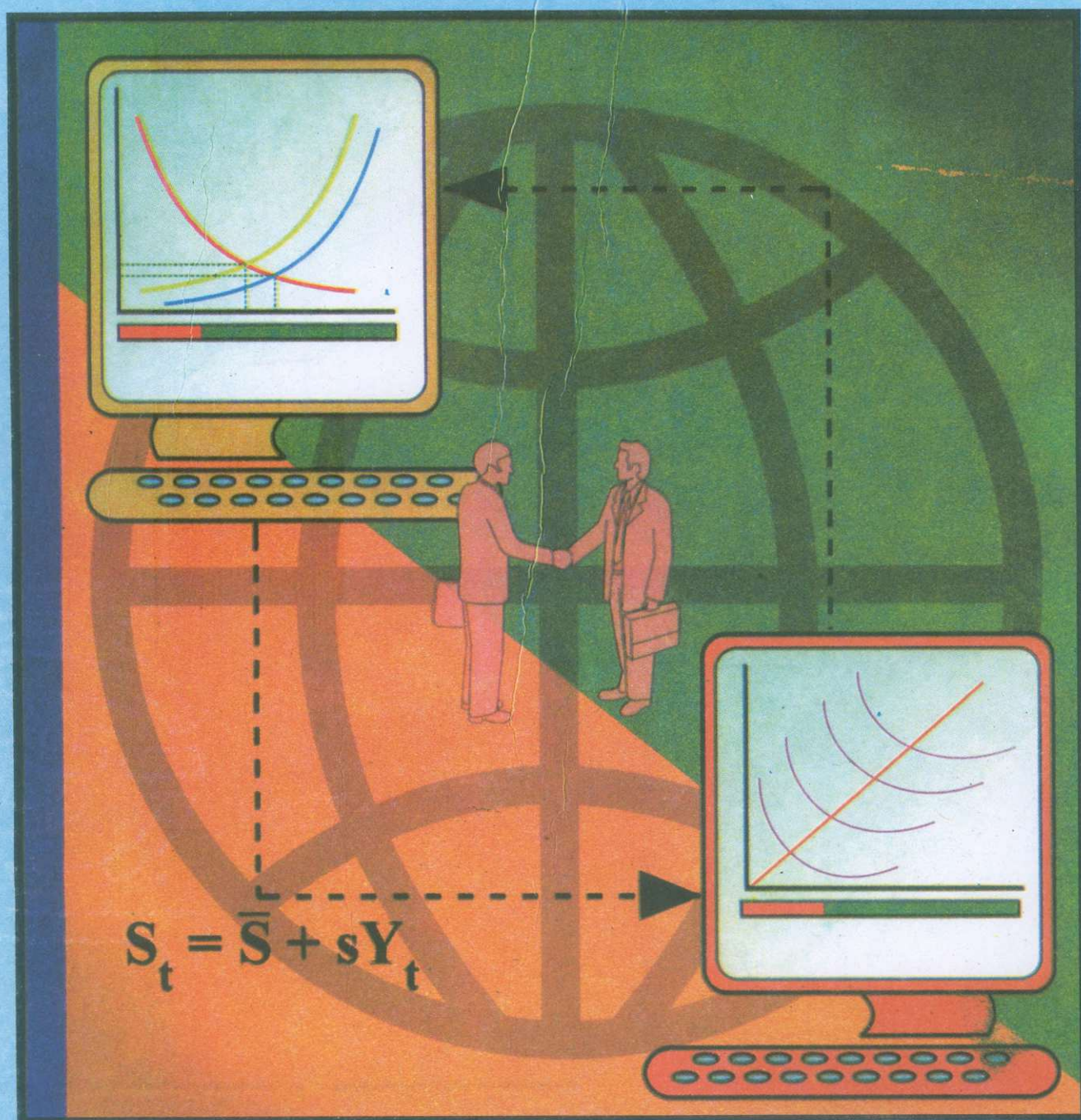




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Indira Gandhi National Open University
School of Social Sciences

MEC - 101
MICROECONOMIC
ANALYSIS



Producer Behaviour

2

“शिक्षा मानव को बन्धनों से मुक्त करती है और आज के युग में तो यह लोकतंत्रा की भावना का आधार भी है। जन्म तथा अन्य कारणों से उत्पन्न जाति एवं वर्गगत विषमताओं को दूर करते हुए मनुष्य को इन सबसे ऊपर उठाती है।”

- इन्दिरा गाँधी

“Education is a liberating force, and in our age it is also a democratising force, cutting across the barriers of caste and class, smoothing out inequalities imposed by birth and other circumstances.”

- Indira Gandhi

Block

2

PRODUCER BEHAVIOUR

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BLOCK 2 PRODUCER BEHAVIOUR

Introduction

The underlying theory behind the production is discussed in this block. It starts with the production function as a technical relation between input and output (**Unit 4**) and brings in the features of short and long periods, which have a bearing the production process. The choice available to a producer in the use of factors of production in the producing of a specified level of output without violating the technical requirements is deliberated upon. In the process, the returns to a factor and the possible impact of simultaneous change in factor inputs have been examined. Different categories of costs to be incurred by a producer over short and long periods have been taken up in **Unit 5**. The traditional thinking on the features of cost in production have been discusses and the requirements introduced through modern theories have been contrasted. The last unit of the block, (**Unit 6**), takes up empirical estimation of production and cost functions along with the producers' decision problems when the objectives of profit maximisation and cost minimisation are incorporated. Moreover, it discusses the basic techniques available for measuring technical progress, besides the optimisation problem in the process of production.

UNIT 4 THEORY OF PRODUCTION

Structure

- 4.0 Objectives
- 4.1 Introduction
- 4.2 Short Period Analysis
 - 4.2.1 Returns to a Factor
- 4.3 Long Period Analysis
 - 4.3.1 Iso-quant
 - 4.3.2 Elasticity of Substitution
- 4.4 Returns to Scale
- 4.5 Homogeneous Production Function
- 4.6 Let Us Sum Up
- 4.7 Key Words
- 4.8 Some Useful Books
- 4.9 Answer or Hints to Check Your Progress

4.0 OBJECTIVES

After studying this unit, you will be able to:

- appreciate the relationship between input and output in production;
- measure productivities of inputs;
- analyse laws of production with single input variation and multiple inputs variation;
- assess of production in short-and long-run; and
- measure to degree of substitution between factors of production.

4.1 INTRODUCTION

Production activities related to goods and services require inputs. Typically, the set of inputs includes labour, capital equipments and raw materials. The producing unit usually has to solve the choice problem as a given amount of output can be produced from various combinations of inputs. Firms, therefore, look for production possibilities that are technologically feasible. A production function describes the relation between input and output with a given the technology. More formally, it shows the maximum amount of output that can be produced from any specified set of inputs, given the existing technology. If we assume that there are only two factors of production – labour (L), capital (K) and a single output q , mathematically a production function can be written as, $q = F(L, K)$

4.2 SHORT PERIOD ANALYSIS

Short period in production refers to a time when some inputs remain fixed. A fixed input is one, whose quantity cannot be changed readily, whereas, a variable input varies with production. Inputs like land, building and major pieces of machinery cannot be varied easily and, therefore, can be called fixed inputs. On the other hand, inputs like labour (labour hours), raw materials, and processed materials can be easily increased or decreased. Therefore, these are categorised as variable inputs. Depending on whether inputs can be kept fixed or not, we have a short period or a long period. To put this more precisely, if inputs being used in the production process have just enough time such that they can not be varied, then the analysis pertains to the short-run. On the other hand, if the inputs employed have enough time such that they are amenable to variation, then the analysis is based on the frame work of long-run.

Generally, the firms do not readily change their capital, which could be land, machinery, managerial and technical personnel. Therefore, these are fixed input in the short-run. When we treat these under $\bar{K}$, the production function can be written as

$$q = F(L, \bar{K})$$

where L = Labour, a variable factor

$\bar{K}$ = Capital, a fixed factor

Marginal Product (MP) of a Factor

From the above mentioned production function, immediately we can study the effect on total output when there is a variation in labour utilisation, keeping the other factor $\bar{K}$, fixed. Thus, we have the marginal physical product, which shows the change in output quantity for a unit change in the quantity of an input, (L), when all other inputs (K) are held constant. Mathematically, it is given by the first partial derivative of a production function with respect to labour. Thus,

$$q = F(L, \bar{K}) = \text{Total Product (TP).}$$

$$\therefore \text{MP of Labour} = MP_L = \frac{\partial q}{\partial L} = f_L$$

$$\text{If } L = \bar{L}, \text{ then marginal product of capital} = MP_K = \frac{\partial q}{\partial K} = f_K$$

It is reasonable to expect that the marginal product of an input depends on the quantity used of that input. In the above example, use of labour is made keeping the amount of other factors (such as equipments and land) fixed. Continued use of labour would eventually exhibit deterioration in its productivity. Thus, the relationship between labour input and total output can be recorded to show the declining marginal physical productivity. Mathematically, the diminishing marginal physical productivity is assessed through the second-order partial derivative of the production function.

Thus, change in labour productivity can be presented as:

$$\frac{\partial MP_L}{\partial L} = \frac{\partial^2 q}{\partial L^2} = f_{LL} < 0$$

Similarly, change in productivity of capital is denoted by

$$\frac{\partial MP_k}{\partial K} = \frac{\partial^2 q}{\partial K^2} = f_{kk} < 0$$

Average Product (AP) of a Factor

The productivity of a factor is often seen in terms of its average contribution. Although not very important in the theoretical discussions, where analytical insight is tried to be drawn from marginal productivities, average productivity finds a platform in empirical evaluations. Deriving it from the total product is relatively easy. It is the output per unit of a factor.

So, AP of labour = $AP_L = \frac{TP}{L} = \frac{q}{L}$. Similarly,

$$AP \text{ of capital} = AP_K = \frac{TP}{K} = \frac{q}{K}.$$

Relation between TP and MP

Graphically, given the total product curve, MP is the slope of the tangent at any point on the TP curve. This is shown in Figure 4.1.

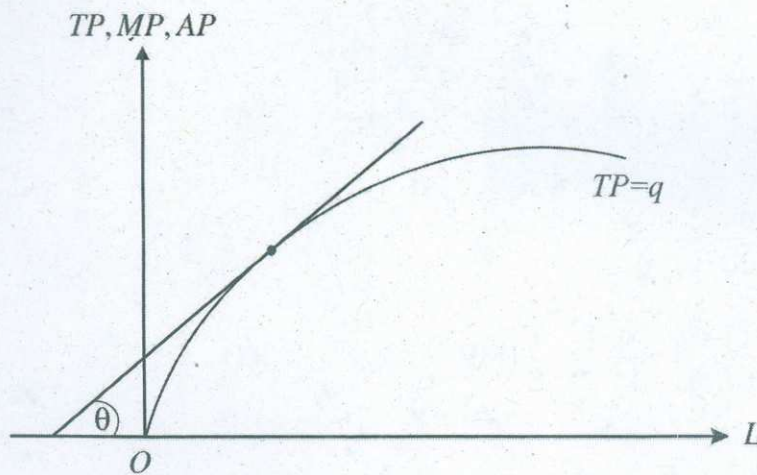


Fig. 4.1: Total Product Curve and Marginal Product

$$\text{See that } MP_L = \tan \theta = \frac{\partial TP}{\partial L}$$

Relation between TP and AP

Given the TP curve, AP is the slope of a ray from the origin to any point on the TP curve.

$$AP_L = \tan \alpha = \frac{QL}{OL} = \frac{q}{L}$$

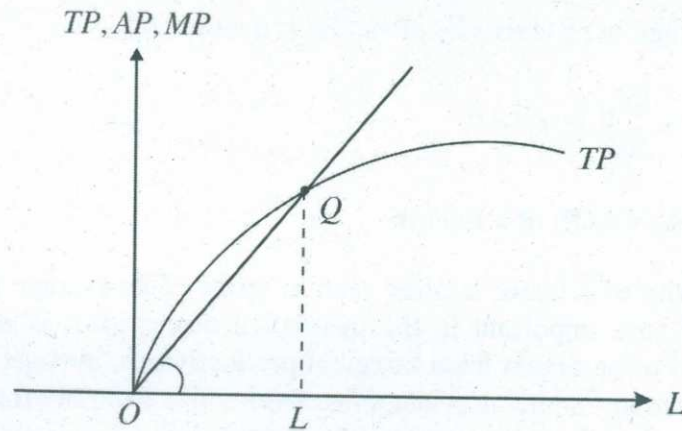


Fig. 4.2: Derivation of AP from TP Curve

Relation between AP and MP

This relation between AP and MP is true for all “average” and “marginal” productivity conditions. When AP is rising, $MP > AP$; when AP is maximum, $MP = AP$ and when AP is falling, $MP < AP$.

Proof:

Since $AP_L = \frac{q}{L}$, differentiating AP_L partially with respect to L , we get,

$$\frac{\partial}{\partial L}(AP_L) = \frac{\partial}{\partial L}\left(\frac{q}{L}\right) = \frac{L \frac{\partial q}{\partial L} - q \frac{\partial L}{\partial L}}{L^2} = \frac{L \frac{\partial q}{\partial L} - q}{L^2}$$

$$\text{or, } \frac{\partial}{\partial L}(AP_L) = \frac{\partial q}{\partial L} \cdot \frac{1}{L} - \frac{q}{L^2}$$

$$\text{or, } \frac{\partial}{\partial L}(AP_L) = \frac{1}{L} \left(\frac{\partial q}{\partial L} - \frac{q}{L} \right) = \frac{1}{L} (MP_L - AP_L) \quad \dots(1)$$

From equation 1,

$$\frac{\partial}{\partial L}(AP_L) > 0 \text{ when } MP_L > AP_L \quad (\text{since } L > 0);$$

$$\frac{\partial}{\partial L}(AP_L) = 0 \text{ when } MP_L = AP_L;$$

$$\frac{\partial}{\partial L}(AP_L) < 0 \text{ when } MP_L < AP_L \quad (\text{since } L > 0)$$

It is an empirically observed feature that all inputs have positive but diminishing marginal products. If L and K are the only factors of production, then $F_L > 0$, $F_K > 0$, $F_{LL} < 0$, $F_{KK} < 0$.

Thus, for any factor, initially, MP is positive. Then a situation arises when MP is zero, and further on it falls to negative as more of the factor is employed in production.

Check Your Progress 1

1) What is a production function?

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2) In short period why you cannot increase the managerial inputs?

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3) Define marginal productivity of labour

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4.2.1 Returns to a Factor

This is a short-run concept which deals with the variability of only one factor keeping the others constant. There are three kinds of returns:

- i) Increasing returns: when the AP of a factor rises and $MP > AP$
- ii) Constant returns: when AP is constant and $MP = AP$
- iii) Diminishing returns: when AP is falling $MP < AP$

The concept of returns to a factor can also be expressed in terms of the “partial input elasticity of output”.

Partial input elasticity of output

Partial input elasticity of output is also called elasticity of output with respect to a factor. It is the percentage change in output quantity for one per cent change in the quantity of a factor when all other factors remain constant.

Elasticity of q with respect to L is given by,

$$E_{q,L} = \frac{\frac{\partial q}{\partial L} \cdot L}{q} = \frac{\frac{\partial q}{\partial L} \cdot L}{q} = \frac{MP_L}{AP_L}$$

$$\text{Similarly, } E_{q,K} = \frac{\frac{\partial q}{\partial K} \cdot K}{q} = \frac{MP_K}{AP_K}$$

Let us now relate this to returns to scale.

$$\text{Since } AP_L = \frac{q}{L},$$

$$\begin{aligned} \therefore \frac{\partial}{\partial L}(AP_L) &= \frac{\partial}{\partial L}\left(\frac{q}{L}\right) = \frac{L \frac{\partial q}{\partial L} - q \cdot \frac{\partial L}{\partial L}}{L^2} = \frac{L \frac{\partial q}{\partial L} - q}{L^2} \\ &= \frac{1}{L} \frac{\partial q}{\partial L} - \frac{q}{L^2} \\ &= \frac{q}{L^2} \left(\frac{\partial q}{\partial L} \cdot \frac{L}{q} - 1 \right) = \frac{q}{L^2} (E_{q,L} - 1) \quad \dots(2) \end{aligned}$$

$$\text{As } \frac{q}{L^2} > 0 \text{ from equation (2),}$$

$$\text{increasing returns} = \frac{\partial}{\partial L}(AP_L) > 0 = E_{q,L} > 1$$

$$\text{constant returns} = \frac{\partial}{\partial L}(AP_L) = 0 = E_{q,L} = 1$$

$$\text{diminishing returns} = \frac{\partial}{\partial L}(AP_L) < 0 = E_{q,L} < 1$$

Besides the above mentioned three returns, there can be another type known as the 'non-proportional returns to a variable factor'. Under it, initially, there is increasing returns to a factor up to a certain level beyond which there is diminishing returns.

Graphical Representation of Various Returns

Diminishing Returns: If the TP curve is as shown in the adjacent Figure 4.3, then the MP_L given by $\tan \theta$ is throughout less than the AP_L given by $\tan \alpha$.

