia) respectively show how the co-state and the state variables change over time along the optimal path. The fourth condition, known as the ***transversality condition,*** implies that as the economy approaches its terminal time (which in this case is infinity), either the value of capital goes to zero (in which case it does not matter for the economy if it leaves a positive capital stock at the end), or, if the value of the capital stock is positive, then the economy uses up all its capital stock (i.e., k goes to zero).

Differentiating (ia) with respect to t and using (iia) to eliminate λ_t, we get the following differential equation: $\dfrac{dc}{dt} = \left(\dfrac{u'(c_t)}{-u''(c_t)}\right)(f'(k_t - n - \rho)$. Noting that the term $\dfrac{-cu''(c)}{u'(c)} \equiv \sigma$ denotes the elasticity of marginal utility with respect to consumption, the above equation can be written as:

va) $\dfrac{dc}{dt} = \left(\dfrac{c_t}{\sigma}\right)(f'(k_t - n - \rho)$

Equations (iiia) and (va) together form a system of differential equations which, along with the transversality condition, determine the movements of per capita consumption and per capita capital stock of the economy along the optimal path. The qualitative characterization of this path is shown in the phase diagram given at Fig. 8.1.

The phase diagram traces the lines/curves along which $\dfrac{dc}{dt} = 0$ and $\dfrac{dk}{dt} = 0$. The point of intersection of these two lines is called the ***steady state***. A steady state is equivalent to long run equilibrium whereby the values of the variable remain constant overtime.

From (va), $\frac{dc}{dt} = 0$ implies either $c = 0$ or $f'(k) = n + \rho$. On the other hand, from (iiia), $\frac{dk}{dt} = 0$ implies $c = f(k) - nk$. Plotting all these lines and curves in the c-k plane with c along the vertical axis and k along the horizontal axis, we get a diagram as shown in Fig. 8.1.

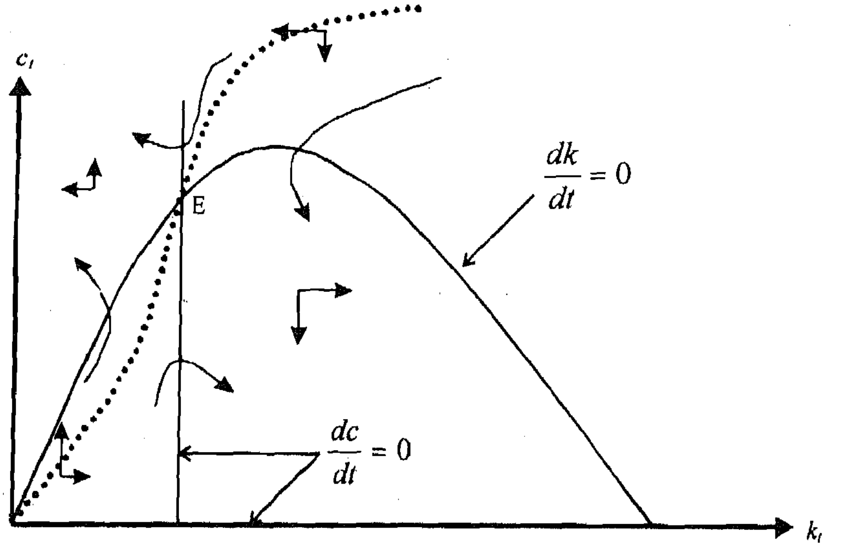

Fig. 8.1: **Phase Diagram**

The direction of arrows in the phase diagram shows the direction of movements of c and k when they are not on the $\frac{dc}{dt} = 0$ and $\frac{dk}{dt} = 0$ lines respectively. From Fig. 8.1 it is clear that there exits one steady state, denoted by point E in the diagram, which is associated with positive values of c and k. It can be shown (though we make no attempt to prove it here) that this steady state point is a 'saddle point' such that there exits a **unique** path which approaches the steady state point; all the other paths move away **from** the steady state point. This unique path is denoted by the dotted line in the **diagram.** This is the only path **which** satisfies all the first order conditions of **optimality** (conditions **(ia)**-(iva) given above) including the **transversality** condition. This path is therefore the optimal path which satisfies the social planner's dynamic optimization problem. Given any initial capital-labour **ratio,** the social planner will choose the initial per **capita consumption** so **as** to be on this optimal path, which will eventually take the economy to **its** long run equilibrium (or steady state) point E.

At this juncture it is important to point out certain important features of the steady state point E. As was mentioned before, at the steady state both the per capita capital stock, k, and per capita consumption, c, are constant. Also, at this non-zero steady state, the k-value **and** the corresponding c-value **are** defined respectively by the equations: $f'(k) = n + \rho$ and $c = f(k) - nk$. The condition that $f'(k) = n + \rho$ is called the ***modified golden rule.*** It states that the steady state value of the capital-labour ratio is such that the marginal product of capital is **equal** to the **sum** of the rate of population growth and the **rate** of time preference. In any dynamic model of **capital accumulation and** growth the concept of golden rule plays an important role (see Unit 3). **The *golden rule*** itself refers ***to*** that steady **state** value of **capital-labour ratio** where $f'(k) = n$, i.e., the **marginal product** of capital is exactly **equal** to the rate of growth of **population. The significance** of the golden rule obtains from the fact **that** it is that steady state value of capital-labour ratio which corresponds to the maximum steady state value of per capita consumption. Note that when households' rate of time preference is zero, the modified golden rule coincides with the golden rule and the steady state of the Ramsey model is attained at the golden rule point. However, when people have positive rate of time preference, they are unwilling to sacrifice current consumption for future consumption beyond a point; as a result they accumulate less and reach a lower level of steady state consumption than in the golden rule.

8.3 DECENTRALISED HOUSEHOLDS' PROBLEM

The Ramsey model described above can be easily transformed into the problem of a decentralized competitive market economy consisting of many households – all with identical preferences. We shall see that under certain assumptions the optimal solution path in the two cases turns out to be identical.

Instead of assuming the existence of a central planner, let us consider a market economy where the households are price takers. There are two competitive factor markets which determine the wage rate (w_t) and the interest rate or the rate of return on capital (r_t) at each point of time.

There exist many identical competitive firms which hire in labour and capital from the households at the above mentioned wage rate and rental rate. Using labour and capital these firms produce the final output, using the same technology as the central planner. Competition ensures that the wage rate and the rental rate are equal to the respective marginal products of labour and capital at full employment, i.e., $r_t = f'(k_t)$ and $w_t = f(k_t) - k_t f'(k_t)$.

Each household maximizes its welfare given as before by $W = \int_0^\infty u(c_t)\exp^{-\rho t}\, dt$. Each household starts with a certain amount of capital stock and labour stock. Additionally households can borrow from one another. Let $a_t \equiv k_t - b_t$ denote the per capita asset stock of an household at period t, which consists of the per capita capital stock owned by the household at period t minus the amount of debt (per capita) at period t. (If the household is a net lender, then b_t would be negative). Arbitrage condition in the asset market ensures that physical capital and lending earns the same rate of return. Hence the per capita income of the household at period t is $w_t + r_t a_t$, which the household spends on consumption and further asset accumulation. Thus the budget constraint faced by the household in per capita terms is given by: $c_t + \frac{da}{dt} + na_t = w_t + r_t a_t$.

The complete dynamic optimization problem for the household can now be written in terms of the two time dependent variables per capita consumption (c_t) and per capita asset stock (a_t):

Maximize $W = \int_0^\infty u(c_t)\exp^{-\rho t}\, dt$ subject to $\frac{da}{dt} = w_t + (r_t - n)a_t - c_t$; a_0 given.

The corresponding Hamiltonian functionand the first order conditions in terms of the control variable (c_t), state variable (a_t) and co-state variable(λ_t) are given by:

$$H = u(c_t)\exp^{-\rho t} + \lambda_t\left[w_t + (r_t - n)a_t - c_t\right]$$

ib) $$u'(c_t)\exp^{-\rho t} = \lambda_t$$

iib) $$\frac{d\lambda}{dt} = -\lambda_t(r_t - n)$$

iiib) $$\frac{da}{dt} = w_t + (r_t - n)a_t - c_t$$

ivb) $$\lim_{t\to\infty} \lambda_t a_t = 0$$

Note that these conditions look quite similar to the first order conditions that we obtained for the central planner's problem (conditions (ia)-(iva) in the previoussection). However, for the household's problem there is an additional condition which has to be satisfied if we want to get a meaningful solution. This condition is called the ***No-Ponzi-Game condition*** which we elaborate below. First note that we have allowed for intra-household borrowing which means that a household *can* maintain a consumption stream above its income by borrowing. Now if a household could go on borrowing indefinitely then it will be optimal for the household to always maintain an infinitely high (or maximum possible) consumption stream and finance such high level of consumption simply by borrowing more and more. Of course, such a strategy would also imply that the net per capita borrowing of the household would increase exponentially at the rate (r_t - n) (since the household has to borrow not only for consumption, but also to pay back the interest rate as well) and the present discounted value of the net debt of the household will approach infinity. Such a financing scheme is called *Ponzi-Game financing*[4]. In order to rule out such infinite indebtedness by any family we specify the condition that the even though the household can be temporarily a net debtor (i.e., the present discounted value of its asset is temporarily negative), over a sufficiently longer horizon, it must eventually repay all its debt and hold non-negative asset stock. Formally, as t goes to infinity, the present discounted value of the (per capita) asset stock of the household must be non-negative:

vb) $$\lim_{t\to 0} a_t \exp^{-\int_0^t (r_\tau - n)d\tau} \geq 0 \cdot$$

condition (vb) is called the No-Ponzi-Gamecondition. For a decentralized household conditions (ib)-(vb) specify the optimal path the solves the household's dynamic optimization problem.