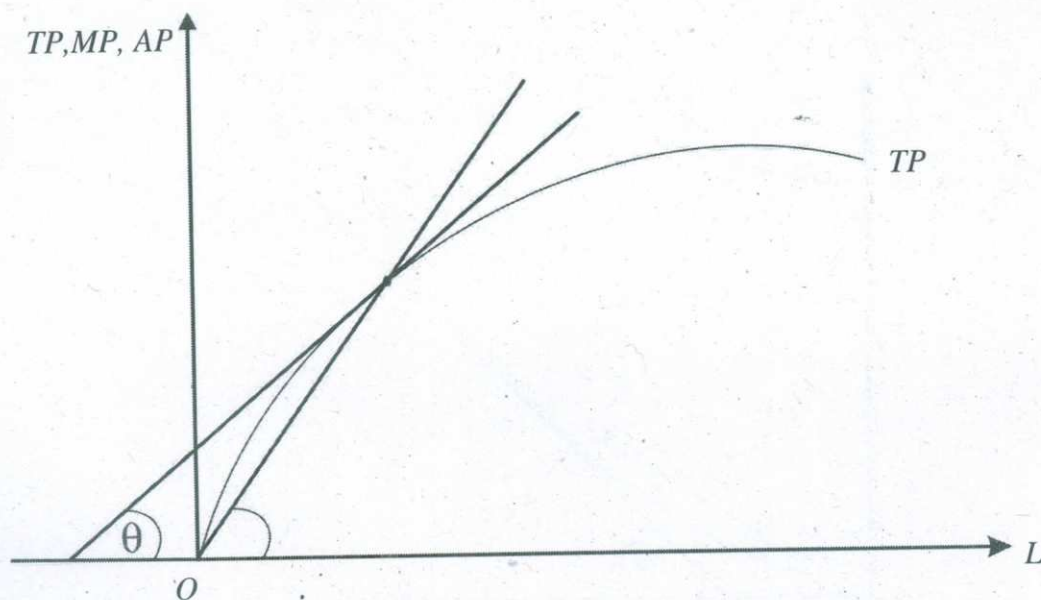
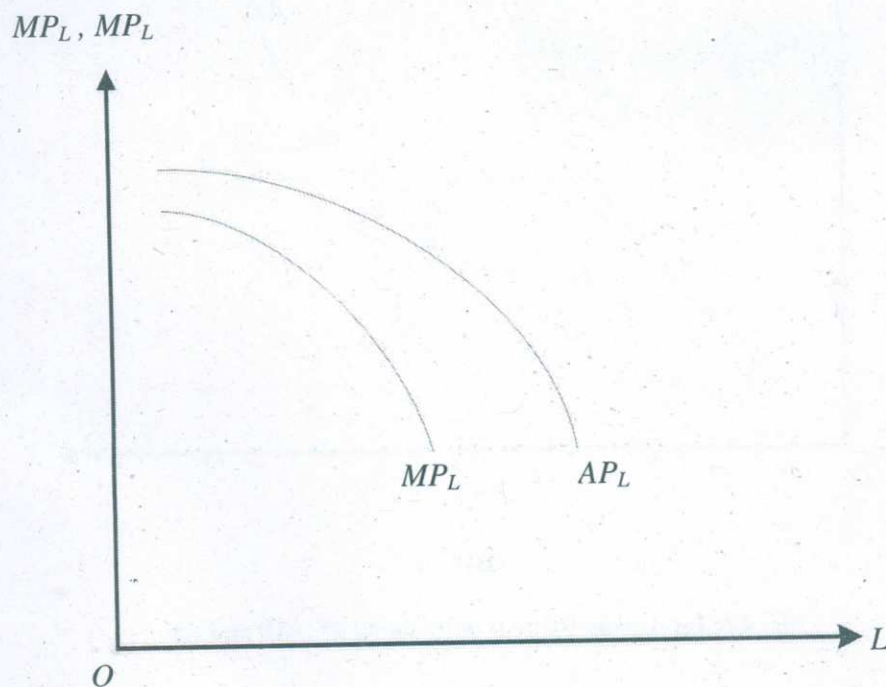


Fig. 4.3: Diminishing MP and AP

Increasing Returns

Here AP_L rises and $\tan \alpha < \tan \theta$, for all L . Therefore, $MP > AP$. This is shown in the adjacent Figure 4.5.

Fig. 4.4: Derivation of MP_L from TP

As AP_L is falling from the relation between MP and AP, $MP < AP$ we have the adjoining Figure 4. 4.

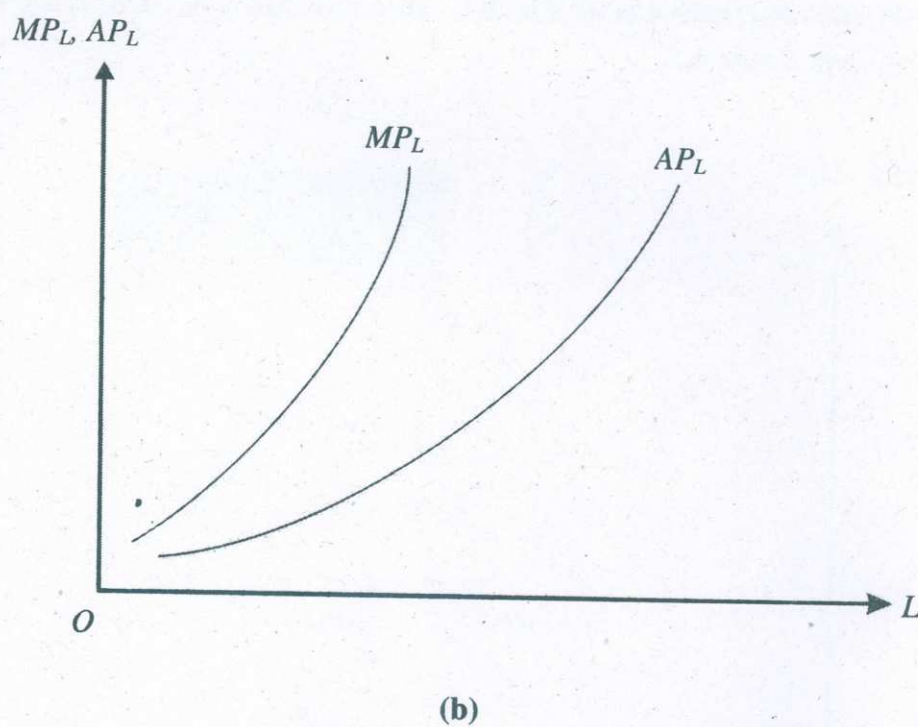
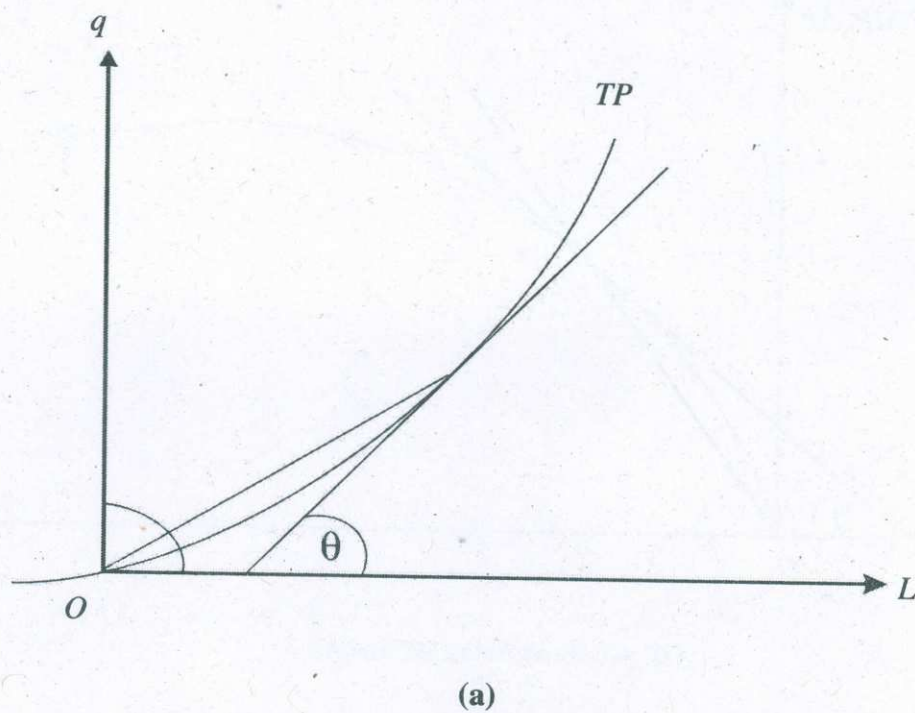
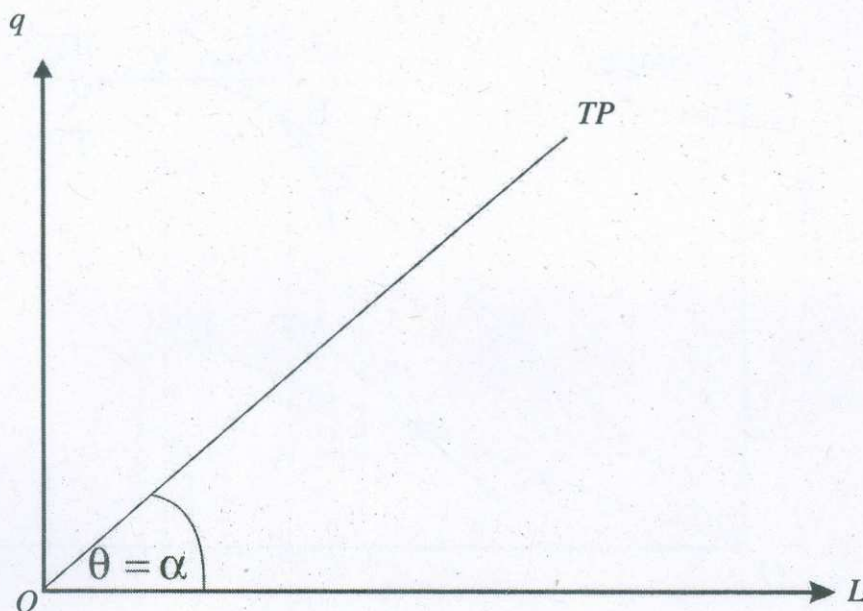


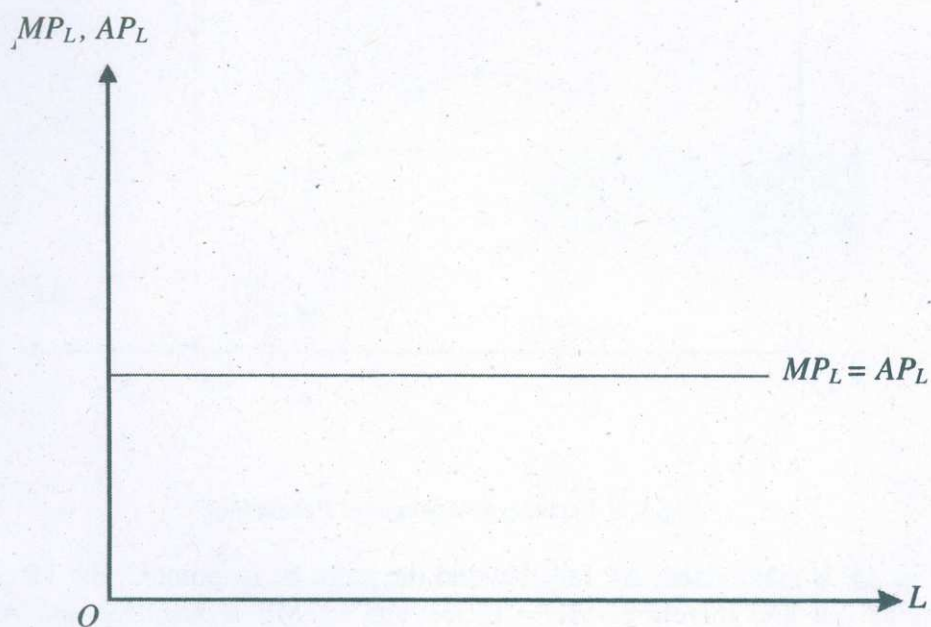
Fig. 4.5: Increasing Returns as given by TP, MP and AP

Constant Returns

Here, AP_L is constant and $\tan \theta = \tan \alpha$, therefore, $MP_L = AP_L$ as is shown by a horizontal straight line in the next Figure 4.6(a, b)



(a)



(b)

Fig. 4.6: Constant Returns as given by TP, MP and AP

Non-Proportional Returns

The TP curve is such that upto point A, MP is rising and so is AP and $MP > AP$, as shown in the diagram below. Beyond point A, MP falls but AP rises, till the two are equated at point B.

In stage I, by adding one more unit of L, the producer can increase the average productivity of all the units. Thus, it would be unwise for the producer to stop production in this stage.

In stage III, $MP < 0$, so that by reducing the L input, the producer can increase the total output and save the cost of a unit of L. Therefore, it is impractical for a producer to produce in this stage.

Hence, stage II represents the economically meaningful range. This is so because here $MP > 0$ and $AP > MP$. So that an additional L input would raise the total production. Besides, it is in this stage that the TP reaches a maximum.

Check Your Progress 2

- 1) Why marginal productivity of labour declines?
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- 2) The producer must choose the second stage from the total production curve. Why?
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- 3) What is partial input elasticity of output?
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4.3 LONG PERIOD ANALYSIS

Long period refers to a time when all the factors are variable. Earlier in the short period analysis, we had considered capital (K) to be fixed factor. Here in this part, we assume both L and K to be variable factors. Therefore, the production function would be:

$$q = F(L, K)$$

The producer can now employ L and K units at her will to produce output q as per the production technology. Therefore, in the K – L input space the producer can choose any combination of K and L to produce output.

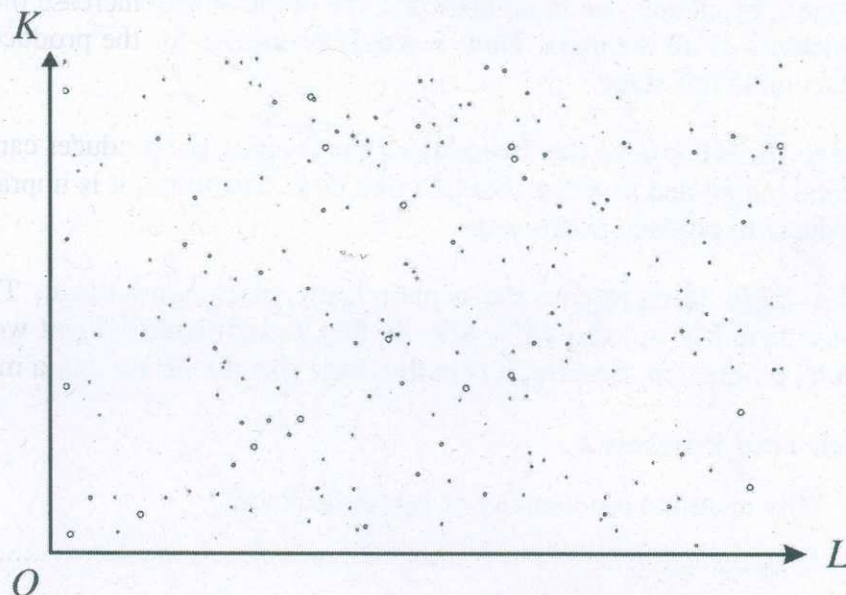


Fig. 4.8: K-L Input Space Available to Producer

4.3.1 Iso-quant

The dots in the above Figure 4.8 denotes the various combinations of (L, K) that the producer can pick up from form to produce. Among these combinations, there can be those, which produce the same level of output. Herein comes the concept of an iso-quant. An iso-quant is a locus of combinations of (L, K) that produce the same level of output 'q' (see Figure 4.9).

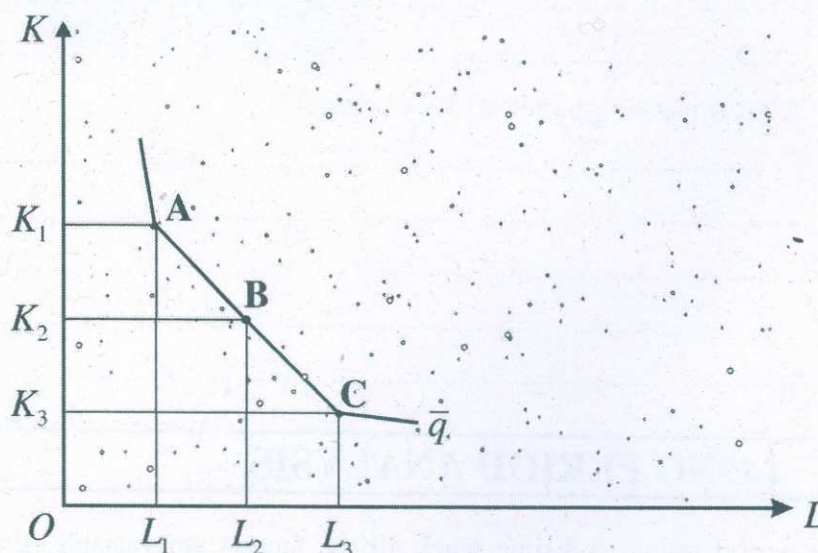


Fig. 4.9: Shape of Iso-quant

In the above figure 4.9, $\bar{q}$ amount of output can be produced by the input combinations — (L_1, K_1) , (L_2, K_2) , (L_3, K_3) . Joining these, we get an iso-quant, which is also denoted by $\bar{q}$. Therefore, we see that the same level of output $\bar{q}$ can be produced using different techniques — either more K, less L (e.g., technique A), or more L and less K (e.g., technique C).

Slope of an iso-quant

Since along an iso-quant the level of output remains the same, if ΔL units of L are substituted for ΔK units of K , the increase in output due to ΔL units L (namely, $\Delta L \cdot MP_L$) should match the decrease in output due to a decrease of ΔK units of K (namely $\Delta K \cdot MP_K$). In other words,

$$|\Delta L \cdot MP_L| = |\Delta K \cdot MP_K|$$

$$\Rightarrow \left| \frac{\Delta K}{\Delta L} \right| = \frac{MP_L}{MP_K}$$

$$\text{or, } \frac{\Delta K}{\Delta L} = -\frac{MP_L}{MP_K}$$

when Δ is a very small amount we can write $\frac{\Delta K}{\Delta L}$ as $\frac{dK}{dL}$. In the $K - L$ input space, $\frac{dK}{dL}$ represents the slope of the iso-quant at any point on it.

$$\therefore \text{Slope of the iso-quant} = \frac{dK}{dL} = -\frac{MP_L}{MP_K}$$

Check Your Progress 3

- 1) Define an iso-quant.

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- 2) Derive the slope of an iso-quant mathematically, if the production function is $q = F(L, K)$.

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The absolute value of $\frac{dK}{dL}$, denoted by $\left| \frac{dK}{dL} \right|$ is known as the marginal rate of technical substitution of L for K. ($MRTS_{LK}$). By definition, it measures the reduction in one input per unit increase in the other that is just sufficient to maintain a constant level of output. It is equal to the ratio of the marginal product of L to the marginal product of K.

Curvature of the iso-quant

An iso-quant is convex to the origin. This is so because as more and more units labour are employed, the producer would prefer to give up less and less of the other input to produce the same amount of output. This is shown in the following Figure 4.10:

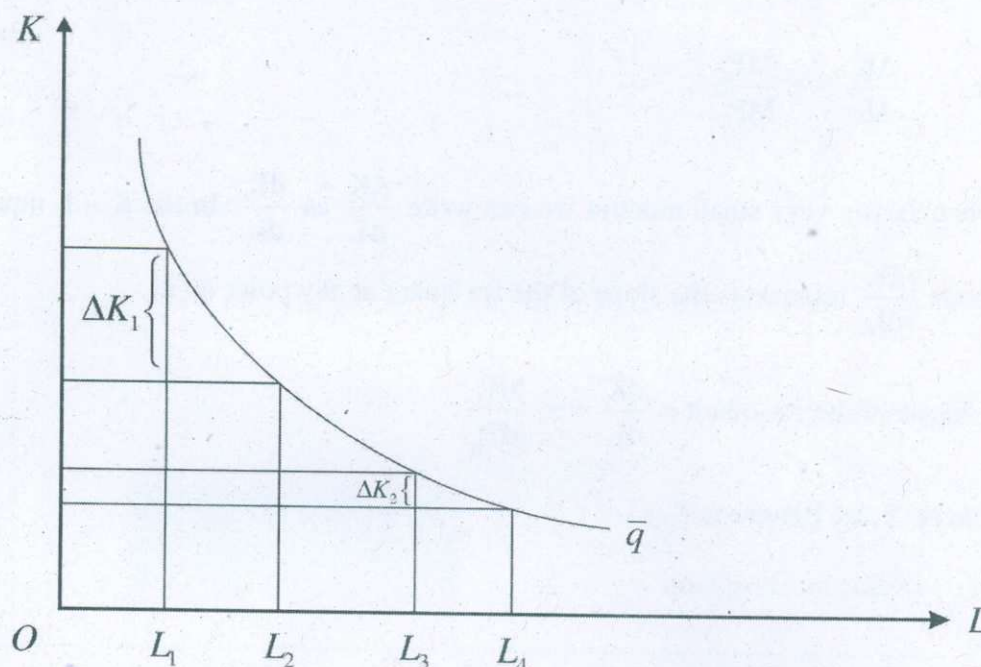


Fig. 4.10: Curvature of an Iso-quant

For a rise in L from L_1 to L_2 , the producer gives up ΔK_1 amount of K. For the same output level $\bar{q}$, as L increases from L_3 to L_4 , she gives up ΔK_2 amount of K. As $\Delta K_2 < \Delta K_1$, it implies that for more units of L, the producer is willing to give up less of K.

A convex iso-quant implies a diminishing $MRTS_{LK}$. As $MRTS_{LK} = \frac{MP_L}{MP_K}$, a diminishing MRTS means as L increases, MP_L decreases and MP_K increases.

Economic Region of Iso-quant

With help of production function, we generate iso-quant maps as shown in the following Figure 4.11. The only difference these have with the previous iso-quant is that presently we have positively sloped segments.

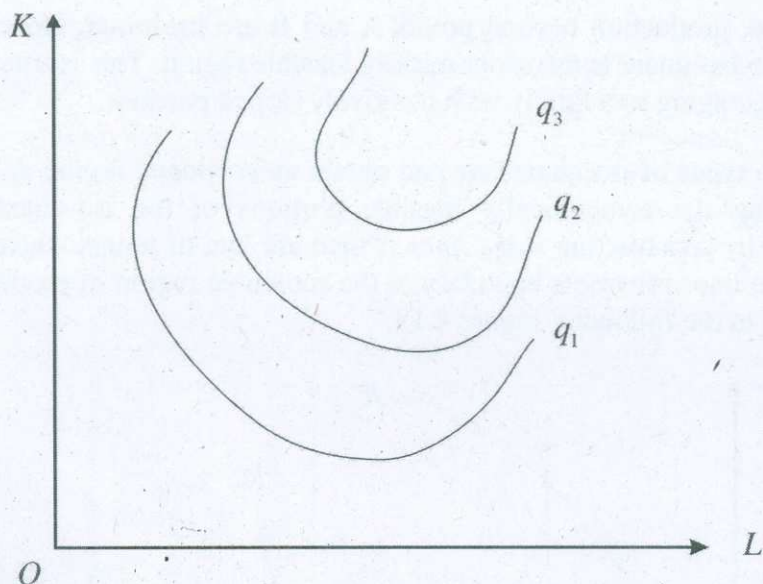


Fig. 4.11: Economic Region of Production and Iso-quants

Let us examine the characteristic of one such iso-quant with the help of the following Figure 4.12.

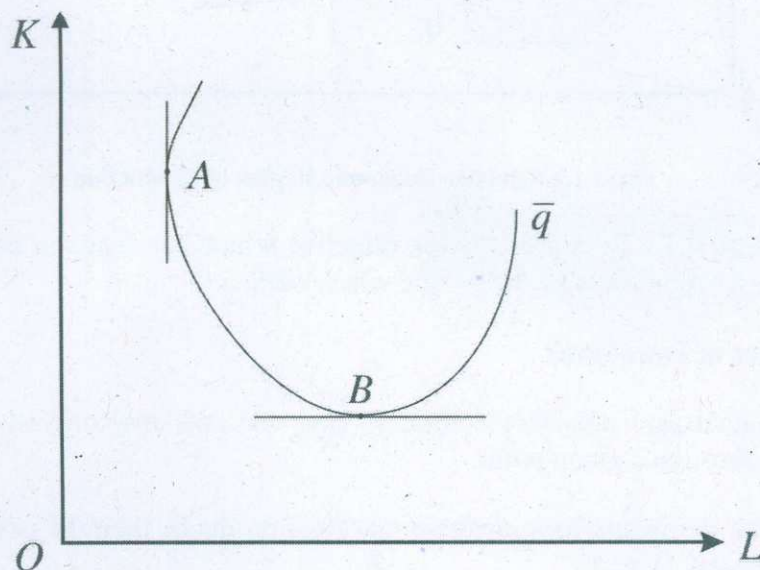


Fig. 4.12: Economic Region of Production

We know that the slope of an iso-quant is given by, $\frac{dK}{dL} = \frac{MP_L}{MP_K}$.

From the figure 4.12, at point A, the slope is infinite or undefined. This implies that at A, $MP_K = 0$. Beyond A, $\frac{dK}{dL} > 0$, implying $MP_K < 0$. As we had seen in the short-run analysis, no producer has an incentive to undertake production at this portion (this zone is similar to stage III of short run analysis).

At point B, $\frac{dK}{dL} = 0$, implying $MP_L = 0$. Beyond point B, MP_L falls. By similar logic, no producer would want to employ L beyond point B.

Therefore, production beyond points A and B are irrelevant. Hence, segment AB of the iso-quant is the economically feasible region. This is true for all iso-quant belonging to a family with positively sloped portions.

For these types of iso-quant we can obtain an *economic region of production* comprising the economically feasible portions of the iso-quant. This is obtained by constructing *ridge lines*, which are loci of points where $MP_K = 0$. The ridge lines provide a boundary to the economic region of production. This is shown in the following Figure 4.13.

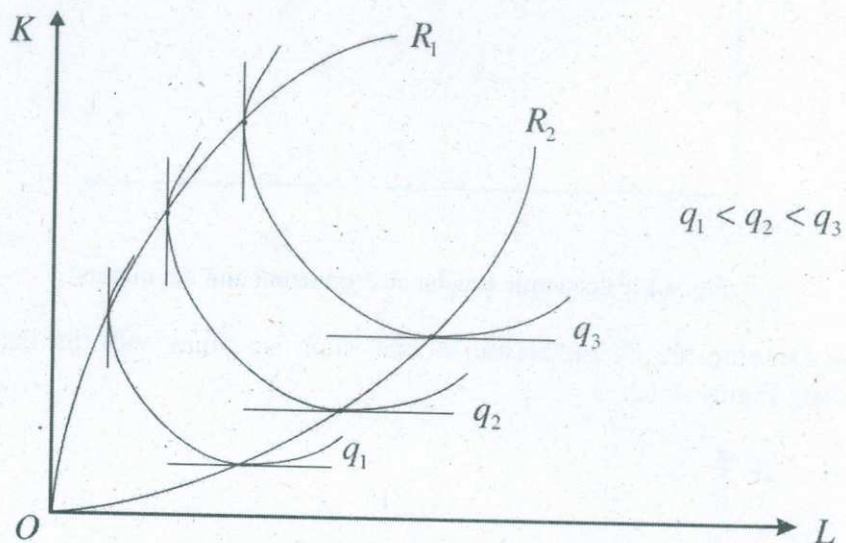


Fig. 4.13: Selection Economic Region of Production

In the figure 4.13, R_1 represents the ridgeline which is a locus of points where $MP_K = 0$. R_2 represents the ridge line where $MP_L = 0$.

Properties of Iso-quant

- 1) Iso-quant are non-intersecting, so that one and only one iso-quant will pass through a given point.
- 2) If both the inputs have positive marginal products, then the iso-quant are negatively sloped.
- 3) Iso-quant are convex to the origin.
- 4) An iso-quant lying away from the origin represents higher quantity of output.

Types of Iso-quant

Besides the smooth convex iso-quant assuming continuous substitutability, we get other types as follows:

- 1) **Linear:** This type of iso-quant assumes perfect substitutability between the factors of production. A given output quantity may be produced by using either only L or only K or by a combination of both (see Figure 4.14)

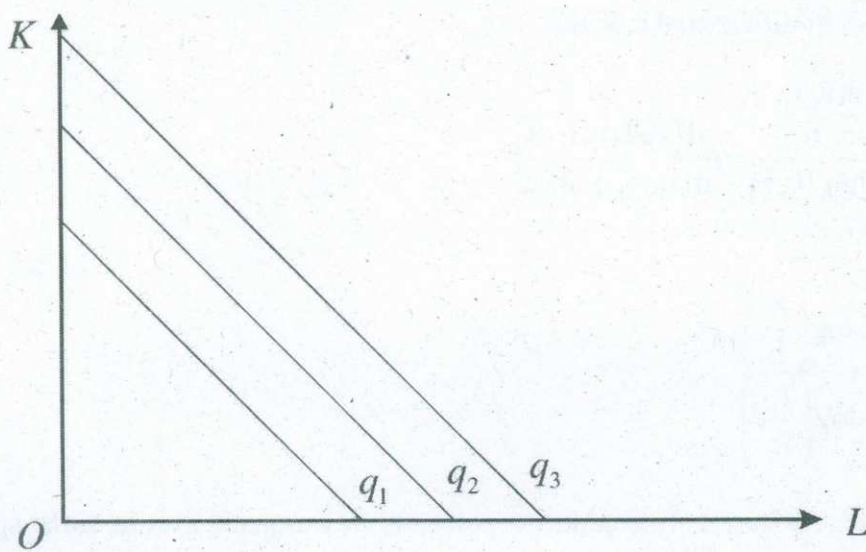


Fig. 4.14: Linear Iso-quant

- 2) **L-shaped:** Assuming strict complementarity between the inputs i.e., no substitutability of factors of production, we get L-shaped iso-quant (see Figure 4.15). It implies there is only one method of production.

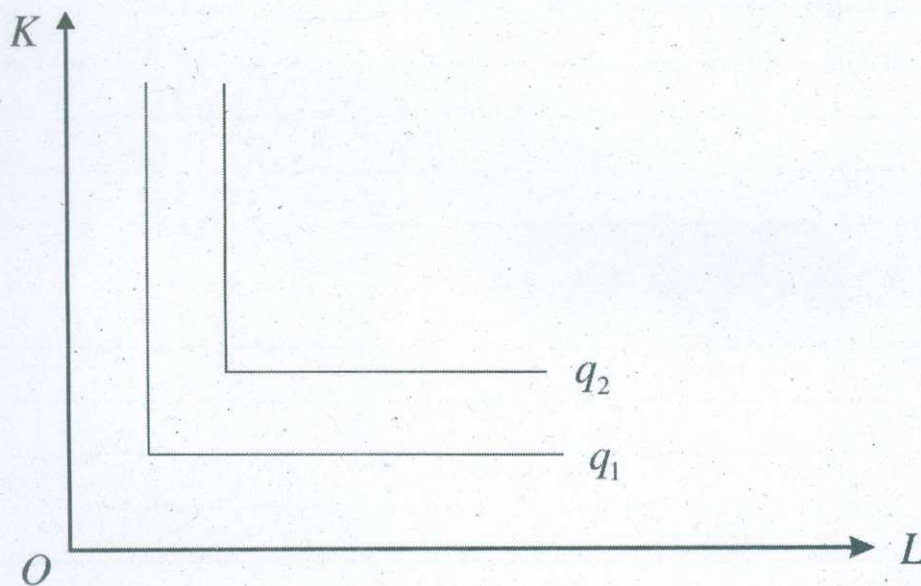


Fig. 4.15: L-shaped Iso-quants

4.3.2 Elasticity of Substitution

MRTS has a drawback as a measure of the degree of substitutability because it depends on the units of measurement of factors. Therefore, we need to consider the concept of elasticity.

The elasticity of substitution denoted by σ , is a pure number that measures the rate at which substitution takes place. It is defined as the percentage change in $K - L$ ratio for one percentage change in the $MP_L - MP_K$ ratio (or $MRTS_{LK}$).

$$\text{Thus, } \sigma = \frac{\% \text{ change in } K/L}{1\% \text{ change in MRTS}}$$

In terms of differential calculus,

$$\sigma = \frac{\frac{d(K/L)}{(K/L)}}{\frac{d(f_L/f_K)}{(f_L/f_K)}} = \frac{d(K/L)}{d(f_L/f_K)} \cdot \frac{f_L/f_K}{K/L}$$

$$\sigma = \frac{d \log \left(\frac{K}{L} \right)}{d \log \left(\frac{f_L}{f_K} \right)}$$

Generally, σ varies from point to point on an iso-quant. But in some special cases $\sigma = 0$ or $\sigma \rightarrow \infty$.

Check Your Progress 4

- 1) What is MRTS ?

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- 2) What is elasticity of substitution?

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- 3) Find for which iso-quant $\sigma = 0$ and for which $\sigma \rightarrow \infty$.

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- 4) What do you mean by returns to scale?

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- 5) What is economic region of production?

4.4 RETURNS TO SCALE

It is a typical long-run concept and involves the effect of change in inputs on the quantity of output produced. There are three types of returns to scale.

For 1% change in all the factors, if correspondingly, output changes by

- 1%, then there is said to be Constant Returns to Scale (CRS)
- Less than 1%, then there is decreasing Returns to Scale (DRS)
- More than 1%, then there is Increasing Returns to Scale (IRS).

Diagrammatic Representation

Constant Returns to Scale

As there is CRS, along a ray through the origin, the distance between the consecutive iso-quants remains the same (see Figure 4.16).

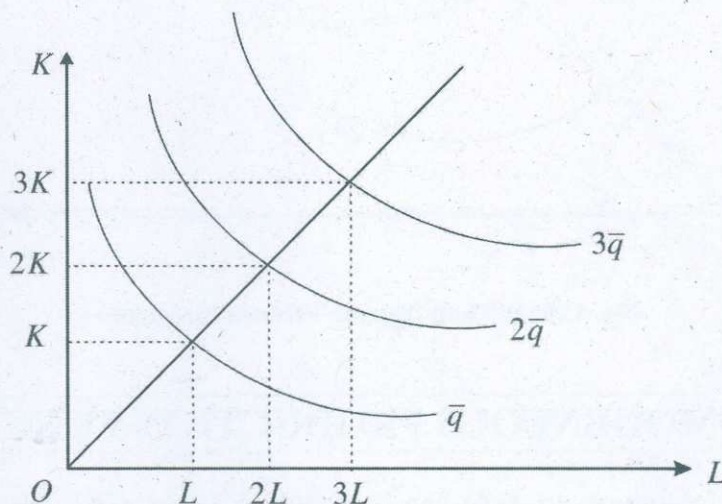


Fig. 4.16: Derivation of Line of Scale

Decreasing Returns to Scale

As there is DRS, along a ray through the origin, the distance between the consecutive iso-quants increases (see Figure 4.17).

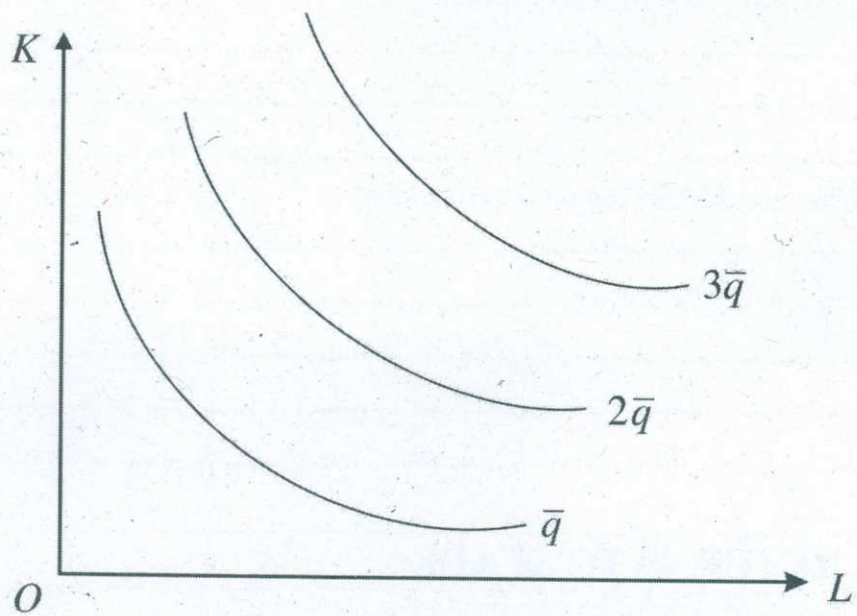


Fig. 4.17: DRS and Distance between Iso-quants

Increasing Returns to Scale

As there is IRS, along a ray through the origin, the distance between the consecutive iso-quants is less (see Figure 4.18).

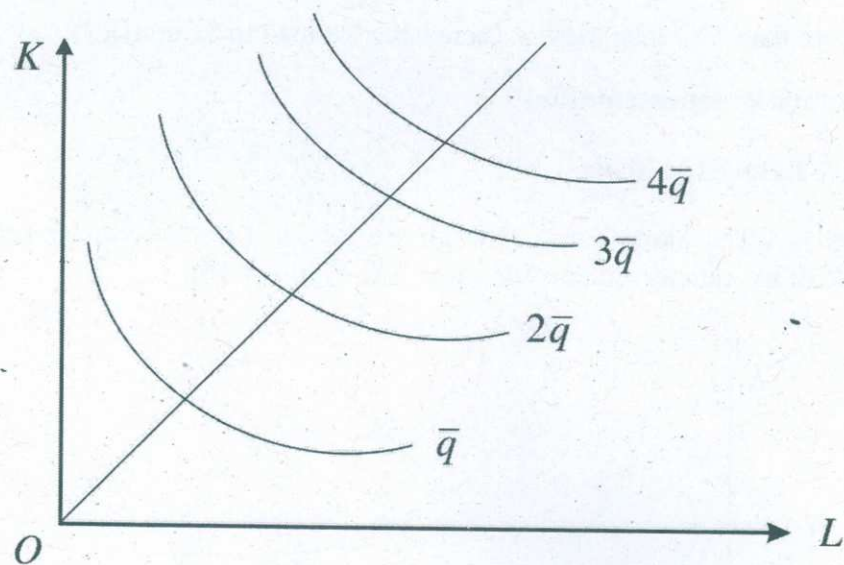


Fig. 4.18: IRS and Distance between Iso-quants

4.5 HOMOGENEOUS PRODUCTION FUNCTION

The concept of returns to scale can be captured easily using the concept of homogeneity of production functions.

A production function say, $z = f(x, y)$ is said to be homogeneous of degree n in x and y , if and only if,

$$f(tx, ty) = t^n z \text{ for any } t > 0.$$

If $n = 1$, then $f(tx, ty) = tz$, then the function is said to exhibit CRS. It is also known as a linearly homogeneous production function.

If $n > 1$, then the function exhibits IRS and when $n < 1$, it exhibits DRS.

Example

Suppose we have a production function,

$$q = AL^\alpha K^\beta$$

If L and K are increased by a factor $t(>0)$ then, we have, $A(tL)^\alpha (tK)^\beta$

$$= At^\alpha L^\alpha t^\beta K^\beta$$

$$= t^{(\alpha+\beta)} AL^\alpha K^\beta$$

$$= t^{(\alpha+\beta)} q$$

$\therefore$ The function is homogeneous of degree $(\alpha+\beta)$. It is said to exhibit

CRS, if $\alpha+\beta = 1$; DRS if $\alpha+\beta < 1$; IRS if $\alpha+\beta > 1$

When $\alpha+\beta = 1$, the function is also called a linearly homogeneous function.