As before, differentiating (ib) with respect to t and using (iib) to eliminate λ_t, and noting that the term $\frac{-cu''(c)}{u'(c)} \equiv \sigma$ denotes the elasticity of marginal utility with respect

[4] **Named after Charles Ponzi (1877-1949) who raised considerable amount of money promising a high rate of interest (50% for 45 days, 100% for 90 days). As long as new lenders were attracted to these returns, he was able to repay previous debt. In eight months he ended up with 10 million dollars of certificates and 14 million of debt. Criminal charges were finally levied on him in the US and died a pauper.**

to consumption, we can derive the following differential equation:

vib) $$\frac{dc}{dt} = \left(\frac{c_t}{\sigma}\right)(r_t - n - \rho)$$

Equations (iiib) and (vib) together form a system of differential equations which, along with the transversality condition and the no-Ponzi-Game condition, determine the movements of per capita consumption and per capita asset stock of the household along the optimal path.

Note that while for the economy as a whole the wage rate and the rental rate are given by $r_t = f'(k_t)$ and $w_t = f(k_t) - k_t f'(k_t)$, the price-taking households take these as given while optimizing. However we assume that the households are endowed with perfect foresight, which implies that though they ***do not know*** the relationship between w and r with the per capita capital stock, they can exactly ***guess*** the values of w and r at every point of time.

Also note that while the households can borrow from one another, in equilibrium for the economy as a whole net borrowing must be zero. Since households are all identical, this implies that net borrowing of each household would be zero, i.e., $a_t = k_t$.

Thus replacing $r_t = f'(k_t)$, $w_t = f(k_t) - k_t f'(k_t)$, and $a_t = k_t$ in equations (iiib) and (vib), we get a system of differential equation which is exactly identical to the system of differential equations that we obtained for the central planner. Hence their solution must also be the same. In other words, under the assumption of perfect foresight on the part of the households, the household's optimal consumption and accumulation paths are exactly identical to the consumption and accumulation paths that would be chosen by a social planner. To put it differently, the decentralized market economy's solution path coincides with the solution path of the centralized command economy, and the solution paths can once again be characterized by the diagram given at Fig. 8.1. This is a very strong result which shows the equivalence between the market equilibrium and the socially optimal solution. This equivalence result however depends crucially on the assumption of perfect foresight.

Check Your Progress 1

1) What are the steady state values of per capita consumption and per capita capital stock in the centralised version of the Ramsey optimal growth model?

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2) Would these values change if we consider a decentralized economy instead of a centralised planner's problem? Explain your answer.

8.4 GOVERNMENT IN RAMSEY MODEL: RICARDIAN EQUIVALENCE

The decentralized equilibrium that we discussed above does not incorporate government as a separate economic agent. It is however easy to introduce a role for the government in the de-centralized Ramsey model, whereby the government finances its own expenditure either by imposing a lump-sum tax on the household's income in every period. Since the government expenditure is unrelated to the households' consumption expenditure, the utility function of the households remain unchanged. However since the government is taking away a part of the household's income, the budget constraint of the household changes. To be more precise, if the government imposes a tax τ on the household income then the disposable income of the household reduces to $w_t + r_t a_t - \tau$; consequently the budget constraint of the household becomes: $\frac{da}{dt} = w_t + (r_t - n)a_t - c_t - \tau$. The rest of the optimal condition remains the same. In the phase diagram therefore, the $\frac{dk}{dt} = 0$ curve shifts downward by the amount τ, as shown in Fig. 8.2. The new steady state is obtained at the point E′ in Fig. 8.2, where the steady state value of the per capita capital stock remains the same, but steady state value of per capita consumption is lower. The corresponding optimal path of the household is once again denoted by the dotted line in the diagram.

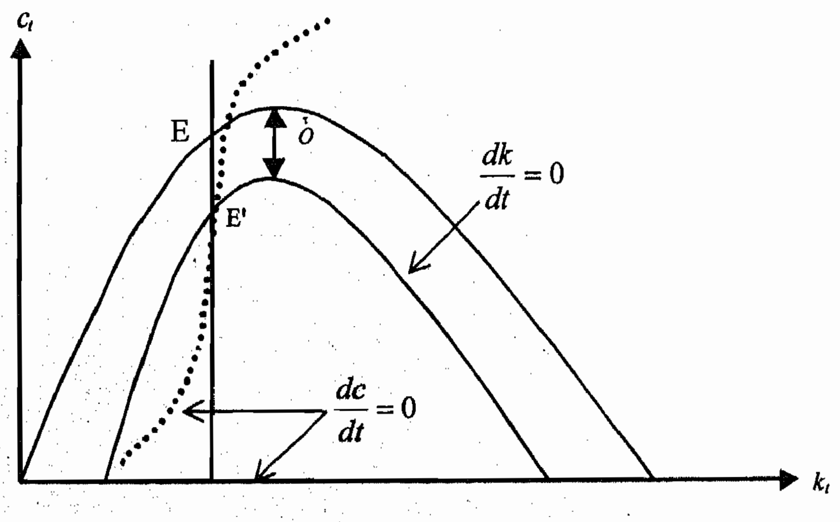

Fig. 8.2: Phase Diagram (with Government)

Now suppose the government decides to finance its expenditure by borrowing from the households, instead of taxation. Will that make any difference to the households' optima? At first glance it seems that if the government borrows from the households by distributing government bonds, then the government bonds would constitute an asset for the households (because the government would pay an interest on these loans). So the household's disposable income is not reduced; only the composition of its asset portfolio changes. So at first glance it seems that the household's optima under tax financing and under debt financing would be different. However, a more careful consideration of the problem reveals that the mode of financing of the government expenditure really **does not** make any difference to the household's optima. As long the government is forced to maintain a balanced budget in the long run, that is, it cannot go on borrowing indefinitely, at some point of time in future the government has to impose a tax in order to repay its debt incurred earlier. Since the households have perfect foresight, they would know that at some point of time they would have to pay the tax. Thus even though they are not paying the taxes today, they will adjust their current consumption and capital accumulation decisions accordingly which will enable them to bear the tax burden in future. This is a strong result which states that for optimizing households endowed with perfect foresight (or rational expectation), the mode of financing of government expenditure does not matter: tax financing and debt financing are exactly equivalent as far as the household optima is concerned. This result is known as the *Ricardian Equivalence Result*.

Check Your Progress 2

1) In the decentralized version of the optimal growth model, how would the equilibrium values of per capita capital stock and the per capita consumption change if the government imposes a lump sum tax to finance its own expenditure? In this context, explain the Ricardian Equivalence proposition.

8.5 LET US SUM UP

In this unit, we have looked at the Ramsey-Cass-Koopmans optimal growth model where infinitely lived individuals maximize their life-time utility. We identify the optimal consumption-accumulation path as the saddle path which takes the economy to its steady state in the long run. The steady state point is identified with the modified golden rule point. Although the problem is identified as a social planner's problem, its basic conclusion remains unchanged even when the economy is characterized by competitive decentralized households. Moreover, introducing a government sector which finances its expenditure through lump sum taxation does not have any effect on the steady state values of the model.

8.6 KEY WORDS

Infinite Horizon Dynamic Optimization Problem : An optimization exercise where the objective function is defined over several time periods starting from time zero to infinity, and where the relevant variables are dynamically linked from one period to another through a dynamic (difference or differential) equation.

Rate of Time Preference : The rate at which future utility is discounted vis-à-vis current utility.

Steady State : A long run equilibrium point where all the relevant dynamic variables remain constant over time.

Phase Diagram : A diagram which plots the dynamic (difference or differential) equations in order to characterize the direction of movements of the variables over time.

Golden Rule : When the marginal product of capital in an economy is equal to the rate of population growth. This is also the point where the steady state value of per capita consumption is at its maximum.

Modified Golden Rule : When the marginal product of capital in an economy is equal to the sum of the rate of population growth and the rate of time preference.

No-Ponzi-Game Condition : A condition that rules out the situation where an individual keeps on borrowing to meet previous debt obligations and eventually in the log run ends up with negative assets.