Properties of a homogeneous production function.

- 1) If $q = F(K, L)$ is a homogeneous production function, then we can write it as,

$$F(tK, tL) = t^n q \text{ for any } t > 0.$$

$$\text{Let } t = \frac{1}{L} \text{ for any } L > 0.$$

Then the function can be written as,

$$F(tK, tL) = F\left(\frac{K}{L}, 1\right) = \frac{q}{L^n}$$

$$\text{Let } k = \frac{K}{L} \text{ and } F\left(\frac{K}{L}, 1\right) = f(k).$$

$$\therefore \text{ we have } f(k) = \frac{q}{L^n}$$

$$\text{or, } q = L^n f(k)$$

- 2) $q = L^n f(k)$

$$MP_L = \frac{\partial q}{\partial L} = nL^{n-1}f(k) - KL^{n-2}f'(k)$$

$$MP_K = \frac{\partial q}{\partial K} = L^{n-1}f'(k).$$

Both MP_L and MP_K are functions of $k \left(= \frac{K}{L} \right)$.

It may be useful to remember that homogenous production function is a special case of **homothetic production** functions. To take note of the concept, you must look for the ratio MP_L/MP_K , which does not change with any proportionate change in L and K in case of homothetic production function. The difference this formulation has with that of homogenous production function can be seen from the iso-quants given in Figure 4.19. The production function is homothetic if

slope of Q_1 at A_1 = slope of Q_2 at A_2

and

slope of Q_1 at B_1 = slope of Q_2 at B_2

In contrast to this feature, the homogenous production function would have $OA_2/OA_1 = OB_2/OB_1$.

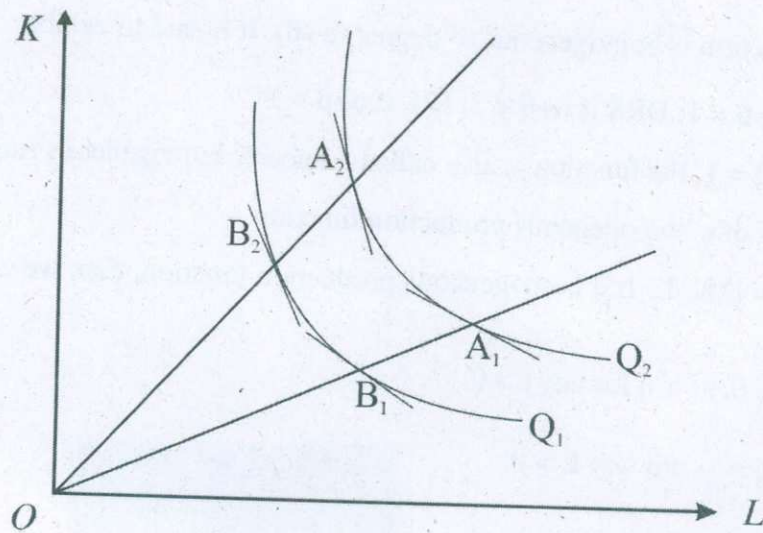


Fig. 4.19: Homothetic Production Function

Check Your Progress 5

- 1) What is a homogeneous production function?

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- 2) Prove for $q = L^n f(k)$ $MP_L = nL^{n-1}f(k) - KL^{n-2}f'(k)$; $MP_K = L^{n-1}f'(k)$.

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3) When MP_L and MP_K are functions of (K/L) ?

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4.6 LET US SUM UP

This unit covers theoretical insights on production process. It starts with production function and points out that it is a technical relation between inputs and output. Production decisions are based on short-run and long-run considerations. While the short-run offers enough time to a producer to change the variable inputs like labour and raw materials, the long-run allows changing all inputs, including machinery and building etc. To carry on production in both the periods, it helps understand the productivity concepts of average and marginal products. The average product is the output produced per unit of an input. Marginal product, on the other hand, gives change in the total product due to a unit change in one of the inputs. It is convenient to speak in terms of average product of labour (AP) and marginal product of labour (MP_L). The production process brings out the relationship between AP and MP. It is seen that when AP is rising $MP > AP$, when AP is maximum, $MP = AP$ and when AP is falling $MP < AP$. The total production passes through three distinct stages (I, II and III). In stage –I, the total product curve shows an increasing trend, while in stage –III it declines. A producer usually chooses to operate in stage-II of the product curve when AP reaches the maximum point.

A single level of output can be produced by different combinations of inputs. The curve that captures this feature is called an iso-quant. The slope of an iso-quant allows one to analyse the process of input substitution for a given level of output. When there are two inputs, labour and capital, the slope of the iso-quant is given by $\frac{dK}{dL} = -\frac{MP_L}{MP_K}$, where $\frac{dK}{dL}$ is called marginal rate of technical substitution of L for K

($MRTS_{LK}$) and MP_L and MP_K are marginal productivities of labour and capital. The rate at which the substitution between the factors takes place is given by the elasticity of substitution. It is usually denoted by σ . Thus

$$\sigma = \frac{\% \text{ change in } K/L}{\% \text{ change in } MRTS}$$

Simultaneous change of all inputs results in a change in output in proportion to greater, equal or lower scale. Such a feature is studied through returns to scale. The last part of the discussion is focussed on returns to scale through homogenous production function. The degree of homogeneity is related to an assessment of returns to scale and the properties of the homogenous

4.7 KEY WORDS

Average Product: Total product per unit of an input

Cobb-Douglas Production Function: A production function of the form $Q = f(aK^bL^c)$ where a , b and c are constants, Q is output, and L and K are inputs.

Constant Returns to Scale: The case where a proportionate change in all inputs changes output by the same proportion.

Decreasing Returns to Scale: The case where a proportionate increase in all inputs leads output to increase by a small proportion.

Economic Region of Production: The downward sloping segment of an iso-quant.

Elasticity of Substitution: A measure of the responsiveness of the input ratio to a change in the input-price ratio.

Homogeneous Production Function: A special case of homothetic production function in which a proportionate change in inputs causes output to change by a proportion which does not vary changes in the inputs.

Homothetic Production Function: A production function where the ratio of marginal product is unaffected by a proportionate change in inputs.

Increasing Returns to Scale: A situation where proportionate increase in all inputs causes output to increase by a large proportion.

Iso-quant Line: The locus of points representing various combinations of inputs yielding a specified output.

Diminishing Marginal Rate of Substitution: The declining marginal rate of substitution as one input is substituted for another.

Long Period Production: A period of time sufficient for altering the quantities of all inputs into the production process.

Marginal Rate of Technical Substitution: The rate at which one input can be substituted for another without affecting the level of output.

Production Function: The functional relationship between inputs and output.

4.8 SOME USEFUL BOOKS

Ferguson and Gould (1989), *Microeconomic Theory*, Irwin Publications in Economics; Homewood, IL: Irwin.

Koutsoyiannis, A. (1979), *Modern Microeconomics*, Second edition, London: Macmillan.

Ferguson, C. E. (1969), *The Neoclassical Theory of Production and Distribution*, Cambridge: Cambridge University Press.

4.9 ANSWER OR HINTS TO CHECK YOUR PROGRESS

Check Your Progress 1

- 1) A technical relation between inputs and output
- 2) It is likely to conflict with the scale of production and may contribute to the increased cost.
- 3) Change in output due to a unit change in labour with level of other factors kept constant.

Check Your Progress 2

- 1) Because contribution of other factors to production cannot be perfectly substituted by labour.
- 2) At this state AP reaches the maximum.
- 3) It is elasticity of output with respect to a factor (show the derivation).

Check Your Progress 3

- 1) Different combinations of inputs producing a fixed level of output.
- 2) See section on slope of an iso-quant.

Check Your Progress 4

- 1) Rate of substitution between inputs allowed by the production function.
- 2) Define as given in the text and explain the meaning.
- 3) Consider the iso-quants which are not of regular shape, e.g., complementary inputs.
- 4) See the change in output by changing all inputs simultaneously.

Check Your Progress 5

- 1) Change in inputs changes the value of function to a certain degree.
- 2) See the derivation given the text in section on Homogenous Production Function.
- 3) In Homogenous Production Function (see, the derivation in the text on homogenous production function).

UNIT 5 THEORY OF COST

Structure

- 5.0 Objectives
- 5.1 Introduction
- 5.2 Concept of Short-Run and Long-Run
 - 5.2.1 Long-Run Costs
 - 5.2.2 Short-Run Costs
- 5.3 Traditional Theory of Cost
 - 5.3.1 Theory of Cost in the Short-Run
 - 5.3.2 Theory of Cost in the Long-Run
 - 5.3.3 Economies of Scale
- 5.4 Modern Theory of Cost
- 5.5 Let Us Sum Up
- 5.6 Key Words*
- 5.7 Some Useful Books
- 5.8 Answer or Hints to Check Your Progress
- 5.9 Exercises

5.0 OBJECTIVES

After going through this unit, you will be able to:

- identify the types of cost incurred in the process of production;
- consider the nature of cost in short-and long-run;
- assess the theoretical insights on the shape of the cost curves; and
- examine the traditional and modern theories of cost.

5.1 INTRODUCTION

In a production process, cost is involved due to employment factors of production. In this regard, it is essential to know various types of costs a producer faces while deciding upon the employment of factors. We can think of two types of costs – explicit and implicit. The “explicit” cost is incurred for using a particular factor. For example, if a producer employs 20 units of labour at Rs.5 per unit, the explicit cost is Rs.100. This is also called the accounting cost. The “implicit” or “opportunity cost” is defined as the value of a resource in its next best use; suppose a Miss Sen quits her job of Rs.5, 00,000 per year and opens her own small business. Although, the accounting cost to her from her own business is zero, the opportunity cost is Rs.5, 00,000 per year.

In following, we would deal with “explicit” costs of production. These costs could be of two types – fixed and variable. Fixed cost is incurred on account of a fixed factor. For example, office building, a piece of land etc.

Variable cost, on the other hand, is incurred from a variable factor. For example, payment according to labour hours, purchasing certain raw materials, whose volume of employment can be easily changed.

However, no factor can be strictly fixed or variable. The definition of a fixed or variable factor depends, therefore, on the time frame in which the production process is being considered. If the time frame is very short, a particular input cannot be varied whereas it becomes variable if the time period is sufficiently long.

5.2 CONCEPT OF SHORT-RUN AND LONG-RUN

The short-run is defined as a period of time in which amount of certain types of inputs cannot be changed, regardless of the level of output. The short-run is a nebulous concept. It can be a day, a month, a year or even a nanosecond! In one nanosecond, virtually nothing can be changed in the production process. In a day, it may be possible to intensify the usage of certain machines. In a month the entrepreneur may be able to rent some additional equipment, in a year she may build a new plant. There are, therefore, many "short-runs", and the longer the time the greater are the possibilities for factor substitution and adjustment and greater is the scope of treating fixed costs as variable costs. Accordingly, the long-run is defined as a period of time when all inputs, used in the production process, are variable.

Cost of producing a given output clearly depends on the time available to make adjustments in the amount used of the productive factors. Let us now look at how the costs change in long and short periods.

5.2.1 Long-Run Costs

Suppose we have a production function,

$Q = F(L, K)$, where Q = quantity of output produced

L = units of labour employed

K = units of capital employed

If we take w = wage rate per unit of labour and

r = rent per unit of capital,

total cost (TC) is:

$$C = wL + rK$$

Assuming total cost C remains constant i.e., $C = \bar{C}$, we get,

$$\bar{C} = wL + rK \quad \dots (1)$$

$$\text{or, } K = \frac{\bar{C} - wL}{r}$$

Equation (1) gives the idea of iso-cost and can be used to obtain what is known as an iso-cost line. In the iso-cost line because, we can have different combinations of L and K employed at the same cost outlay. Graphically, this is represented as a downward sloping straight line with slope $\frac{w}{r}$ in the K - L space (Figure 5.1).

Different levels of cost outlays give rise to different iso-cost lines, with the highest cost being represented as the one lying farthest from the origin.

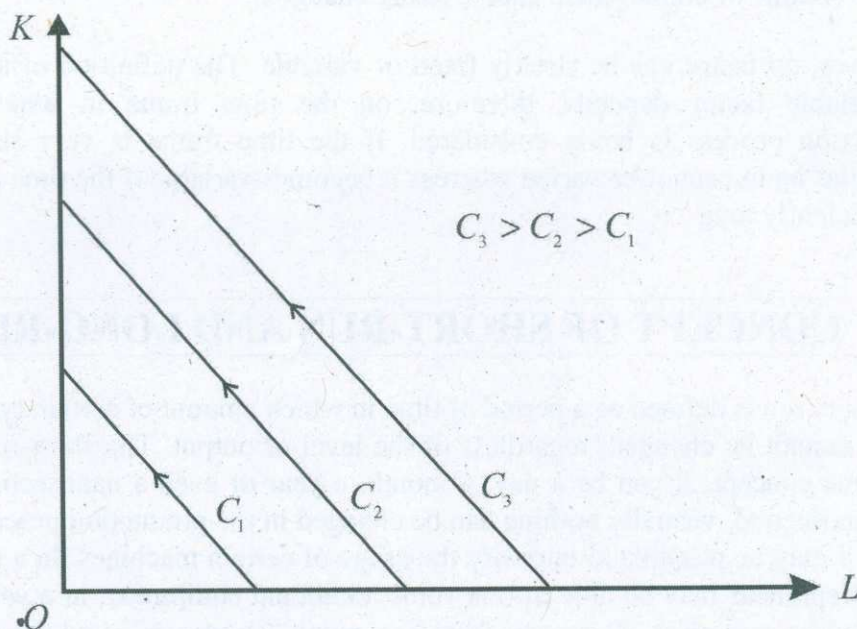


Fig. 5.1: Iso-cost lines

Given an output level say $\bar{Q}$, the optimal L and K , is obtained from the point of tangency between the lowest iso-cost and the iso-quant, as shown in Figure 5.2

From the figure, with cost-outlay C_0 , $\bar{Q}$ cannot be produced. In addition, C_2 becomes too costly for producing $\bar{Q}$. Thus, C_1 is the optimum cost-outlay for production of output level $\bar{Q}$.

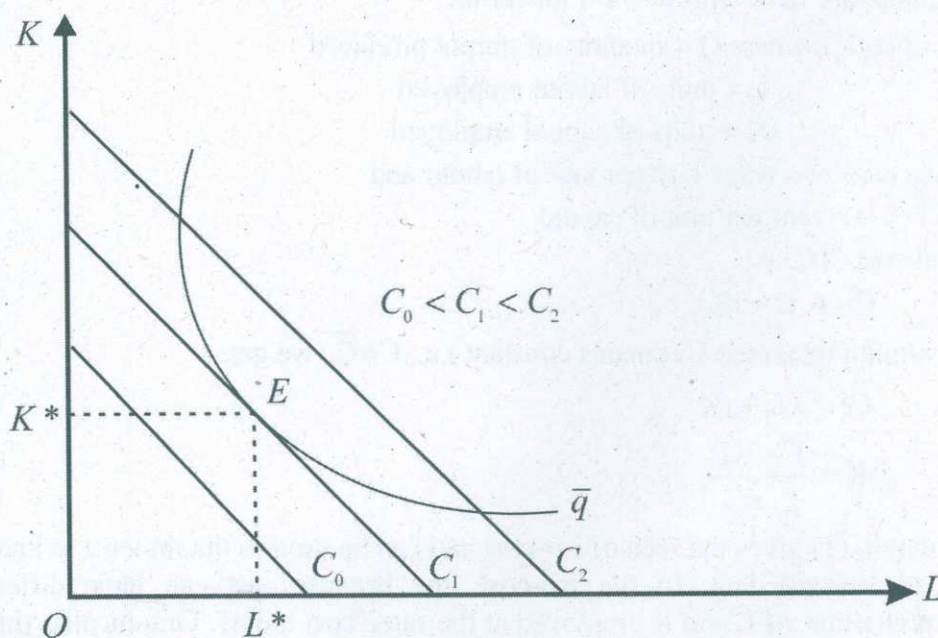


Fig. 5.2: Optimal Cost-output Condition

With C_1 level of cost outlay, $\bar{Q}$ is just producible.

Therefore, C_1 is the associated cost for output level $\bar{Q}$

In the long-run, L and K are both variable and hence higher levels of output are producible. And as above, the optimum cost would be given by the tangency between the iso-cost and the iso-quants, as shown in Figure 5.3.

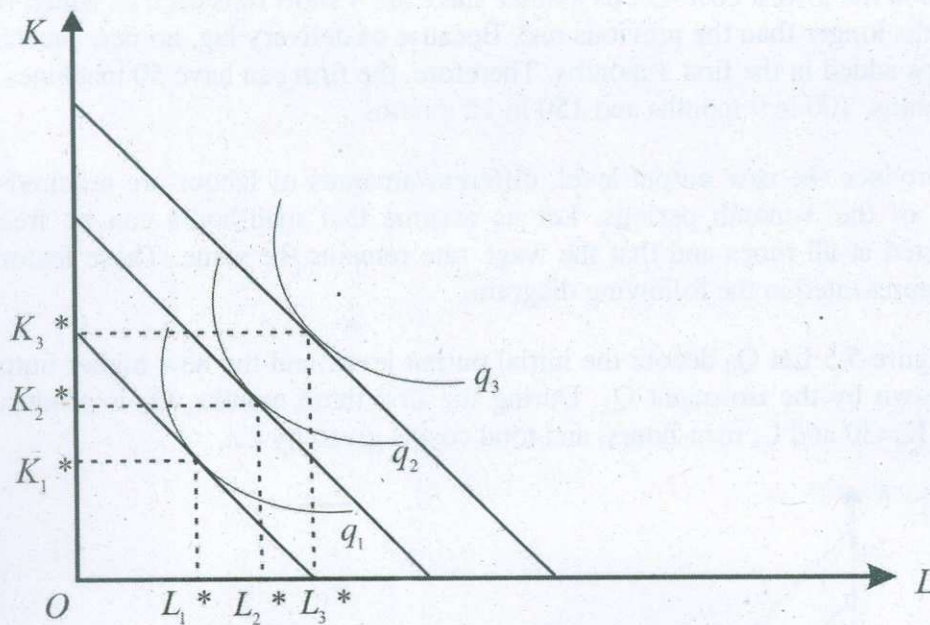


Fig. 5.3: Map of Optimal Cost-output Combination

Thus, for output Q_1 , cost is C_1 , for Q_2 it is C_2 and for Q_3 it is C_3 . Accordingly, in the C - Q plane we get a long-run total cost curve (LRTC) as the one shown in Figure 5.4, which is upward rising.