Ricardian Equivalence : A situation where the government's policy to influence some economic outcome is nullified by the actions of the private agents so that there is a equivalence between the pre-policy (competitive) and post-policy economy.

8.7 SOME USEFUL BOOKS

The standard references for the optimal growth model of section 8.2 are the following two classic articles by Ramsey and Cass:

Ramsey, Frank P., 1928, "A Mathematical Theory of Savings", *Economic Journal*, Vol. 38, pp. 543-559. Reprinted in Sen, Amartya (ed): *Growth Economics*, Penguin, 1970.

Cass, David, 1965, "Optimum Growth in an Aggregate Model of Capital Accumulation", *Review of Economic Studies*, Vol. 32, pp. 233-240.

All the relevant topics, including the Ricardian Equivalence proposition, can be found in the following textbook:

Blanchard, Olivier and Stanley Fischer, 1989, *Lectures on Macroeconomics*, MIT Press, chapter 2.

8.8 ANSWER/HINTS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress 1

1) Read Section 8.2 and write your answer.

2) Read Section 8.3 and write your answer.

Check Your Progress 2

3) Read Section 8.4 and write your answer.

8.9 MATHEMATICAL APPENDIX

Dynamic Optimisation in Continuous Time (Optimal Control)

A.1 Finite Horizon Problem

Consider the following optimisation problem which is defined over a finite time horizon from 0 to T:

$$\text{Max. } W = \int_0^T F(u_t, x_t, t)\,dt, \text{ subject to } \frac{dx}{dt} = f(u_t, x_t, t) \qquad \ldots(1)$$
$$x_0 = \bar{x};\ u_t \in U.$$

The objective function here is an ***integral***, and our task is to find out a time path of the time dependent variable u from the choice set U, (i.e., to choose a $u \in U$ for each point of time t starting from 0 to T) such that this integral is maximized.

But our choice is not unconstrained. (Had it been so, a simple point-by-point static optimization exercise would have given us the required solution path). Note that the F function (which can be called the ***instantaneous*** objective function) depends not only on u but also on another time dependent variable x. And our choice of u at each point of time affects the next period's value of x through the given differential equation. Thus our choice of u affects the instantaneous objective function directly, as well as indirectly through x. So when we are choosing the optimal time path for u that maximizes the integral W, we have to take into account how x is changing due to our choice of u – this explains the presence of a constraint function in the form of a differential equation in x.

Since we are choosing the variable u directly (i.e., we have direct control over this variable) it is called the ***control variable***. The other time dependant variable, x, which changes over time due to our choice of u, but we do not have direct control over its change, is called the ***state variable***. The function f in the given differential equation that describes the change in x over time is called the ***state transition function***.

Pontryagin's Maximum Principle: Let u_t^* be a solution path to the problem specified in (1), and let x_t^* be the associated path for the state variable, where u_t^* is a piece-wise continuous function of t and x_t^* is a strictly continuous but piece-wise differentiable in t. Then there exists a strictly continuous and piece-wise differentiable variable λ_t, and a function H defined as:

$$H(u,x,\lambda,t) = F(u_t,x_t,t) + \lambda_t f(u_t,x_t,t),$$

such that

i) H is maximised with respect to u at u^* for all $t \in [0,T]$

ii) $$\left.\frac{\partial H}{\partial x}\right|_{(u^*,x^*,\lambda,t)} = -\frac{d\lambda}{dt}$$

iii) $$\left.\frac{\partial H}{\partial \lambda}\right|_{(u^*,x^*,\lambda,t)} = \frac{dx}{dt}$$

iv) $\lambda_T x_T = 0$

The function H is called the ***Hamiltonian Function*** associated with the given dynamic optimizations problem, and the newly introduced time dependent variable λ is called the ***co-state variable*** associated with the state variable x.

The co-state variable λ_t measures the change in the value of the objective function W associated with an infinitesimal change in the state variable x at time t. If there were an exogenous tiny increment to the state variable at time t, and if the problem were modified optimally there after, then the increment in the total value of the objective would be λ_t. Thus it is the marginal valuation of the state variable at time t, and is therefore often referred to as the ***shadow price*** of the state variable at time t.

Pontryagin's Maximum Principle gives us four ***first order necessary conditions*** for the optimization problem defined in (1). The first three F.O.N.C's are defined in terms of the Hamiltonian function. Note that if the Hamiltonian function is non-linear in u, then (i) can be replaced by the condition $\left.\frac{\partial H}{\partial u}\right|_{(u^*,x,\lambda,t)} = 0$, **provided the second order check** $\left.\frac{\partial^2 H}{\partial u^2}\right|_{(u^*,x,\lambda,t)} < 0$ **is verified.**

The last condition of the Maximum Principle (condition (iv)), which specifies a terminal condition for λ, is called the ***Transversality Condition***.

A.2 Infinite Horizon Problems

Consider the following infinite horizon dynamic optimization exercise:

$$\text{Max. } W = \int_0^\infty F(u_t, x_t, t)\,dt, \text{ subject to } \frac{dx}{dt} = f(u_t, x_t, t) \qquad \ldots(2)$$
$$x_0 = \overline{x};\ u_t \in U.$$

This problem is almost the same as the problem specified in (1); only now the time horizon is no longer finite.

When the time horizon stretches from 0 to ∞, additional complications may arise: Firstly, the integral W may not converge. In that case it does not make sense to talk about optimal paths, because any paths (u_t, x_t) will eventually make the integral infinitely large, and no comparison between one set of (u_t, x_t) with another is possible. Hence for the dynamic optimization exercise to be meaningful, we must impose additional restrictions so that the integral converges to a finite value.

Secondly, there is some controversy regarding the appropriate transversality condition in infinite horizon problems. Usually some limiting conditions, which are similar to the transversality conditions applied in the finite horizon problems, are applied.

Consider problem (2) and suppose the integral converges. The first order non-essential conditions for this problem are given in the following theorem:

Let u_t^* be a solution path to the problem specified in (5), and let x_t^* be the associated path for the state variable, where u_t^* is a piece-wise continuous function of t and x_t^* is strictly continuous but piece-wise differentiable in t. Then there exists a strictly continuous and piece-wise differentiable variable λ_t, and a function H defined as:

$$H(u, x, \lambda, t) = F(u_t, x_t, t) + \lambda_t f(u_t, x_t, t),$$

such that

i) H is maximised with respect to u at u^* for all $t \in [0, \infty)$

ii) $\left.\dfrac{\partial H}{\partial x}\right|_{(u^*, x^*, \lambda, t)} = -\dfrac{d\lambda}{dt}$

iii) $\left.\dfrac{\partial H}{\partial \lambda}\right|_{(u^*, x^*, \lambda, t)} = \dfrac{dx}{dt}$

iv) $\lim_{t \to \infty} \lambda_t x_t = 0$

UNIT 9 THE OVERLAPPING GENERATIONS MODEL

Structure

9.0 OBJECTIVES

After going through this Unit you should be able to exlain:

- the standard two-period overlapping generations model with production;
- the concept of dynamic efficiency and examine whether the dynamic efficiency property holds for an overlapping generations economy; and
- the role of social security system in the overlapping generations framework in eliminating dynamic inefficiency.

9.1 INTRODUCTION

In the context of intertemporal decision making on the part of the households, you have come across the optimal growth model in the previous unit. There is another important framework that also considers households' intertemporal decision making – although over finite time horizon. This framework is called the overlapping generations framework. The framework was first developed by Samuelson (1954) which has subsequently been widely used in macro dynamics.

In the overlapping generations framework, individuals have a finite time horizon (in the standard case, a two-period time horizon), but the society lives forever. The term 'overlapping generations' implies that at each point of time the life-time of two generations overlaps. To clarify, let us take the standard case where each generation lives exactly for two periods. Thus for the cohort of individuals who are born at the beginning of period 't', they are alive in period 't' (when they are young) and in period '$t+1$' (when they are old). On the other hand, a new set of individuals are born at the beginning'of period $t+1$, who would be alive in period '$t+1$' and period '$t+2$' respectively. Thus between the lifetimes of these two successive generations, there is an exact overlap of one period. In the following discussion we shall concentrate on the two-period overlapping generations framework only.

9.2 STRUCTURE OF THE MODEL

The set of people who are born at the beginning of period t will be called 'generation t'.

Let us now look at the activities of a representative member of generation t. Each person is born with an endowment of one unit of labor. In the first period of his life, when he is young, he works and earns a wage income of which he consumes a part and saves (and invests) the rest. In the next period of his life, when he is old, he does not work anymore. He nonetheless earns an interest income on his previous period's savings (investment). He also gets back the principal amount that he invested, which he consumes in the second period along with the interest earning. Thus his first and second period budget constraints are respectively given by:

i) $c_{1t} + s_t = w_t$;

ii) $c_{2t} = (1 + r_{t+1})s_t$,

where c_{1t} and c_{2t} are the first and second period consumption of the representative member of generation t, and w_t and r_{t+1} are the wage rate at period t and the **rate** of interest at period $t+1$ respectively.[1] The representative member maximizes his two-period utility function $U(c_{1t}, c_{2t})$ subject to these two budget constraints. Noting that $s_t = \frac{c_{2t}}{(1+r_{t+1})}$, we can combine the two budget constraints by eliminating s_t, which gives us the following single equation that represents the ***life-time budget constraint*** of the agent: $c_{1t} + \frac{c_{2t}}{(1+r_{t+1})} = w_t$.

Maximizing $U(c_{1t}, c_{2t})$ subject to the life-time budget constraint generates the following two first order conditions:

1) $$\frac{U_1}{U_2} = (1 + r_{t+1}),$$

2) $$c_{1t} + \frac{c_{2t}}{(1+r_{t+1})} = w_t.$$

From these two equations we can write the optimal c_{1t} and c_{2t} as functions of w_t and r_{t+1}. Since $s_t = \frac{c_{2t}}{(1+r_{t+1})}$, the optimal value of s_t also becomes a function of w_t and r_{t+1}.

We shall assume that all members of all generations are identical in terms of tastes and preferences, i.e., they have similar utility functions.[2]

Let us now look at the overall macroeconomic picture. As we have mentioned before, at each period there are two generations who are simultaneously alive. Thus at period t, there is a set of people who belong to generation *t-1* (these are the people who are currently old) and a set of people who belong to generation *t* (these are the people who are currently young). The generation t people are the workers in period t, each of

[1] **Note that though the member of generation *t* made his saving and investment decision at period *t*, he earns the interest on that savings only in the next period. Hence r_{t+1} is the relevant rate of interest.**

[2] **Similar, but not identical. To be more precise, the time subscripts in the utility function will be different for different generations.**

whom earn a wage income w_t. The generation t-1 people are the interest earners who earn an interest income on their previous period's savings (i.e., savings made in period t-1) at the rate r_t.

Suppose the population in successive generations grows at a constant rate n. Thus if L_t denotes the number of people belonging to generation t, and L_{t-1} denotes the number of people belonging to generation t-1, then $L_t = (1+n)L_{t-1}$. Total population in the economy at period t is given by $L_t + L_{t-1}$.

The production side of the economy is like any standard neoclassical growth model that you have seen before. A single final commodity is produced which is used both as a consumption good as well as an investment good. Technology for final commodity production is given by a neoclassical production function: $Y_t = F(K_t, L_t)$, where F is continuous, concave, exhibits constant returns to scale (CRS) with respect to its two factors – capital (K) and labour (L). As in the Ramsey model, the CRS property of the production function implies that per capita output (y) can be written as the function of the capital-labour ratio (k) in the following way:

$$y \equiv \frac{Y}{L} = \frac{F(K,L)}{L} = \frac{L\,F(K/L,1)}{L} = F\left(\frac{K}{L},1\right) \equiv f(k).$$

Moreover the marginal products of capital and labour can also written as the following functions of the capital-labour ratio:

$$\frac{\partial F}{\partial K} = f'(k)\ ;\ \frac{\partial F}{\partial L} = f(k) - kf'(k).$$

The continuity and concavity properties of $F(L, K)$ ensure that f (k) is also continuous and concave. Additionally we assume that the Inada conditions hold: f (0) = 0; $f'(0) = \infty$; $f'(\infty) = 0$.

In a market economy with perfect competition, the wage rate and the rate of interest are equated with the respective marginal products of labour and capital. Thus $w_t = f(k_t) - k_t f'(k_t)$ and $r_t = f'(k_t)$.

For the economy as a whole, savings-investment equality (which generates goods market equilibrium for the aggregate economy) tells us that $C_t + I_t = Y_t = F(K_t, L_t)$. Noting that investment is nothing but the augmentation of the capital stock (assuming no depreciation), i.e., $I_t = K_{t+1} - K_t$, and also noting that due to the property of CRS $F(L_t, K_t) = w_t L_t + r_t K_t$, we can write the above savings-investment equality condition as: $C_t + K_{t+1} - K_t = w_t L_t + r_t K_t$. Notice that C_t denotes the aggregate consumption at period t, which includes consumption by the old group and consumption by the young group. There are L_{t-1} number of old people who are alive at period t and each of them consume an amount c_{2t-1}.[3] On the other hand, there are L_t number of people

[3] c_{2t-1} is the 2nd period consumption of the representative member of generation t-1.

at period t belonging to the young generation, each of them consuming an amount c_{1t} Hence $C_t = L_{t-1}c_{2t-1} + L_t c_{1t}$.

At this p i n t it is important to recall that each young person is born with an endowment of one unit of labour. There is no bequest; therefore the young people do not own any capital stock at period t (capital stock being the only asset in this economy). Thus the entire capital stock is owned by the older generation. And since the older generation is going to die at the end of this period, they consume their entire interest earning plus the capital stock (which, in the one good world, is directly consumable). Hence $L_{t-1}c_{2t-1}$ = K, $+ r_t K_t$. Again, each young person earns a wage w,, of which he consumes c_1, and saves s_t. Thus $L_t c_{1t} = L_t$(w, $- s_t$). Using all these information, we can write the savings-investment equality condition as:

$$K_t + r_t K_t + L_t(w_t - s_t) + K_{t+1} - K_t = w_t L_t + r_t K_t .$$

Simplifying, we get the goods market equilibrium condition for the aggregate economy as: $K_{t+1} = L_t s_t$. Recall that s_t is a function of w_t and r_{t+1}. w_t and r_{t+1} are in turn functions of k, and k_{t+1} respectively. Thus s_t can be written as a function of k, and k_{t+1}: $s(w(k_t), r(k_{t+1}))$. This allows us to write the goods market equilibrium condition in terms of the capital labour ratio as follows:

$$K_{t+1} = L_t s(w(k_t), r(k_{t+1}))$$

$$\Rightarrow \frac{K_{t+1}}{L_{t+1}} = \frac{L_t s(k_t, k_{t+1})}{L_{t+1}} = \frac{L_t s(w(k_t), r(k_{t+1}))}{(1+n)L_t}$$

$$\Rightarrow k_{t+1} = \frac{s(w(k_t), r(k_{t+1}))}{(1+n)}.$$

This last line above represents the basic dynamic equation of this overlapping generations model which specifies the relationship between the capital-labour ratio of today and the capital-labour ratio of tomorrow. Tracing this dynamic equation will tell us how the capital-labour ratio of the economy changes over time. It is easy to see that this equation is a first order non-linear difference equation in k. To characterize the solution path we shall use the phase diagram technique. The phase diagram plots k,,, on the vertical axis and k, on the horizontal axis. The intersection of the k_{t+1} line with the 45° line denotes the steady state. Let us now determine the slope of this line. The slope is given by the derivative $\frac{dk_{t+1}}{dk_t}$. As you can see, the LHS of the dynamic equation is also a function of k_{t+1}. Hence total differentiating both sides,

$$dk_{t+1} = \frac{s_w \frac{dw_t}{dk_t} dk_t + s_r \frac{dr_{t+1}}{dk_{t+1}} dk_{t+1}}{(1+n)} .$$

Noting that $\frac{dw}{\mathrm{dk}} = -kf''(k)$ and $\frac{dr}{\mathrm{dk}} = f''(k)$ we can write the above equation as:

$$\frac{dk_{t+1}}{dk_t} = \frac{s_w[-k_t f''(k_t)]}{(1+n) - s_r f''(k_{t+1})}.$$

Notice that as yet we do not know the signs of s_w and s_r; hence we cannot say whether the k_{t+1} line is positively sloping or negatively sloping. It is easy to see that under the assumption that consumption in both periods are normal goods (i.e., both c_1 and c_2 increases with an increase in w), $0 < s_w < 1$. The sign of s_r however is ambiguous. Since an increase in r implies that the relative price of future consumption in terms of current consumption falls. Hence due to substitution effect current consumption should fall, which means that with unchanged wage rate, savings would rise. However, a fall in relative price will also be associated with a positive income *effi* ton current consumption. Thus whether current consumption increases due to an increase in r depends crucially on whether the income effect dominates the substitution effect. If the income effect dominates the substitution effect, thenc, rises and consequently $s_r < 0$. On the other hand, if the substitution effect dominates the income effect, then c, falls and consequently $s_r > 0$. We shall assume here that the latter holds, i.e., $s_r > 0$. Under this assumption the k_{t+1} line is positively sloping. We still do not know the curvature of this line. Depending on the curvature multiple equilibria (i.e., multiple steady states) are possible, as shown in Fig. 9.1.