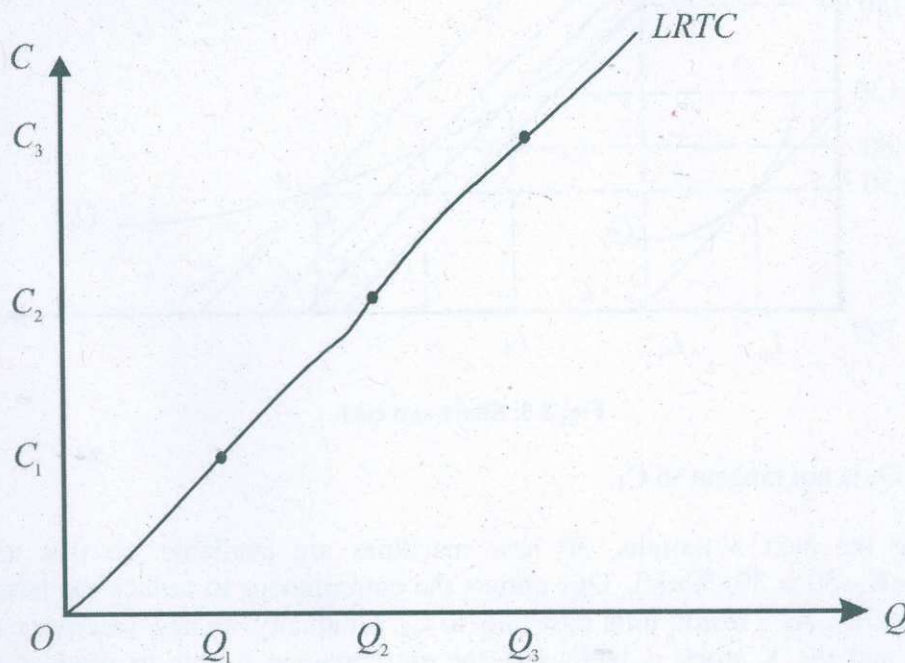


Fig. 5.4: Long-run Total Cost Curve

5.2.2 Short-Run Costs

There are a large number of "short-runs" depending on the time period involved. Each such short-run is characterised by the fact that not all factors of production can be fully adjusted in the given time period. We would illustrate this with the help of an example.

Suppose a firm wishes to raise output and must acquire 150 more machines to do so at the lowest cost. Let us assume there are 4 short runs each of which is 3 months longer than the previous one. Because of delivery lag, no new machine can be added in the first 3 months. Therefore, the firm can have 50 machines in 6 months, 100 in 9 months and 150 in 12 months.

To produce the new output level, different amounts of labour are required in each of the 3-month periods. Let us assume that man-hours can be freely adjusted at all times and that the wage rate remains the same. These features are represented in the following diagram.

In Figure 5.5 Let Q_0 denote the initial output level, and the new higher output is shown by the iso-quant Q_1 . During the first three months, Q_1 is produced with $K_0=30$ and L_1 man-hours, and total cost is given by C_1 .

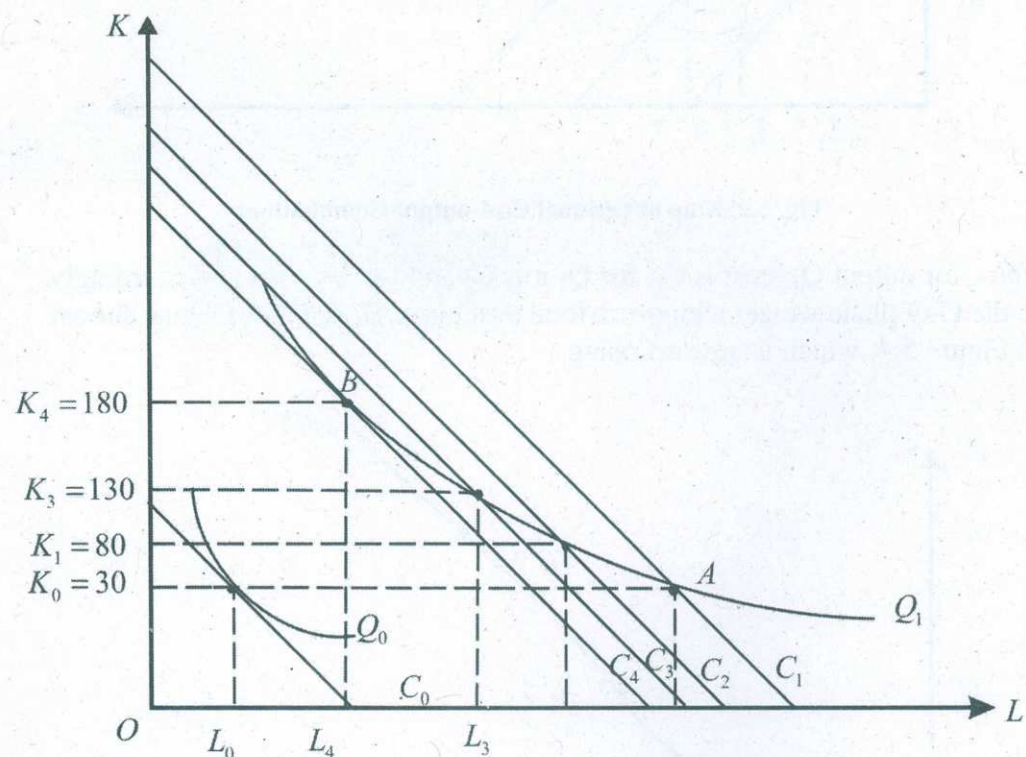


Fig. 5.5: Short-run cost

Note: Q_1 is not tangent to C_1 .

During the next 3 months, 50 new machines are available, so that total $K=K_2=K_1+50=30+50=80$. This allows the entrepreneur to reduce the labour cost to OL_2 . As a result, total cost falls to C_2 . Gradually, as new machines are added and the K stock is increased, the entrepreneur is able to produce Q_1 optimally at a total cost C_4 , because C_4 is the lowest possible cost at which Q_1 can be produced.

The relation between short-run cost and output is shown in the adjacent diagram (Figure 5.6). Point A in this figure corresponds to A in Figure 5, so does B' corresponding to B . LRTC gives the long run total cost (when all factors vary), and SRTC the short-run total cost, when only labour is the variable input.

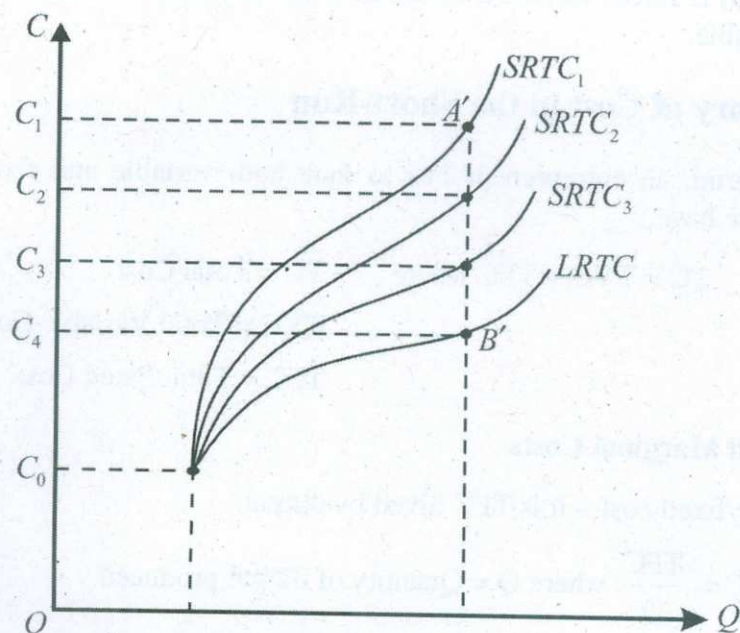


Fig. 5.6: LRTC and SRTC.

It is seen that SRTC is greater than LRTC.

Check Your Progress 1

- 1) What is the difference between a short-run and a long-run?

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- 2) Given that there are 2 factors of production – L and K , can the SRTC curve lie above the LRTC curve, if K is fixed in the short-run?

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5.3 TRADITIONAL THEORY OF COST

As seen above, theory of cost distinguishes between two time periods, the short-run and long-run. We know that the short-run is the period during which some factor(s) is fixed, while the long-run is the period over which all factors become variable.

5.3.1 Theory of Cost in the Short-Run

In the short-run, an entrepreneur has to face both variable and fixed costs. Therefore, we have,

$$TC = TVC + TFC \text{ where } \begin{array}{l} TC = \text{Total Cost} \\ TVC = \text{Total Variable Cost} \\ TFC = \text{Total Fixed Cost.} \end{array}$$

Average and Marginal Costs

- 1) Average fixed cost – It is TFC divided by output.

$$\text{i.e., } AFC = \frac{TFC}{Q} \text{ where } Q = \text{Quantity of output produced.}$$

- 2) Average variable cost – It is TVC divided by output.

$$\text{i.e., } AVC = \frac{TVC}{Q}$$

- 3) Average total cost – It is TC divided by output

$$\text{i.e., } ATC = \frac{TC}{Q}$$

- 4) Marginal cost – It is the addition to TC attributable to the addition of one unit of output, i.e.,

$$MC = \frac{\partial TC}{\partial Q} = \frac{\partial}{\partial Q} TVC$$

Thus, in the short-run MC gives the slope of both TC and TVC.

Geometry of Average and Marginal Cost Curves

We will derive the average and marginal cost curves from the TC curve, using the above definitions.

- i) AFC – This curve is derived from the TFC curve, which is a horizontal straight line in the C-Q plane, as shown below.

As $AFC \cdot Q = TFC$, which is a constant, AFC is a rectangular hyperbola. It is downward sloping because as Q increases AFC must decrease in order to maintain a constant TFC. This is shown in panel B of the following Figure 5.7.

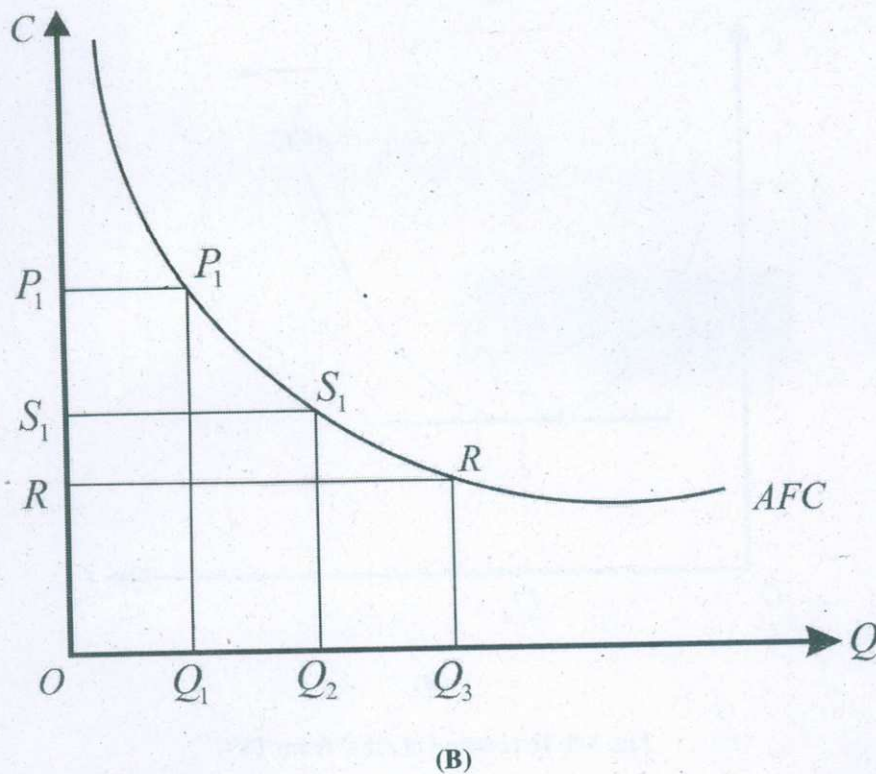
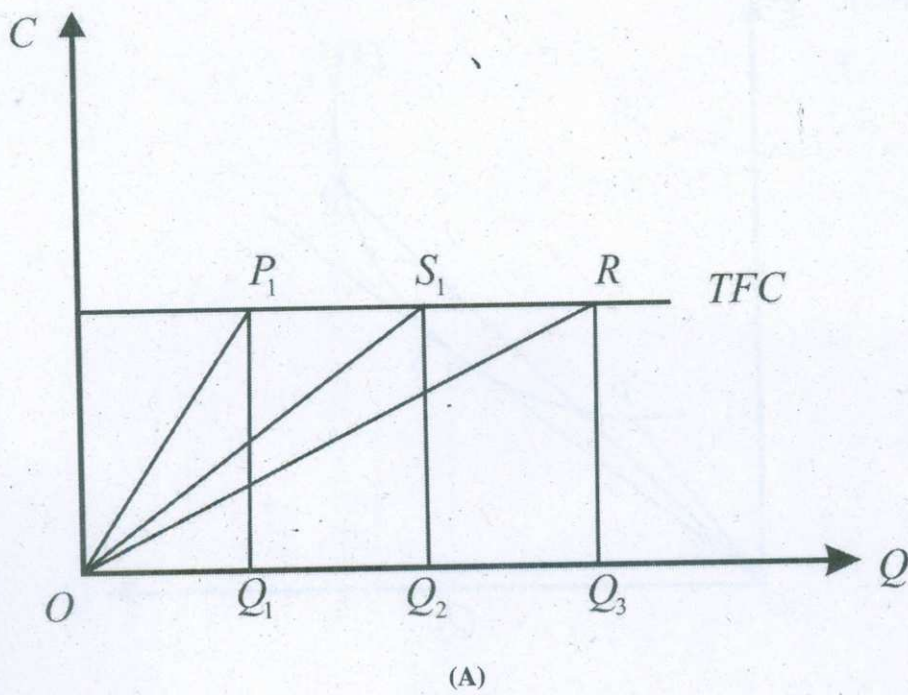


Fig. 5.7: Derivation of AFC from TFC

In the C-Q plane, TFC is a horizontal straight line and AFC is given by the slope of a ray from the origin to any point on the TFC. The points P_1S_1R on TFC correspond to P_1S_1R on AFC.

- ii) AVC – This is derived from the TVC. AVC is given by the slope of a ray drawn from the origin to a point on the TVC, for any level of output.

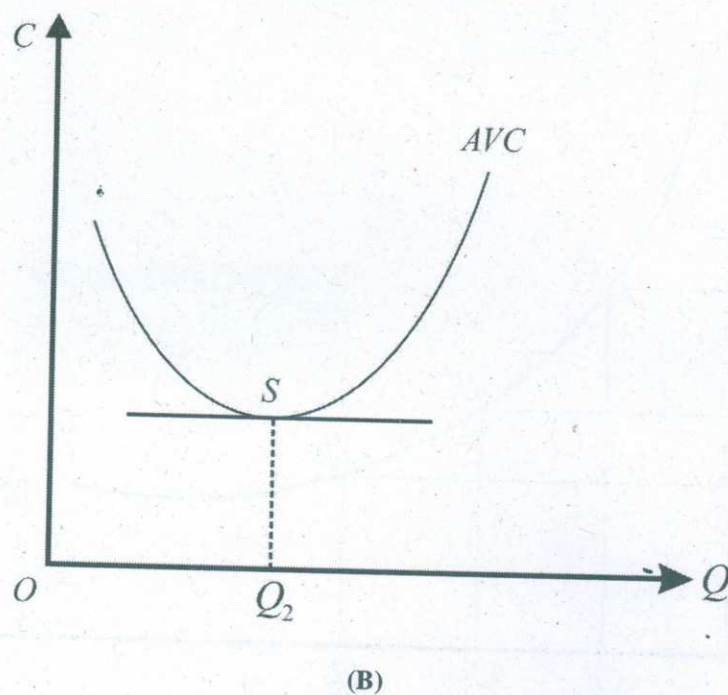
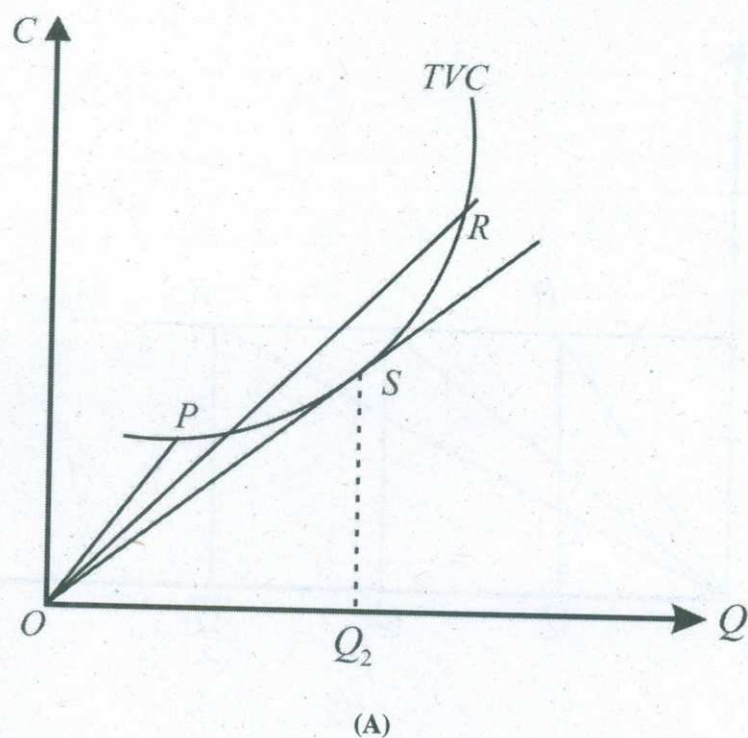


Fig. 5.8: Derivation of AFC from TVC

As is seen in panel A of Figure 5.8, the slope of a ray from the origin to the TVC curve steadily decreases as one passes through point such as P until it becomes tangent to the TVC curve at point S (associated with output OQ_2). Thereafter the slope increases, as one moves from S towards point, such as R. All this is shown in panel B and by constructing a U-shaped AVC, which is downward sloping till output OQ_2 and upward sloping beyond that.

iii) ATC – This curve is derived from the TC. As is the case of AVC, ATC is given by the slope of a ray from the origin to any point on the TC curve.

Figure 5.9, shows the derivation of ATC from TC. We find that in panel A, the slope of the ray to TC diminishes as we move along TC until we reach points and beyond which the slope rises. Accordingly, we get a U-shaped ATC, which initially falls until output level OQ_2 and thereafter rises. OQ_2 in panel A is the same as OQ_2 in panel (B).