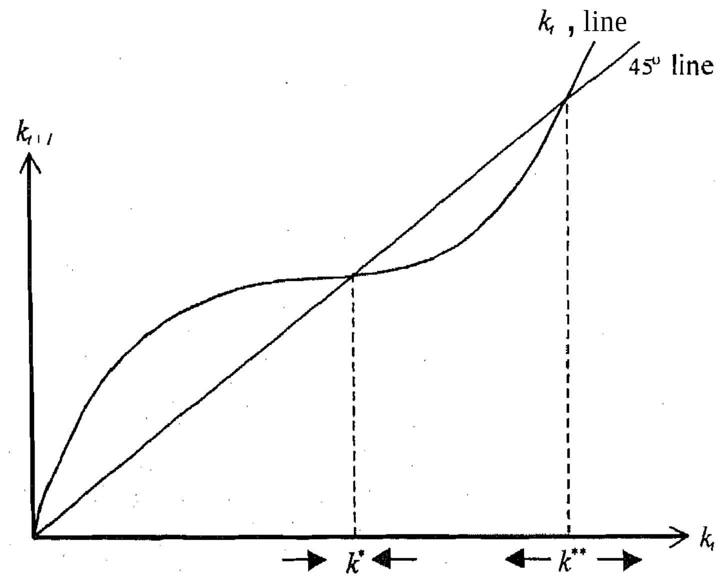

Fig. 9.1 : Multiple Steady State

Local stability of a steady state depends on whether the k_{t+1} line intersects the 45° line from above or fiom below. In Fig. 9.1 we find that k^*, where the k_{t+1} line intersects the 45° line from above, is a locally stable equilibrium. On the other hand, k*', where the k_{t+1} line intersects the 45° line fiom below, is a locally unstable equilibrium.

9.3 DYNAMIC INEFFICIENCY IN OVER LAPPING GENERATIONS MODEL

In the following discussion we shall assume that the parametric conditions are such that there exists a unique equilibrium which is locally stable.

Even when the steady state is unique and stable, in an overlapping generation framework the equilibrium can still be characterized by dynamic inefficiency. This is a serious shortcoming of the overlapping generations framework in comparison to the optimal growth framework. Before we elaborate this point further it is important to know what we mean by dynamic efficiency or inefficiency.

The concept of dynamic efficiency is closely related to the concept of Pareto efficiency. To understand the concept consider a situation where we are comparing between various possible steady states or equilibrium points. Each of these steady states is characterized by a different capital-labour ratio and corresponding steady state levels of per capita consumption for the old and the young. Since utility depends on the consumption during youth and during old-age, each of these steady states is therefore associated with a different level of steady state utility. Now among all these steady states, the one which provides maximum steady state value of utility is called the 'golden rule' point and the corresponding capital-labour ratio is called the **'golden rule'** capital labour ratio. This point is the best possible steady state point which provides maximum life-time utility to each person. One can show that steady state utility is maximized at the point where $f'(k) = n$. Thus golden rule capital-labour ratio is defined by k_g such that $f'(k_g) = n$. Fig. 9.2 depicts the golden rule capital labour ratio as the point which maximizes steady state utility.

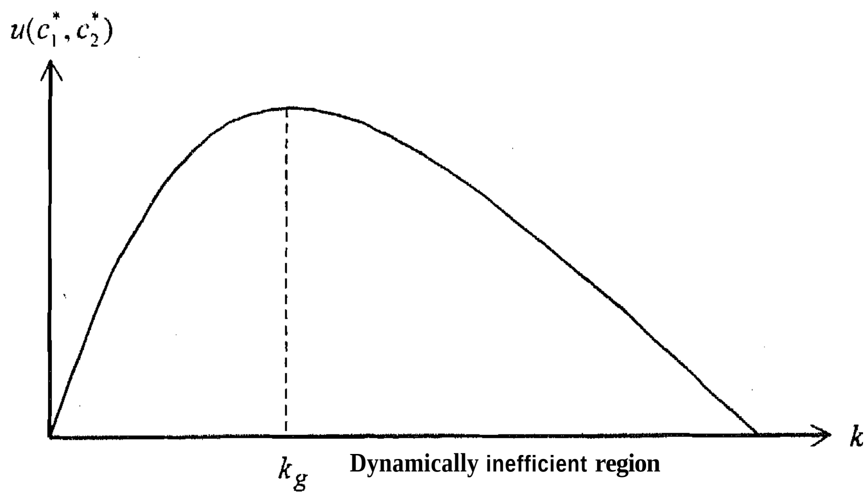

Fig. 9.2: Golden Rule Capital Labour Ratio

Now consider all the points which lie to the right of k_g. Here the capital-labour ratio is more than optimal. In other words people are over-saving and over-investing. If instead of saving, people consume a part of their savings in the first period of their life, then the capital-labour ratio will fall to k_g and at the same time their life time utility will increase. Thus all the points lying to the right of k_g are Pareto inferior to k_g: one can improve the current consumption without reducing future consumption and thus improve total life-time utility. These points are called dynamically inefficient points. Now consider all the points to the left of k_g. Here also the utility level is less than k_g; hence by moving to k_g one can improve the steady state value of utility. However such a move is not costless anymore. If one wants to move from a point to the left of k_g to k_g, then he has to save more. In other words he has to forgo some amount of current consumption. Thus his current utility would *fall*, even though he would be better off in the future (once he reaches k_g). Since such a move from the left to right involves a current utility loss, we cannot say for sure whether these points are better or worse then k_g. All these points are Pareto efficient or dynamically efficient.

We have seen that in the optimal growth framework the steady state is defined by $f'(k) = n + \rho$. Thus the steady state point in that framework is always to the left of the golden rule point and is therefore dynamically efficient. In the case of the overlapping generations framework however dynamic efficiency of the equilibrium point cannot be guaranteed. In fact under very reasonable parametric values the steady state could be dynamically inefficient.

To see how, consider the following example where we assume a specific utility function and a specific production function. Let the utility function of the representative member of generation t be $U(c_{1t}, c_{2t}) \equiv \log c_{1t} + \log c_{2t}$. Also let the per worker production function be $f(k_t) \equiv Ak_t^{\alpha}$, $0 < \alpha < 1$. Both the log utility function and the Cobb-Douglas production function are known to be well-behaved which satisfy all the standard neoclassical properties.

With the log utility function as specified above, one can easily verify that the first order conditions for individual's utility maximization exercise are given by:

i) $$\frac{c_{2t}}{c_{1t}} = (1 + r_{t+1})$$

ii) $$c_{1t} + \frac{c_{2t}}{(1 + r_{t+1})} = w_t.$$

These two first order conditions generate a savings function which is given by: $s_t = \frac{1}{2} w_t$, Also, with Cobb-Douglas production function $w_t = (1 - \alpha)Ak_t^{\alpha}$. Thus the basic dynamic equation in this example is given by: $k_{t+1} = \frac{1}{(1+n)}(1 - \alpha)Ak_t^{\alpha}$. At steady state $k_t = k_{t+1} = k^*$. Hence putting k^* at the LHS and RHS of the above equation, we can solve for the steady state value as: $k^* = \left[\frac{A(1-\alpha)}{(1+n)}\right]^{1/(1-\alpha)}$

Let us now compare this steady state value of capital-labour ratio with the corresponding golden rule capital labour ratio. Note that with the Cobb-Douglas production function, the golden rule capital labour ratio in this example is defined by: $A\alpha(k)^{\alpha-1} = n$. Solving, we get k_g as: $k_g = \left[\frac{A\alpha}{n}\right]^{1/(1-\alpha)}$. Thus whenever $\frac{(1-\alpha)}{\alpha} > \frac{(1+n)}{n}$, $k^* > k_g$, i.e., the steady state will be dynamically inefficient. For example, the equilibrium will be dynamically inefficient if $\alpha = 1/4$ and $n = 1$.

An obvious question that arises here is: why is it that the steady state could be dynamically inefficient in the overlapping generations framework, while such possibility is ruled out in the optimal growth framework? The answer lies in the fact that in the overlapping generations individuals are selfish (no bequest). Since they do not have to share the benefits of their investment with their successive generations (who are growing at the rate n), when they consider the possible future return to their investment, the return is **not** net of the population growth. To put it differently, the relevant return for them is not $(f'(k) - n)$ but just $f'(k)$. Hence they would be interested in investing as long as this return is positive, even if it falls short of n. This is the reason for their possible over-saving.

Check Your Progress 1

1) Derive the basic dynamic equation of the standard two period overlapping generations model with production. Explain the intuition behind this equation.

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2) What is dynamic efficiency? Is the steady state under the overlapping generations structure necessarily dynamically efficient? Give an example to elaborate your answer.

9.4 SOCIAL SECURITY

Given that the overlapping generations equilibrium could be characterized by dynamic inefficiency, one can ask if there is any role for the government here. In other words, does there exist any government policy which could check the over-saving by the households? Typically the government could influence the savings decision through some kind of social security programmes, What is a social security programme? A social security programme is meant to provide some income to individuals after retirement. There are usually two types of social security programmes which *are* in vogue – a fully funded system and a pay-as-you-go system, In a fully funded system a stipulated amount is taken by the government from each young person at period t as social security contribution. The same amount is invested by the government and returned with interest to the old people at period $t+1$. Since the young at period t are the old at period $t+1$, in a fully funded system the set of people who made the contributions today and the set of people who will receive the corresponding benefit tomorrow are the same. If d_t is the contribution made by each young person at period t, and b_{t+1} is the benefit received by each old person at period $t+1$, then $b_{t+1} = (1+r_{t+1})d_t$.

In an unfunded pay-as-you-go system, a stipulated amount is taken by the government from each young person at period t and the entire amount is immediately distributed to the old people at period t. In other words, the pay-as-you-go system involves a transfer of funds from the current young to the current youth. Note that population is growing at the rate n; therefore at each point of time there are n more people belonging to the currently young group compared to the currently old group. Thus if d_t is the contribution made by each young person at period t, and b_t is the benefit received by each old person at period t, then $b_t = (1+n)d_t$.

The type of social security system has important implications for the savings decisions of the young. Note that in a filly funded system the government is doing part of the savings on behalf of the individuals. The representative individual is effectively saving an amount equal to $(s_t + d_t)$ and in the next period earning an interest income equal to $(1+r_{t+1})(s_t + d_t)$. Since each person knows that this is the amount that he would receive in the next period, in his own savings decision, the individual will optimally adjust (cut back) his own savings so that his total effective savings remains the same as in the pre-social security economy. Thus a fully funded social security system has no impact on the total savings and capital accumulation. If the economy was in a dynamically inefficient steady state in the pre-social security economy, it will remain so even after introducing a fully funded social security system.

In contrast, in a pay-as-you-go social security system, the relevant rate of return on government securities for an individual is n. On the other hand, the rate of interest on capital accumulation is r. Thus as long as $r<n$, it pays to save less and contribute more in the form of social security. Hence if the economy was in a dynamically inefficient steady state in the pre-social security economy (which implies that r is indeed less than n), introducing a pay-as-you-go type of social security system will reduce private savings and capital accumulation. As a result the economy will move to the dynamically efficient region.

Check Your Progress 2

1) Does any type of social security system necessarily eliminate the dynamic inefficiency problem in the overlapping generations framework? Elaborate your answer with the two types of social security systems.

9.5 LET US SUM UP

In this unit, we discussed at the standard overlapping generations framework with production. We have derived the basic dynamic equation which basically states that the tomorrow's capital stock is equal to the savings by the young generations today. We have derived the steady state conditions. It has been shown that the steady state under the OLG framework may not be dynamically efficient. The government can play an important role here to ensure dynamic efficiency by introducing a social security system. However the type of the social security system is important: apay-as-you-go type of social security system is effective in eliminating inefficiency while a fully funded social security system is not effective.

9.6 KEY WORDS

Dynamic Efficiency : The situation where an increase in future consumption implies a fall in current consumption so that future utility cannot be increased without reducing current utility.

Overlapping Generations : A framework where agents have finite lifetime and at every point of time more than one generation of agents coexist.

Social Security System : A provision for retirement benefit such that an agent receives certain monetary benefit from the government when he is old.

9.7 SOME USEFUL BOOKS

Blanchard, Olivier and Stanley Fischer,1989, *Lectures on Macroeconomics*, MIT Press, chapter 2.

9.8 ANSWER/HINTS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress 1

1) Read Section 9.2 and Mite your answer.

2) Read Section 9.3 and write your answer.

Check Your Progress 2

1) Read Section 9.4 and write your answer.

UNIT 10 MONEY AND THE ROLE OF MONETARY POLICY

Structure

10.0 OBJECTIVES

After going through this Unit you should be able to explain:

- the role of money in facilitating the transaction in a standard two period overlapping generations model of exchange;
- why money has positive value in an economy where money is the only medium of exchange; and
- the role of money in a model where people derive direct utility by holding money.

1 1 INTRODUCTION

As you know, money is the medium of exchange. In old days gold and silver were used as the medium of exchange. These were commodity money, which had some value apart from their role as medium of exchange. However, in the modern world, money is typically paper money which does not have any worth on its own: its only value is in terms of the commodities that you can buy in exchange.

Money also serves a second purpose. In so far as money is needed to carry out any exchange, it is also used as a store of value. However, as a store of value, money is typically dominated by all other assets in the sense that holding money involves zero return in nominal terms, whereas all other assets carry some positive return in nominal terms. Then why is paper money valued at all? Here we shall seek the answer to this question.

The neoclassical growth models that you have studied so far are based on the assumption that there is a single final commodity. Hence all payments–including the wage rate and the rate of interest – are made in terms of this final commodity. There is no money. In this chapter we shall see how the presence of money affects the real decisions and the dynamic equilibria in these models.

10.2 MONEY IN THE OVERLAPPING GENERATIONS MODEL

For money to be valued it must facilitate the exchange process. Samuelson's overlapping generations (henceforth OLG) set up can illustrate the usefulness of money as a medium of exchange.

Let us look at an OLG model of exchange. The structure of the model is very similar to the OLG growth model that you have studied earlier. The only difference is that the present model is an OLG model of exchange'–there is no production.

The household side of the story is exactly same as before. Each generation lives exactly for two periods. Thus for the cohort of individuals who are born at the beginning of period 't', they are alive in period '*t*' (when they are young) and in period '*t+1*' (when they are old). On the other hand, a new set of individuals are born at the beginning of period $t+1$, who would be alive in period '*t+1*' and period '*t+2*' respectively. The set of people who are born at the beginning of period t will be called 'generation *t*'. Population in successive generations grows at the rate n. We shall assume that the initial stock of population (L_0) was unity, so that number of people belonging to generation t is given by: $L_t = (1+n)^t$.

The representative member of generation *t* is endowed with one unit of a consumption good when young and receives no endowment when old. This consumption good is perishable; hence it cannot be stored for future consumption. On the other hand money can be stored. Thus anybody who intends to consume in the next period must sell the commodity in exchange of money in the first period and then again buy back the commodity against money in the next period. The two-period utility function of the representative member is given by $U(c_{1t}, c_{2t})$, where the utility function has all the standard properties.

At any period the consumption possibilities from the society's point of view is shown in the following diagram. The horizontal axis measures the per capita consumption of young and the per capita consumption of the old. Note that at any point of time t, the total endowment of goods is given by L_t (since each young person receives 1 unit and there are L_t number of young persons in the economy at point t). If the entire endowment is consumed by the young, then each of them consume one unit, which is denoted by point A on the horizontal axis. If, on the other hand, the entire amount is consumed by the old, then each of them consume $(1+n)$ units, which is denoted by point B on the vertical axis. The straight line joining these two points represents the society's consumption possibility frontier: each point on this line represents a certain distribution of the total endowment between the young and the old (see Fig. 10.1).

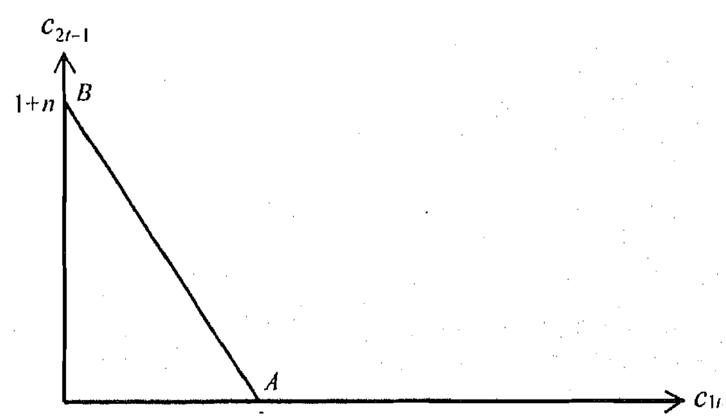

Fig. 10.1: Consumption Possibility Frontier

Let us now see **what** is the best point from an individual's point of view. The individual has an utility function $U(c_{1t}, c_{2t})$. If he could choose between all the points on the consumption possibility frontier then he would have chosen the point which would have maximized his life-time utility. This point is represented by point E in Fig. 10.2.