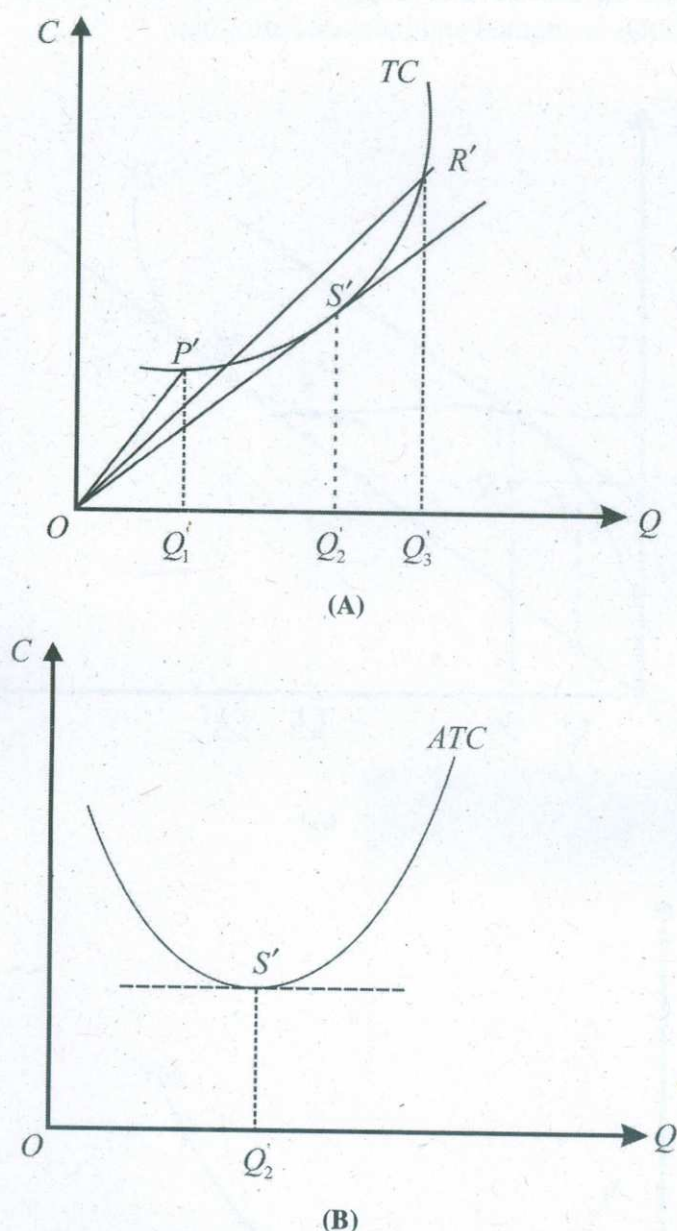


Fig. 5.9: Derivation of ATC from TC

iv) MC – It is derived from the TC curve.

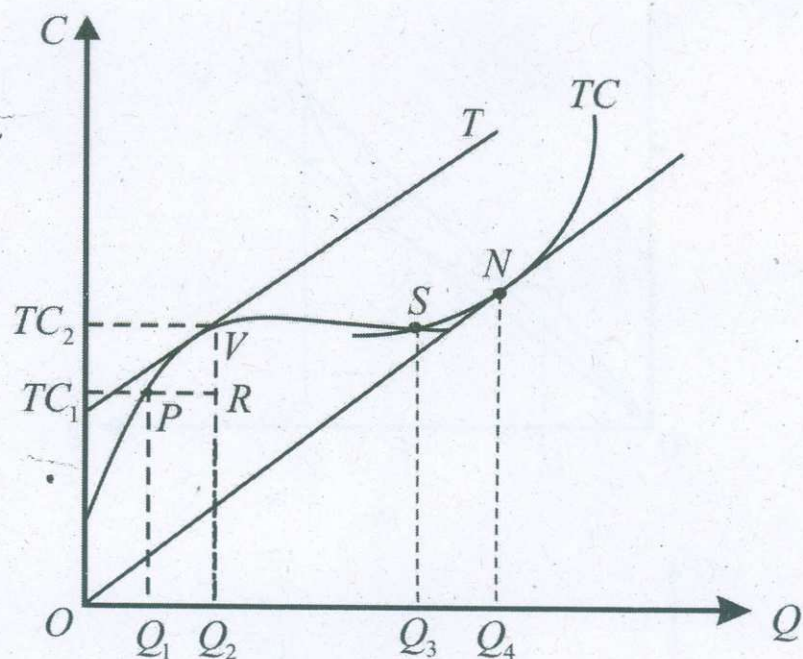
Panel A of Figure 5.10 shows the TC curve. As output increases from OQ_1 to OQ_2 , one moves from point P to point V, the total cost rises from TC_1 to TC_2 . So, MC is given by,

$$MC = \frac{TC_2 - TC_1}{OQ_2 - OQ_1} = \frac{VR}{PR}$$

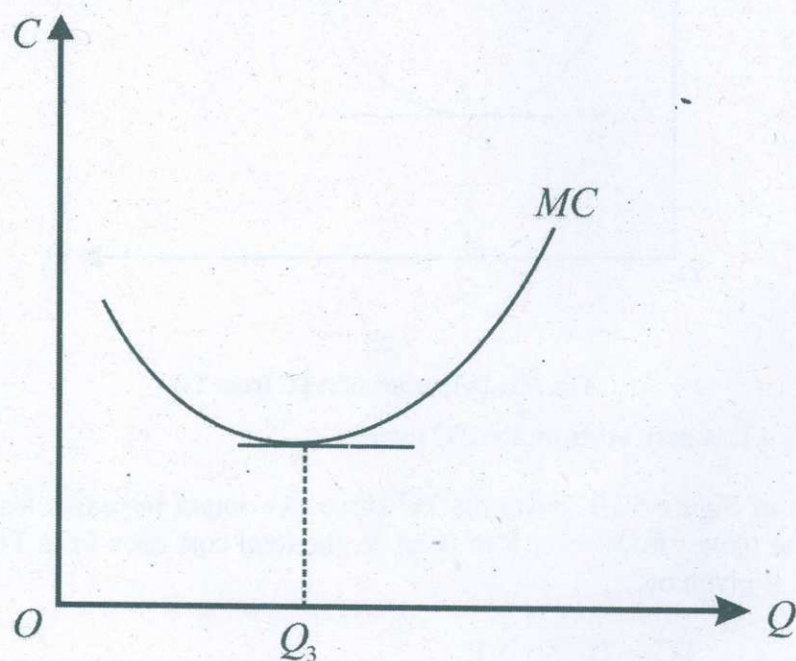
Now, let point P move towards V, along TC. As the distance between P and V becomes smaller and smaller, the slope of the tangent T at point V becomes a

progressively better estimate of VR/PR . In the limit, for movements in a small neighbourhood around point V , the slope of the tangent gives the MC .

As one moves along TC through points such as P and V , the slope of TC diminishes. It continues to diminish until point S is reached at output level OQ_3 and thereafter increases. Accordingly, we have the MC curve to fall until output level OQ_3 is attained and increases after that.



(A)



(B)

Fig. 5.10: Derivation of MC from TC

5.3.2 Theory of Cost in the Long-Run

Long-run is a period of time where all inputs are variable. To explain what long period means, suppose technology is such that plants in a certain industry can have only three different sizes – small, medium and large.

Let the plant sizes give rise to the short-run average cost curve SAC_1 , SAC_2 and SAC_3 .

This is shown in Figure 5.13.

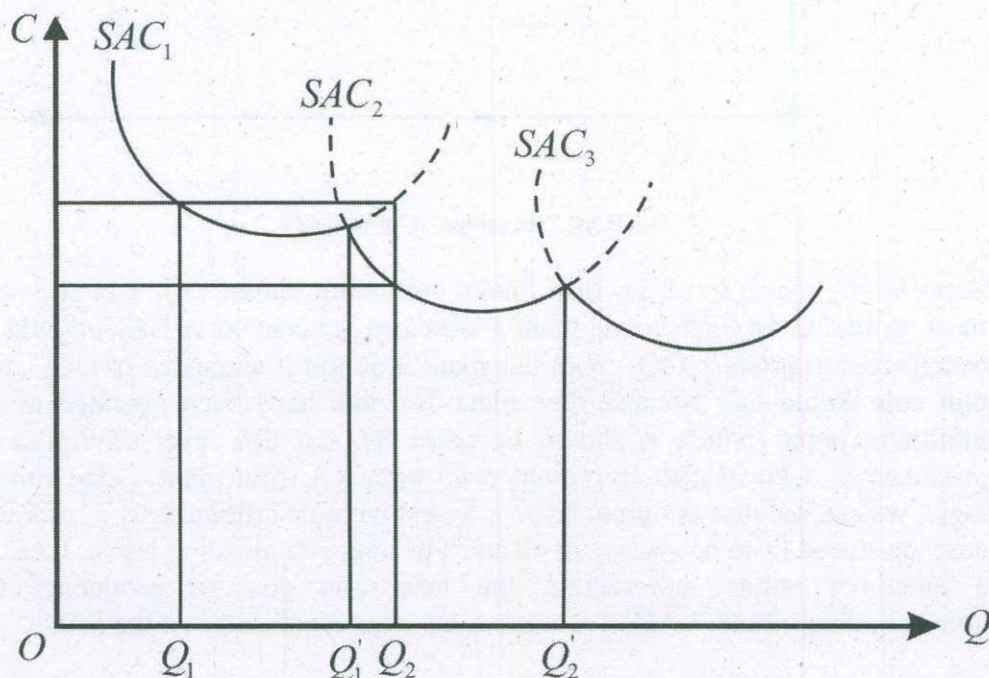


Fig. 5.13: Choice of Plant

In the long-run, a manager has to choose from the three alternative plants. The manager's choice of plant will depend on planned or expected output. To produce output OQ_1 , the firm would choose to do it from plant 1, because it is cheapest to produce there. If output is OQ_2 , then plant 2 is selected. For output levels OQ_1 and OQ_2 , the decision involves some ambiguity. This is because, in these cases, both plants have the same average cost. The manager would choose the bigger plant if she expects demand to rise in future.

Long-Run Average Cost Curve (LRAC)

To derive the LRAC, let us consider six plants at the manager's disposal. These are represented by SAC_1 , SAC_2 , SAC_3 , SAC_4 , SAC_5 and SAC_6 as shown in Figure 15.14.

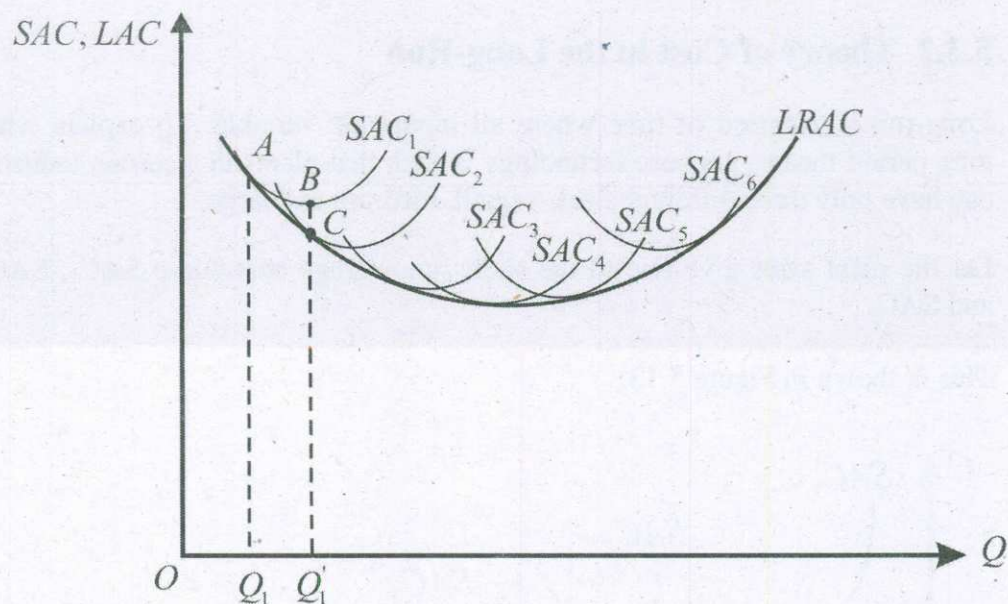


Fig. 5.14: "Envelope" Curve-LAC

Suppose the manager of the firm thinks producing output OQ_1 would be the most profitable. In such a case, plant 1 with average cost curve SAC_1 would be constructed to produce OQ_1 . With this plant if output is expanded to OQ_1' then unit cost would fall; because then plant 1 would have been operated at the minimum point (which is shown by point B). Let this level of output be produced at a lower cost (represented by point C) from plant 2. By similar logic, we can see that as output rises, it would be most efficiently (i.e., at lower cost) produced from newly set up plants. The long-run planning curve, LAC, is a locus of points representing the least unit cost of producing the corresponding output. LAC is, therefore, an "envelope" curve of the SACs.

An interesting point to note here is that at point B, Plant 1 is being most efficiently operated. Yet it pays the manager to shift to plant 2. This is because unit cost is lower at point C than at point B, even though C is not the most efficient point for plant 2. In fact, all plants smaller than type 4 are only used when they are operating at less than the efficient level. Therefore, it is not efficiency for a given plant that matters, but efficiency overall. The LAC is the locus of points with minimum unit cost associated with a particular level of output with respect to all available plants.

Long-Run Marginal Cost Curve (LRMC)

The LRMC is derived from the short-run marginal cost (SRMC) curves. LRMC is formed from the points of intersection of the SRMC curves with the vertical lines (to the X-axis) drawn from the points of tangency of the corresponding SAC curves and LRAC curve. This is shown in Figure 5.15. The LMC must be equal to the SMC for the output at which the corresponding SAC is tangent to the LAC.

For levels of output (Q) to the left of tangency 'a', as shown in Figure 5.15, the $SAC > LAC$. At the point of tangency, $SAC = LAC$. As we move from 'a' to 'a', i.e., from a position of where $SRAC \neq LRAC$, to one of $SRAC = LRAC$

the change in long-run cost becomes greater than that for the short-run i.e.,

$$\frac{\partial}{\partial Q} LAC > \frac{\partial}{\partial Q} SAC \text{ this means } LMC > SMC.$$

For levels of output, to the right of 'a', $SAC > LAC$, which by similar reasoning leads to the result that $LMC < SMC$.

Thus, if we draw a vertical line from 'a' to x-axis, its point of intersection with SMC would be a point on LMC.

Repeating the above procedure for all points of tangency of the SRAC and LAC curves to the left of the minimum point of the LAC, we obtain points of the section of the LMC, which lies below the LAC. At the minimum point M, LMC intersects the LAC. To the right of M, the LMC would lie above the LAC curve and at M, we have,

$$SAC_M = SMC_M = LAC = LMC$$

SAC, SMC, LAC, LMC

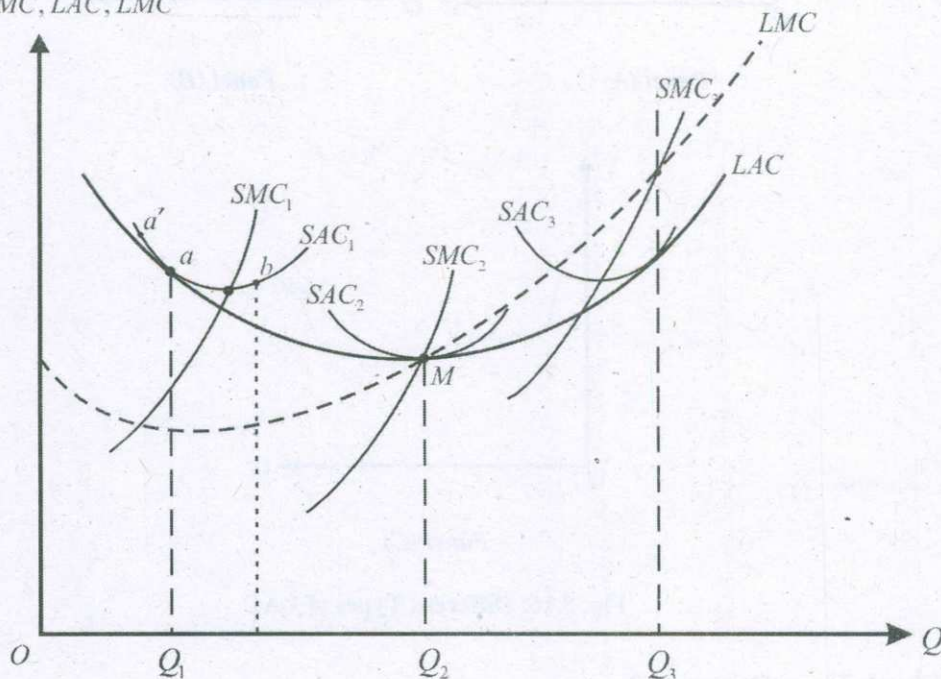


Fig. 5.15: Derivation of the LMC.

5.3.3 Economies of Scale

The economies of scale determine the shape of the long-run average cost curve.

As the size of the plant and the scale of operation become larger, certain economies of scale are usually realized. Factors like specialization and division of labour and other technological improvements enable producers to reduce the unit cost. These factors give rise to a negatively sloped position of the LRAC.

The rising portion of the LAC is usually attributed to the “diseconomies of scale”. Such a feature could take place due to limitations in efficient management. It is difficult to determine just when diseconomies of scale set in and when they become strong enough to outweigh the economies of scale. In

businesses where economies scale are negligible, diseconomies may soon become of paramount importance and cause the LAC to turn up at a relatively small volume of output. Panel A of Figure 5.16 shows the LRAC for such a firm. Sometimes the LAC may not turn upward until a very large volume of output is attained. This is shown in panel B of Figure 5.16. It is the case of 'natural monopolies'. In many situations, a very modest scale of operation may enable firm to capture all the economies of scale. Diseconomies may not be incurred until the volume of output is very high. In such a case, LAC would have a horizontal stretch, as shown in panel (C).

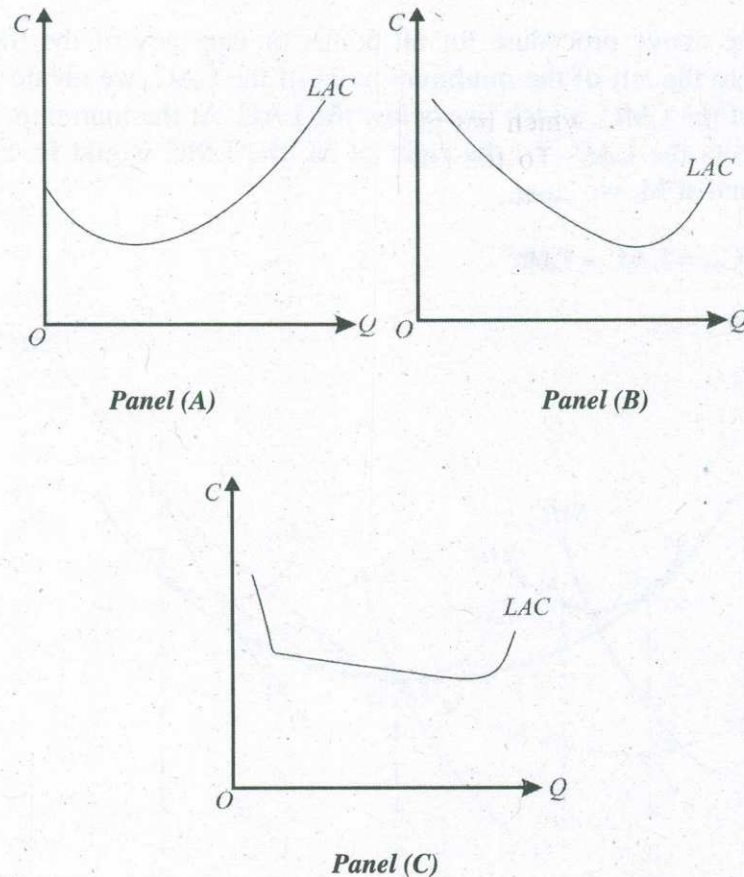


Fig. 5.16: Different Types of LAC

Check Your Progress 3

- 1) Analyse the relation between LMC and SMC to the right of its minimum point.