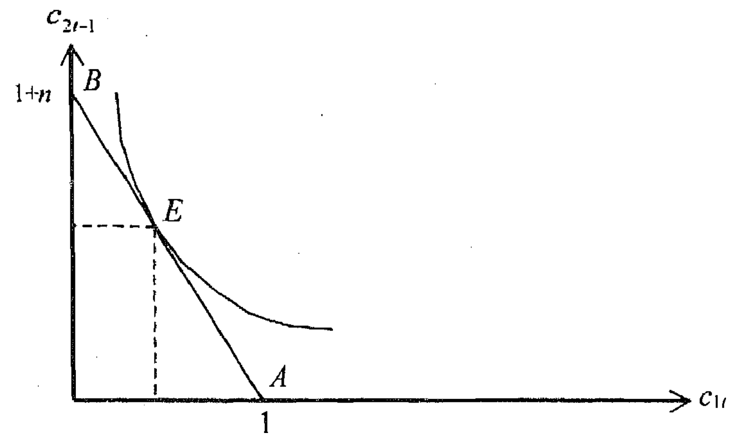

Fig. 10.2: Lifetime Utility Maximisation

It is however important to note that **without money, this point is not reachable by any individual.** In other words, a simple trade between the current young and the current old cannot ensure that point E is reached. Why not? The reason is the following. In order to reach the point E the current young will have to give a part of their endowment to the current old. In exchange they will have to be given part of the endowment of tomorrow's young generation. Since tomorrow's young generation is not present'yet, such a contract cannot be enforced on them tomorrow. So no trade can take place.

Let us now see how introduction of money can help everybody to reach the optimal point. Suppose at time *t* the government gives the current old generation an amount of money, $\overline{M}$, which is equally distributed among all the members of the current old generation. Suppose everybody in the economy (including the future generations) believes that they will be able to exchange money for goods at a certain price. The price level can vary from time to time depending on the demand for and supply of money. Let P_t denote the price level at time *t*. It is obvious that the current old generations are the suppliers of money. But who demands money? The demanders are the current young. Why do they demand money? Since money is **not** perishable (while the consumption good is perishable), the young generation can store the money and in the next period (when they are old) can exchange it for goods so as to reach the optimal point. To ***see*** the argument more clearly, let us consider the maximization problem of a representative member of generation *t*. He will maximize his utility $U(c_{1t}, c_{2t})$ subject to the following two constraints:

i) $P_t(1 - c,,) = M_t^d$; and

ii) $P_{t+1} \cdot c_{2t} = M_t^d$,

where M_t^d is the demand for money by the representative member of generation ***t.*** The underlying intuition behind these two constraints are quite straight forward. The first constraint says that out of his endowment of one unit of final good, the representative member of generation *t* saves an amount $(1 - c,,)$ when young. This amount he exchanges for money at period *t* at the current price P_t. Thus $P_t(1 - c,,)$ constitute his total demand for money in period t. In the next period he sells the stored money M_t^d to the young members of generation $t+1$ at a price P_{t+1}, to get a second period consumption $c_{2t} = M_t^d / P_{t+1}$. Rearranging the terms we get the second constraint. Eliminating M_t^d from the above two constraints, we can generate a single constraint:

$$P_t(1 - c_{1t}) = P_{t+1}.c_{2t}.$$

Maximizing utility subject to this single constraint one can derive the following first order condition: $\frac{U_1(c_{1t}, c_{2t})}{P_t} = \frac{U_2(c_{1t}, c_{2t})}{P_{t+1}}$. This first order condition along with one of the constraints in turn defines the optimal demand for money function in real terms as a function of P_t and P_{t+1}: $\frac{M_t^d}{P_t} = F\left(\frac{P_t}{P_{t+1}}\right)$.

Next we characterize the equilibrium in the money market. At time t each of the current young demands M_t^d amount of money. Hence the total demand for money at time t is given by $L_t M_t^d$, or $(1+n)^t L_0.F\left(\frac{P_t}{P_{t+1}}\right).P_t$. On the other hand, the entire old generation receives a money endowment equal to $\overline{M}$. Thus the total supply of money at period t is given by $\overline{M}$. Equating demand and supply, we get the money market equilibrium condition at period t as:

$$(1+n)^t L_0.F\left(\frac{P_t}{P_{t+1}}\right)P_t = \overline{M}.$$ [1]

Note that $\left(\frac{P_t}{P_{t+1}}\right)$ is the rate of return on money in real terms. It has close link with the rate of deflation. The latter concept is defined as π_t, where $(1+\pi_t) \equiv \left(\frac{P_t}{P_{t+1}}\right)$. Thus we can write the money market equilibrium condition at any period t in terms of the current rate of deflation and the current price level in the following way: $(1+n)^t L_0.F(1+\pi_t)P_t = \overline{M}$.

Notice that the money market equilibrium condition defines a short run equilibrium rate of deflation for every point of time. At time t the equilibrium rate of deflation is given by $(1+n)F(1+\pi_t) = \overline{M}$, and at time $t+1$ the equilibrium rate of deflation is given by $(1+n)^{t+1} L_0.F(1+\pi_{t+1})P_{t+1} = \overline{M}$. Comparing the two we get: $F(1+\pi_t)P_t = (1+n)F(1+\pi_{t+1})P_{t+1}$. Rearranging terms: $\frac{P_t}{P_{t+1}} = (1+n)\frac{F(1+\pi_{t+1})}{F(1+\pi_t)}$,

[1] Since there are only two commodities – the final good and money – money market equilibrium in any period, by Walras' Law, implies that the goods market is also in equilibrium.

i.e., $(1+\pi_t)=(1+n)\dfrac{F(1+\pi_{t+1})}{F(1+\pi_t)}$. At the steady state (long run equilibrium) the rate of deflation is constant which, from the above equation, implies that $\pi_t=\pi_{t+1}=n$. That is, at the long run equilibrium, the rate of deflation is equal to the rate of growth of population.

. With a rate of deflation equal to n, the budget line of an individual (which is represented by the equation, $P_t(1-c_{1t})=P_{t+1}.c_{2t}$) coincides with line AB in fig. 10.2 and therefore the representative individual chooses the optimal point E, which maximizes his utility, Thus introduction of money in this overlapping generations model enables the members to reach the optimal point which was earlier unattainable in the barter economy (without money).

While the overlapping generations model highlights the usefulness of money, the model is inadequate as a theory of money. In the OLG model discussed above, money is the only asset which can be used as a store of value. Had there been any other asset (say, physical capital) with a return higher than money, money would have ceased to have any positive value in this framework. A proper theory of money must explain why money continues to be valued despite being dominated in terms of return by other assets. We now turn to other theories of money which address this issue.

Check Your Progress 1

1) In a barter economy introduction of money may enable people to reach the socially optimal outcome – do you agree? Elaborate in the context of a two-period overlapping generations model of exchange.

10.3 CASH-IN-ADVANCE MODEL

The cash-in-advance model emphasizes the role of money as a medium of exchange. The model starts with the assumption that "money buys goods, goods buy money but goods do not buy goods". This assumption is known as the 'Clower' constraint or the 'cash-in-advance' constraint. The term 'cash-in-advance' implies that to buy goods one must accumulate some cash in advance; one cannot directly exchange goods for goods.

In its simplest form the cash-in-advance model specifies that purchase of goods must be paid for with money held at the beginning of the period. The formal presentation of the model is as follows.

Let the utility function of an infinitely lived representative agent be

$$\sum_{t=0}^{\infty} u(c_t^1, c_t^2, c_t^3, \ldots\ldots, c_t^n),$$

where $c^1, c^2, c^3, \ldots\ldots c^n$ represent the consumption of good1, good2, good 3, ..., good n respectively. The agent can keep his savings either in the form of money, which earns zero nominal return, or in the form of bond which yields a positive nominal return denoted by ρ. The agent starts with a certain stock of money and a certain stock of bond. He also has an income Y at each period Hence the period t budget constraint of the agent is given by:

$$\sum_{i=1}^{n} P_t^i c_t^i + M_{t+1} + B_{t+1} = Y_t + M_t + (1+\rho_{t-1})B_t .$$

It is easy to see that since bonds earns positive return while money earns zero return (that is, money is dominated by bond), maximization of the utility function subject to this budget constraint will generate an optimal solution where holding of money will be zero.

Now suppose we add a cash-in advance constraint to this framework. That is, we specify an additional constraint of the form: $\sum_{i=1}^{n} P_t^i c_t^i \leq M_t$. This constraint says that all the purchases must be carried out from the stock of money. One can show that the solution to the Kuhn-Tucker maximization problem with this additional constraint will imply the optimal money holding is no longer zero. In other words, adding a cash-in-advance constraint immediately generates a demand for money.

Check Your Progress 2

1) What is Clower constraint? How does it ensure that money has positive value?

10.4 MONEY IN THE UTILITY FUNCTION

The cash-in-advance model described above highlights the transaction demand for money. There exists a second approach which emphasizes the fact that money might provide some utility on its own. In the latter approach money, or more precisely the amount of real balances, enters directly into individual's utility function. We describe below a model which uses this framework. The specific model that we shall consider here is due to Sidrauski. The model is an extension of the Ramsey model which allows for holding of money.

As is the Ramsey model we assume that at each point of time t the society consists of L_t number of infinitely lived individuals who have identical preferences. The utility function of the representative individual is given by $W = \int_0^{\infty} u(c_t, m_t) \exp^{-\rho t} dt$, where c_t and m, are respectively the per capita consumption and the per capita holding of real balances in period t, $u(c_t)$ is the associated instantaneous utility in period t, and ρ is the subjective discount rate of the agent. The instantaneous utility function is well-behaved and has all the standard properties: $u_c, u_m > 0; u_{cc}, u_{mm} > 0$. The population in this economy grows at a constant exogenous rate n.