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- 2) Is the LMC also an envelope of the SMCs?

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- 3) Why is the LRAC called the “envelope” of the SACs?

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- 4) Does the LRAC contain the minimum points of each SAC curve?

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5.4 MODERN THEORY OF COST

The U-shaped cost curves of the traditional theory have been questioned both on theoretical as well as empirical grounds. George Stigles suggests that the short-run average variable cost has a flat stretch over a range of output, which reflects the fact that firms build plants with some flexibility in their productive capacity. The reasons for this reserve have been discussed in detail by various economists. The U-shape of LAC has been questioned and is often pointed out that it is L-shaped.

In the following sections, we would look at the short-run and the long-run costs under modern cost theory.

Short-Run

Short-run total cost (SRTC) consists of short-run total variable cost (SRTVC) and the short-run total fixed cost (SRTFC) i.e., $SRTC = SRTVC + SRTFC$.

The corresponding average costs are obtained by dividing each of the above by total output 'Q'.

$$STC = STVC + STFC$$

$$\text{or, } \frac{STC}{Q} = \frac{STVC}{Q} + \frac{STFC}{Q}$$

$$\text{or, } ATC = AVC + AFC$$

Average Fixed Cost

This cost consists of the physical and personal organisation of the firm. These include expenses of maintenance of buildings, land machinery, salaries of the administrative staff etc.

The 'planning' of the firm consists in deciding the 'size' of these fixed factors, which sets a limit to the firm's production (In contrast, variable factors, like labour hours, do not limit the firm's production because these can be hired very easily from the market).

The entrepreneur will plan to produce that level of output, which she anticipates to sell and accordingly she would choose that size of plant, which will allow her to produce that level of output efficiently and with maximum flexibility. The chosen plant would, therefore, have a capacity larger than the 'expected average' level of sales, because the entrepreneur would want to have some **reserve capacity** for various reasons.

The reserve capacity helps to meet the seasonal and cyclical fluctuations in demand. It gives the entrepreneur greater flexibility in repairing broken-down of the production process. Besides, some machinery may be so specialised that they are available only to order, which takes time. In such a cases, the machinery will be bought in excess of the minimum requirement, as a reserve. Generally, some reserve capacities are always allowed in the land and building since expansion of operations may be seriously limited if these have to be acquired. Therefore, an entrepreneur will not necessarily choose the plant, which will give her today the lowest cost, but rather that equipment which will allow the greatest flexibility.

Under these conditions the AFC curve would look like as shown in Figure 18. The firm has some 'larges-capacity' units of machinery, which set an absolute limit to the short-run expansion of output (boundary II). The firm also has small unit machinery, which can set a limit to expansion (boundary I). Boundary I is not an absolute boundary, because the firm can increase its output in the short run, until it is encountered. This can be done in two ways: either by raising the variable factors employed in production or, by buying some additional small-unit types of machinery. If the first method is chosen, then we have the AFC getting extended beyond I, (this then looks like the AFC in traditional theory of cost), if the second is chosen there occurs a break in AFC, whereby we have the portion 'ab'.

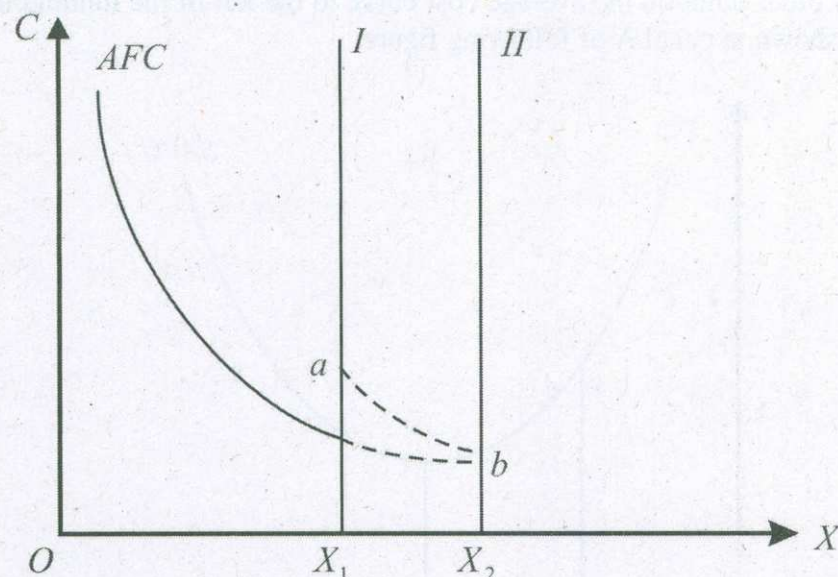


Fig. 5.17: Average Fixed Cost in the Modern Theory

Average Variable Cost

As in the traditional theory, the average variable cost of modern theory consists of the cost of variable inputs like labour and raw materials. The SAVC, in modern cost theory is U-shaped but it also has a flat stretch over a range of output, as shown in Figure 5.18. Over this stretch the $SAVC = MC$, i.e., both being constant per unit of output. As usual, to the left of the flat stretch, MC lies below SAVC and to the right, it lies above SAVC.

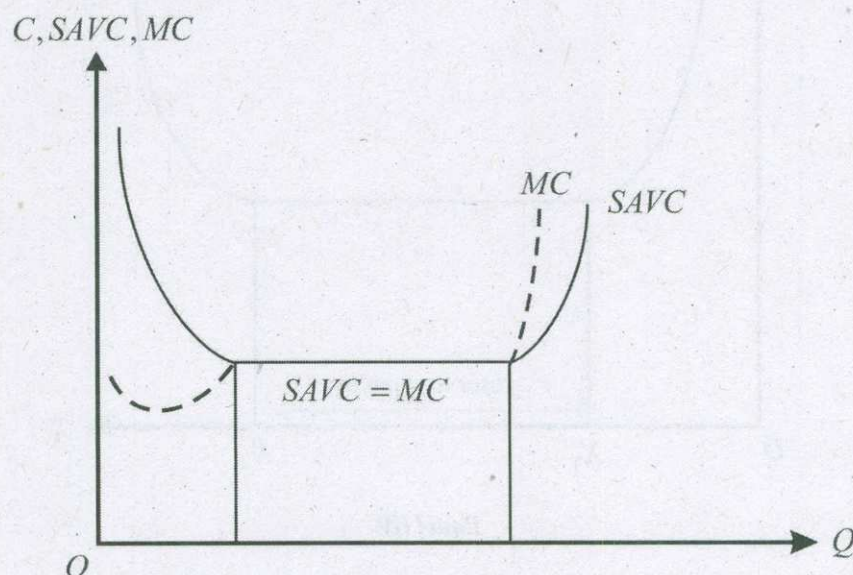
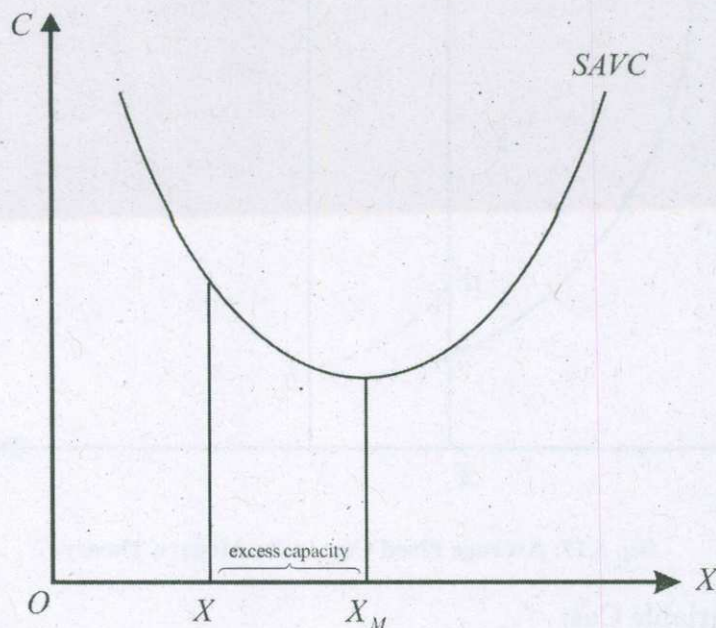


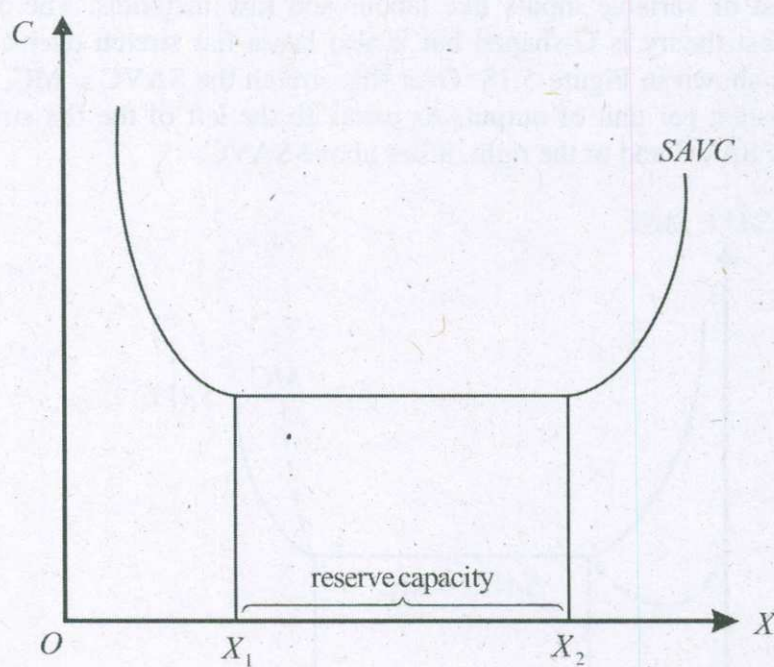
Fig. 5.18: Short-run Average Variable Cost in Modern Theory

The flat stretch in the SAVC is due to the presence of “reserve capacity”. It may be important to take note of features that the reserve capacity is planned in order to give the maximum flexibility in the operation of the firm. It is different from “excess capacity”, which arises with the U-shaped costs of the traditional theory of the firm. By definition, excess capacity is given by the difference in output corresponding to the minimum level of average cost curve

and any other point on the average cost curve to the left of the minimum point. This is shown in panel A of following figure.



Panel (A)

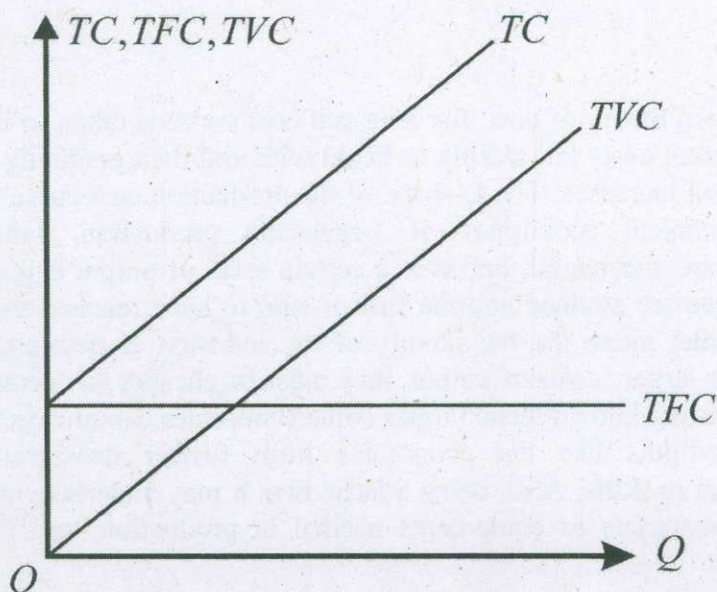


Panel (B)

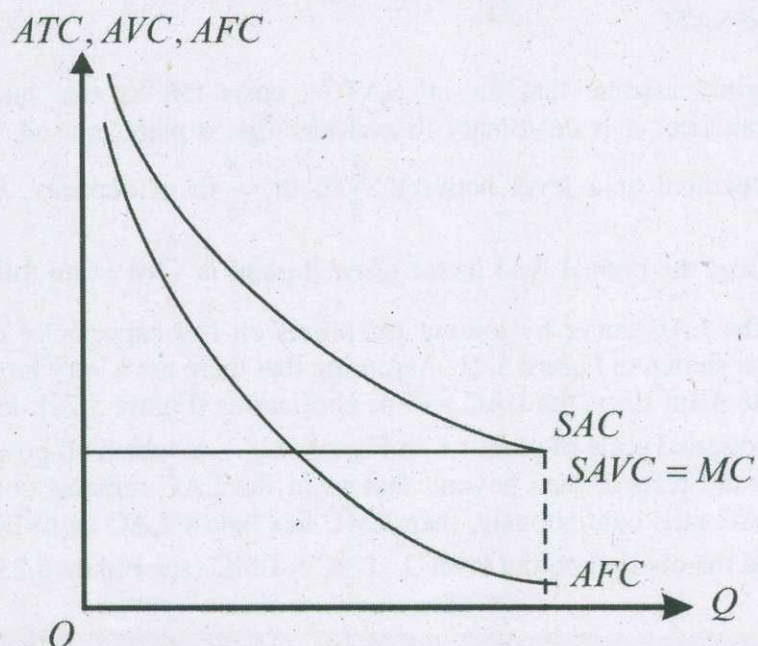
Fig. 5.19: SAVC under Traditional & Modern Theory

The traditional theory assumes that each plant is designed without any flexibility i.e., it is designed to produce optimally only a single level of output (X_M , as shown in panel (A) of Figure 20). If the firm produces an output $X < X_M$ there is excess (unplanned) capacity, equal to the difference $X_M - X$. This excess capacity is undesirable because it leads to higher unit costs.

In the modern theory of costs, the range of output X_1X_2 in panel (B) of Figure 5.19, reflects the planned reserve capacity, which does not lead to increases in costs. The firm sometimes chooses to operate the plant closer to X_1 and at others closer to X_2 . Generally, the entrepreneur expects to operate her plant within the X_1X_2 range.



Panel (A)



Panel (B)

Fig. 5.20: Derivation of Short-run Average Cost Curves from Total Cost Curves

Average Total Cost

The ATC is obtained by adding the AFC and AVC at each level of output. The ATC is shown in Figure 5.20. The ATC curve falls continuously up to the level of output (X_2) at which the reserve capacity is exhausted. Beyond that level, ATC will start rising. The MC will intersect the ATC curve at its minimum point (which occurs to the right of the level of output X_A , at which the flat stretch of the AVC ends).

Long-Run

In the modern theory of cost, the long-run cost curve is taken to be L-shaped. The production costs fall steeply to begin with and then gradually as the scale of production increases. The L-shape of the production cost curve is explained by the technical economies of large-scale production. Initially, these economies are substantial, but after a certain level of output is reached, all or most of these are attained and the firm is said to have reached the **minimum optimal scale**, given the technology of the industry. If new techniques are invented for larger scales of output, they must be cheaper to operate. But even with the existing known technologies some economies can always be achieved at larger outputs like the economies from further decentralization and improvement in skills. Also, being a large firm it may undertake production of some raw materials or equipments needed in production, and thereby may reduce costs.

Derivation of LAC

The LAC as in the traditional theory is derived from the SATC. Let us assume that the firm faces plant sizes represented by short-run costs – SATC₁, SATC₂, SATC₃ and SATC₄.

Let us further assume that for all SATCs, costs fall as size increases. In business practice, it is customary to consider that a plant is used 'normally' when it operated at a level between $\frac{2}{3}$ rd or, $\frac{3}{4}$ th of capacity. Therefore,

assuming that the typical load factor of each plant is $\frac{2}{3}$ rd of its full capacity,

we draw the LAC curve by joining the points on full capacity of each plant size. This is shown in Figure 5.21. Assuming that there are a very large number of available plant sizes, the LAC will be continuous (Figure 5.22). If there is a minimum optimal scale of plant ($\bar{x}$ in Figure 5.23.), at which all possible scale economies are reaped, then beyond that point the LAC remains constant. As usual if LAC falls continuously, then LMC lies below LAC as in Figure 5.22 and beyond the optimal output level $\bar{x}$, LAC = LMC (see Figure 5.23).