People can hold their wealth either in the form of money or in the form of physical capital. At each point of time the government distributes certain amount of new money stock equally among the households. These constitute transfers which in real terms is denoted by X_t. Accordingly the society's budget constraint at period t is given by:

$$C_t + \frac{dK_t}{dt} + \frac{1}{P_t}\frac{dM_t}{dt} = w_t L_t + r_t K_t + X_t.$$

Noting that $\frac{1}{L}\frac{dK}{dt} = \frac{dk}{dt} + nk$ and $\frac{1}{L}\cdot\frac{1}{P}\frac{dM}{dt} = \frac{dm}{dt} + nm + \pi m$, where k is the per capita capital stock and π is the rate of inflation, we can write the budget constraint in per capita terms as:

$$c_t + \frac{dk_t}{dt} + nk_t + \frac{dm_t}{dt} + (n + \pi_t)m_t = w_t + r_t k_t + x_t.$$

Note that per capita household wealth is given by $a = m + k$. Therefore we can write the above budget constraint in terms of per capita wealth as:

$$c_t + \frac{da_t}{dt} = w_t + (r_t - n)a_t + x_t - (\pi_t + r_t)m_t.$$

As in the Ramsey model, the complete dynamic optimization problem for the household can now be written in terms of the three time dependent variables per capita consumption (c_t), per capita real money balances holding (m,) and per capita asset stock (a,):

$$\text{Maximize } W = \int_0^{\infty} u(c_t, m_t) \exp^{-\rho t} \mathrm{dt}$$

subject to $\frac{da}{dt} = w_t + (r_t - n)a_t + x_t - c_t - (\pi_t + r_t)m_t ; a_0$ ***given.*** Also the household treats the wage rate (w_t), the rate of interest (r_t), the rate of inflation (π_t) and the transfer from government (x_t) as exogenously given. Thus for the household now there are two control variables, c_t and m_t , and one state variable a_t. The corresponding Hamiltonian function and the first order conditions in terms of the control, state and co-state variables are given by:[2]

$$H = u(c_t, m_t)\exp^{-\rho t} + \lambda_t\left[w_t + (r_t - n)a_t + x_t - c_t - (\pi_t + r_t)m_t\right]$$

i) $u_c(c_t, m_t)\exp^{-\rho t} = \lambda_t$

ii) $u_m(c_t, m_t)\exp^{-\rho t} = \lambda_t(\pi_t + r_t)$

iii) $\frac{d\lambda}{dt} = -\lambda_t(a_t - n)$

iv) $\frac{da}{dt} = w_t + (r_t - n)a_t + x_t - c_t - (\pi_t + r_t)m_t$

v) $\lim_{t\to\infty} \lambda_t a_t = 0$.

Additionally there is the No-Ponzi-Game condition given by

vi) $\lim_{t\to 0} a_t \exp^{-\int_0^t (r_\tau - n)d\tau} \geq 0$.

Finally note that while the household treats: w_t, r_t, π_t and x_t as exogenous to its decision making process, it nonetheless has perfect foresight so that it can exactly ***guess*** the correct values of these variables at every point of time. Accordingly in equations (i)-(vi) we can put $w_t = f(k_t) - k_t f'(k_t)$ and $r_t = f'(k_t)$. Also as we have mentioned before, the new money is distributed as government transfer (in real terms) to the households, so that $x_t = \frac{1}{L_t}\left(\frac{1}{P_t}\frac{dM}{dt}\right)$. We can write this latter term as

$x_t = \frac{1}{L_t}\frac{M_t}{P_t}\left(\frac{1}{M}\frac{dM}{dt}\right) \equiv m_t\sigma$, where σ is the rate of growth of nominal money stock.

Let us now look at the steady state of this economy. At steady state $\frac{da}{dt} = \frac{dm}{dt} = \frac{d\lambda}{dt} = 0$. The condition $\frac{dm}{dt} = 0$ implies $\pi = \sigma - n$. Putting these and the other steady state conditions in equations (i)-(iv), we get the steady state per capita capital stock and the steady state per capita consumption as: $f'(k^*) = \rho + n$; and $c^* = f(k^*) - nk^*$. Comparing *these* values with the corresponding steady state

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[2] **To know the mathematical technique go back to the Mathematical Appendix of Unit 9 where the infinite horizon Ramsey model has been discussed.**

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subject to $\frac{da}{dt} = w_t + (r_t - n)a_t + x_t - c_t - (\pi_t + r_t)m_t; a_0$ given. Also the household treats the wage rate (w_t), the rate of interest (r_t), the rate of inflation (π_t) and the transfer from government (x_t) as exogenously given. Thus for the household now there are two control variables, c_t and m_t, and one state variable a_t. The corresponding Hamiltonian function and the first order conditions in terms of the control, state and co-state variables are given by:[2]

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iii) $\frac{d\lambda}{dt} = -\lambda_t(a_t - n)$

iv) $\frac{da}{dt} = w_t + (r_t - n)a_t + x_t - c_t - (\pi_t + r_t)m_t$

v) $\lim_{t\to\infty} \lambda_t a_t = 0$.

Additionally there is the No-Ponzi-Game condition given by

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$x_t = \frac{1}{L_t}\frac{M_t}{P_t}\left(\frac{1}{M}\frac{dM}{dt}\right) \equiv m_t\sigma$, where σ is the rate of growth of nominal money **stock.**

Let us now look at the steady state of this economy. At steady state $\frac{da}{dt} = \frac{dm}{dt} = \frac{d\lambda}{dt} = 0$. The condition $\frac{dm}{dt} = 0$ implies $\pi = \sigma - n$. Putting these and the other steady state conditions in equations (i)-(iv), we get the steady state per capita capital stock and the steady state per capita consumption as: $f'(k^*) = \rho + n$; and $c^* = f(k^*) - nk^*$. Comparing these values with the corresponding steady state

2 To know the mathematical technique go back to the Mathematical Appendix of Unit 9 where the infinite horizon Ramsey model has been discussed.

values of the original Ramsey model without money (described in unit 9), we find that they are exactly identical, Thus introduction of money to the Ramsey model does not affect the real equilibrium.

10.5 POLICY IMPLICATIONS

We have so far discussed three different dynamic frameworks with money. Note that the stock of money often constitutes an important policy variable in the hand of the government. By varying the stock or the rate of growth of money the government may influence the real decisions of the agents. Let us now examine the effectiveness of monetary policy in these different frameworks.

Let us first consider the OLG framework. If the government increases the stock of money in each period at a constant rate and distributes it as lumpsum transfer to the old generation, then that increases the second period endowment of the old. An increase in the second period endowment will unambiguously decrease savings which in turn will affect the price level and therefore the rate of inflation. Note that the negative of the rate of inflation (i.e., the rate of deflation) is the real return on money holding. Hence a change in the inflation rate implies a change in the real return to money, which will affect the optimal decision about the other real variables as well. Thus in the OLG framework money is not neutral.

In the Cash-in-advance model also increasing the supply of money eases the Clower constraint and enables the households to buy more goods. Thus as long as the Clower constraint is binding, money affects the real variables.

In the money-in-the-utility-function approach, on the other hand, the steady state values of the real variables are unaffected by the introduction of money. To be more precise, the ***rate of growth of money*** stock does not affect the real equilibrium. This result is known as the 'supemeutrality' of money. Thus monetary policy has no role to play in this model.

Check Your Progress 3

1) If one introduces money in the Ramsey model how does the long run equilibrium values of the real variables change?

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2) Can change in the money supply have any impact on the real variables? Discuss your answer in the context of the different model of money that you have **studied** here.

10.6 LET US SUM UP

In this unit, we have looked at the role of money in an economy. First we have introduced money in the standard overlapping generations framework of exchange. We have seen that in this framework money allows the economy to reach its optimal point. We also discuss the role of money in the presence of Clower constraint when all transactions must be carried out in terms of money. We also discuss the role of money in a model where people derive direct utility from money. This latter model is an extension of the Ramsey model. We find that introduction of money has no impact on the real variables in this model.

10.7 KEYWORDS

Clower Constraint : A situation where money is the only medium of exchange so that any transaction requires money.

Super-neutrality of Money : A situation where a change in the rate of growth of money stock does not have any impact on the equilibrium values of the real variables in the economy.

10.8 SOME USEFUL BOOKS

Blanchard, Olivier and Stanley Fischer, 1989, *Lectures on Macroeconomics, MIT* Press, chapter 2.

10.9 ANSWER/HINTS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress 1

1) Read Section 10.2 and write your answer.

Check Your Progress 2

1) Read Section 10.3 and write your answer.

Check Your Progress 3

1) Read Section 10.4 and write your answer.

2) Read Section 10.5 and write your answer.