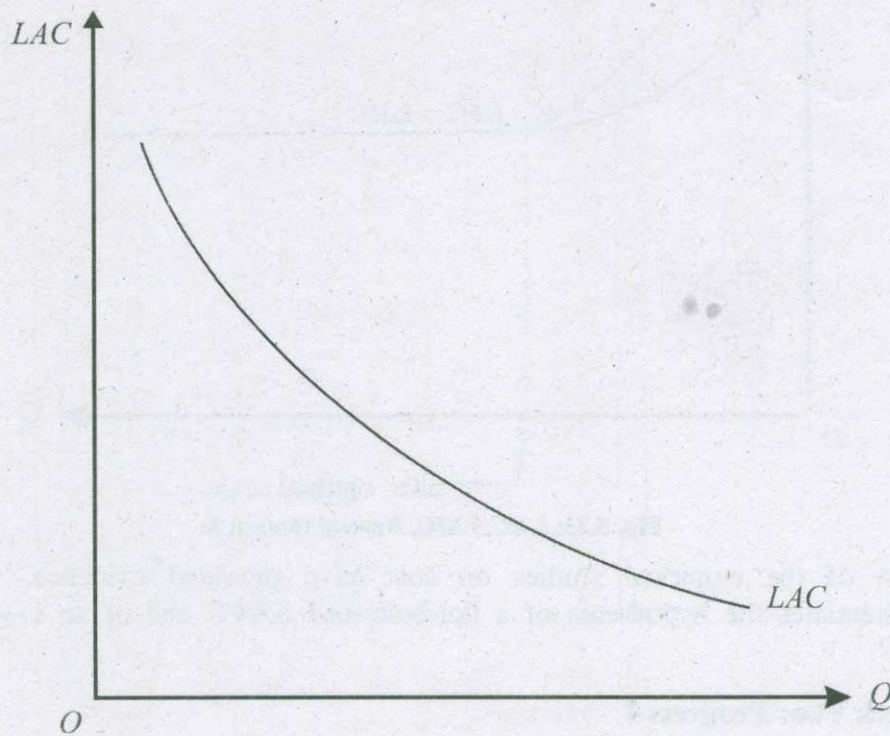
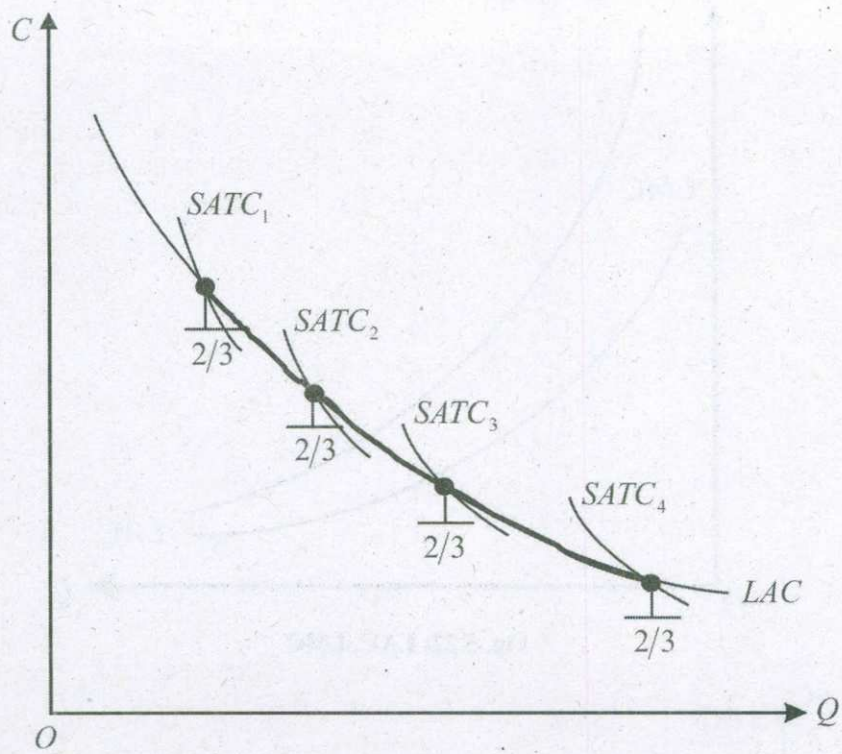


Fig. 5.21: LAC under Modern Theory

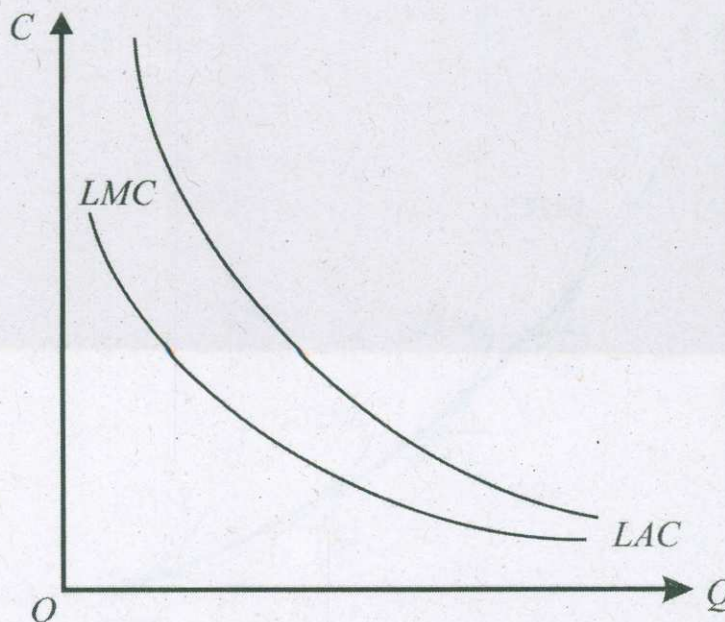


Fig. 5.22: LAC, LMC

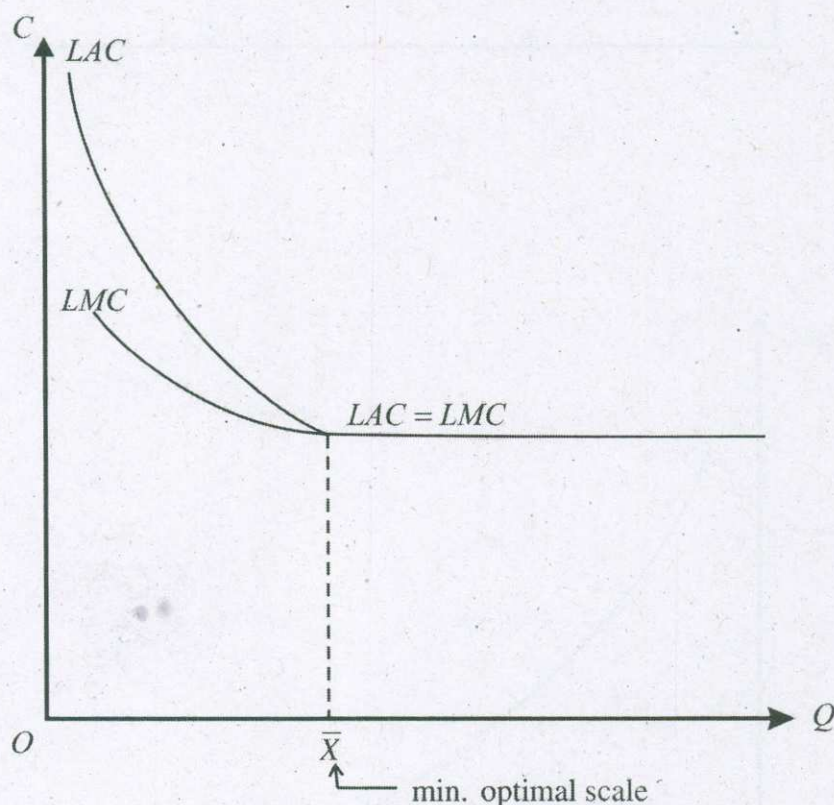


Fig. 5.23: LAC, LMC, Beyond Output $\bar{X}$

Most of the empirical studies on cost have provided evidence, which substantiates the hypotheses of a flat-bottomed SAVC and of an L-shaped LAC.

Check Your Progress 4

- 1) How does the AFC curve in modern cost theory differ (graphically) from that of in traditional cost theory?

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- 2) What is the shape of the SAVC curve in the modern theory?

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- 3) What leads to the flat portion of the SAVC curve?

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- 4) What are the characteristics of the modern LAC?

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5.5 LET US SUM UP

In a production process, costs play an important role. Much of the output to be produced depends on cost conditions. Costs differ with time. Generally, costs are higher if the production occurs in a short period of time due to lack of flexibility in inputs. Therefore, we need to analysis issues for the short-run. This gives rise to concept of short-run cost functions.

In the long-run with greater flexibility, costs conditions are different from that of the short-run. This gives rise to long run cost functions – LAC, LMC, etc. Another important issue associated with the long-run in the concept of “Economies of Scale”, which affects the shape of the LRAC.

Finally, we consider the modern theory of costs, which is more closer to reality. Unlike the traditional theory, which rules out the existence of “reserve capacity”, generally maintained by the firms, the modern theory incorporates it and accordingly, we get somewhat different short-run and long run cost functions.

5.6 KEY WORDS

Average Cost: Cost per unit of output.

Economies of Scale: Factors, which cause to LRAC to decrease as the firms output increases. The firms LRAC is negatively sloped in the presence of scale economies.

Excess Capacity: Difference between the optimum output and the output corresponding to any other point on the AC curve.

Explicit/Accounting Cost: Cost incurred due to the usage of resource in a production process.

Fixed Costs: Costs that the producer has to incur whether she undertakes production with variable inputs or not.

Implicit Cost: It is the opportunity for using the variable inputs in a production process.

Long-Run: A period of time when all inputs are variable.

Marginal Cost: Change in total cost for a unit change in output.

Reserve Capacity: This implies that range of output for which average cost remain same.

Short-Run: A period of time when some inputs remain fixed.

Variable Cost: Cost incurred for using the variable inputs in a production process.

5.7 SOME USEFUL BOOKS

Ferguson and Gould (1989), *Microeconomic Theory*, Irwin Publications in Economics; Homewood, IL: Irwin.

Koutsoyiannis, A. (1979), *Modern Microeconomics*, Second edition, London: Macmillan.

5.8 ANSWER OR HINTS TO CHECK YOUR PROGRESS

Check Your Progress 1

- 1) Short-run is a period when some factors remain fixed; whereas long-run is a period when none remain fixed and all the factors vary.
- 2) In the long-run, the producer can substitute K for L, and thereby choose the optimum K and L that can produce (see Figure 5.24 below).

Check Your Progress 2

- 1) The average cost, $AC = \frac{TC}{Q}$ for any level of output, q, AC is given by the slope of the ray from the origin to any point on the AC curve.

- 2) Yes. This is because of the following:

$$TC = TVC + TFC.$$

$$MC = \frac{\partial(TC)}{\partial Q} = \frac{\partial(TVC + TFC)}{\partial Q}$$

$$= \frac{\partial(TVC)}{\partial Q} + 0 \quad [\because TFC = \text{Constant}]$$

$$\therefore MC = \frac{\partial}{\partial Q} TC = \frac{\partial}{\partial Q} TVC$$

3) $TC = TVC + TFC$

or, $\frac{TC}{Q} = \frac{TVC}{Q} + \frac{TFC}{Q}$

or, $AC = AVC + AFC$

$AC - AVC = AFC$

- 4) It is a rectangular hyperbola. Therefore all along the curve $AFC \cdot Q = TFC$ = some constant.

Check Your Progress 3

- 1) Let us consider SAC_3 , SMC_3 and LAC in Figure 5.15. To the left of the tangency between SAC_3 and LAC , $SAC_3 > LAC$. From that point we arrive at a point where $SAC = LAC$. It implies $SMC < LMC$. Similarly, to the right of the tangency between SAC_3 and LAC , we have $SAC > LAC$. So from a point where $SAC = LAC$, we arrive where $SAC > LAC$, implying $SMC > LMC$.
- 2) No. LMC intersects the $SMCs$.
- 3) By joining the efficient point of operation of all plants we arrive at the $LRAC$. Precisely, $LRAC$ is seen to be tangential to the $SACs$, at some point. On observation it seems as if the $SACs$ are contained in the $LRAC$ curve and also seems as if $LRAC$ envelopes the $SACs$. Therefore, $LRAC$ is also name as the "envelope curve".
- 4) No. Except at the minimum point of the U-shaped $LRAC$, nowhere does the $LRAC$ contain the minimum points of the $SACs$. The efficient point of the $SACs$ does not necessarily give the lowest cost of producing an output given all the plants at the firm's disposal.

Check Your Progress 4

- 1) Graphically, the AFC in traditional theory is a rectangular hyperbola, which is falling throughout and is asymptotic to the two axes. Traditional theory does not allow for usage of reserve capacity.

The modern theory, allows for the usage of reserve capacity, so that even in the short-run capacity can be increased till a certain extent. Accordingly, we get a broken AFC curve as shown in Figure 18.

- 2) The SAVC curve falls up to a certain output level, then flattens out for a particular range of output and then rises beyond.
- 3) Generally firms maintain a 'reserve capacity'. Whereby at the same average variable cost they can produce more output from the given plant (such a plant is planned by the firms from beforehand).
- 4)
 - a) It does not turn up at very large scales of output.
 - b) It becomes flat beyond a certain output level.
 - c) It is not the envelope of the SATCs, but rather intersects them at the level of output defined by the 'typical load factor' of each plant.

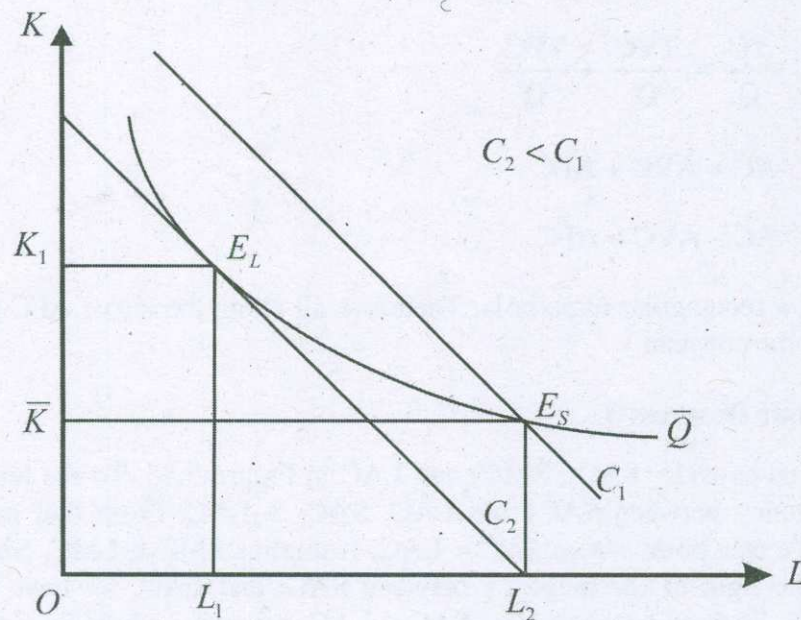


Fig. 5.24

(In the short-run $K = \bar{K}$, so that to produce $\bar{Q}$, more of L has to be employed from the figure we see that equilibrium in the short-run (E_S) occurs at a higher total cost line than E_L (the long-run equilibrium). Therefore, total cost is higher in the short-run. As this will hold for all q and SRTC will always be above LRTC.)

5.9 EXERCISES

- 1) What are the difference between the LAC under traditional theory and that under modern theory?
- 2) Derive the short-run cost function for Cobb-Douglas production function given by:

$$q = AK^\alpha L^\beta, A, \alpha, \beta > 0,$$

where $q \Rightarrow$ quantity of output, $K \Rightarrow$ capital, $L \Rightarrow$ Labour,

[Hint: In the short-run, suppose $K = \bar{K}$]

Show also how the cost curves depend on the parameter β . [Hint: Assume

β to be $\frac{\geq}{<} 1$].

- 3) Let $c = aq^3 + bq^2 + fq + e$ is a short-run cost function. What should be the restrictions on the parameters (a, b, f, e) for a valid cost function?

- 4) Suppose a producer's total cost function is as follows:

$$TC = 300 + 3Q + 0.02Q^2$$

where TC = total cost, Q = units of output produced.

What are the corresponding TFC function, AFC function, TVC function, and AVC function? Plot these curves as well as the MC curve for the first six units of Q .

- 5) If the long-run total cost curve is linear, what do the corresponding average and marginal cost curves look like?
- 6) Examine whether the following statements are true or false. Explain your answer.
- SRATC is never less than LRATC.
 - STMC is never less than LRMC.
 - If the production function exhibits increasing returns to scale everywhere, a firm's LRAC curve must be declining.
 - Is the MC equal to the slope of both TC and TVC curves, in the short-run?

UNIT 6 PRODUCTION ECONOMICS

Structure

- 6.0 Objectives
- 6.1 Introduction
- 6.2 Production Functions
 - 6.2.1 Technical Progress
 - 6.2.2 Classifying Technical Progress
- 6.3 Cost Functions
- 6.4 Profit Maximisation
 - 6.4.1 Profit Maximisation and Input Demand
 - 6.4.2 Supply Function
 - 6.4.3 Hotelling Lemma
 - 6.4.4 The Envelope Theorem
- 6.5 Cost Minimisation and Profit
 - 6.5.1 Cost-Minimising Inputs
 - 6.5.2 Cost-Minimising Factor Demand Functions
 - 6.5.3 Dual Problem: Output Vs Cost in Profit Maximisation
- 6.6 Let Us Sum Up
- 6.7 Key Words
- 6.8 Some Useful Reference
- 6.9 Answer or Hints to Check Your Progress
- 6.10 Exercises

6.0 OBJECTIVES

After going through this unit, you will be able to:

- understand the important theoretical insights in looking at production process empirically;
- formulate cost and production functions for purpose of estimation;
- appreciate the idea of technical process; and
- derive profit maximisation and cost minimisation conditions as a part of dual problem.

6.1 INTRODUCTION

An examination of production processes of a firm requires understanding of the relationship between input and output specified by the technology along with cost conditions that would decide the feasibility of a production plan. Therefore, it is necessary to understand the empirical formulation of production functions and relation between the conditions of profit maximisation and cost minimisation. In view of this, the following discussion focuses on (i) specifications of production and cost functions, (ii) conditions of profit maximisation and (iii) cost minimising input demands that satisfy the profit maximisation requirements. In the process, some important issues related to technical progress, constrained optimisation and duality between cost minimisation and output maximisation would be looked into.

6.2 PRODUCTION FUNCTIONS

In this section, we present four simple production functions that are widely used in empirical estimation. In each case, the discussion aims at depicting the features of input-output functional relation, returns scale envisaged and scope for factor substitution. Two-input cases have been considered in each production function although generalisation to many inputs can easily be accomplished and we will present these separately.

i) Linear Production Function

Suppose that the production function is given by:

$$q = f(K, L) = aK + bL. \quad \dots (6.0)$$

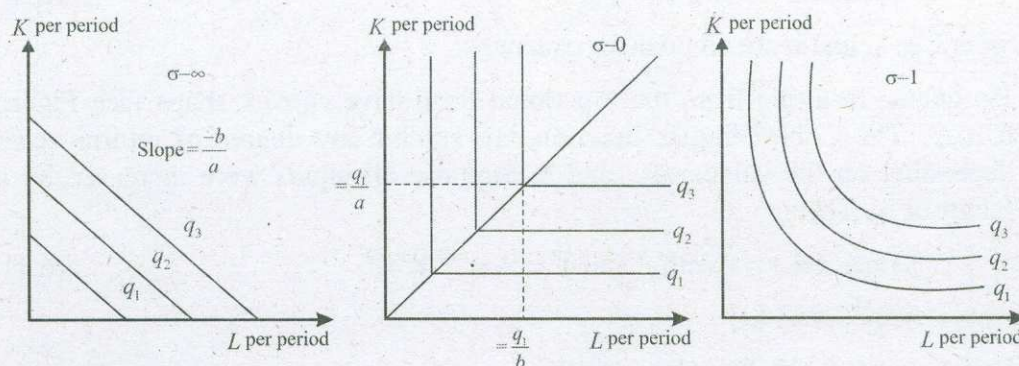


Fig. 6.1: Iso-quant Maps for Production Functions

The production function exhibits constant returns to scale. It can be seen that for any $m > 0$,

$$f(mK, mL) = amK + bmL = m(aK + bL) = mf(K, L) \quad \dots (6.1)$$

The iso-quants of the production function are parallel straight lines with slope $(-b/a)$ (see Figure 6.1(a)). Since along any straight-line iso-quant, the RTS is constant, the denominator of the elasticity of factor substitution σ is equal to 0, and hence σ is equal to infinity. The feature of infinity factor substitution possibility indicates that you can substitute labour for capital or capital for labour without any limit. Such flexibility makes the functional formulation counter intuitive. Perhaps you will appreciate that we require both of capital and labour for any production and cannot think of ignoring either irrespective of their prices.

ii) Fixed Proportions Production Function

The fixed-proportions production function is given by:

$$q = \min(aK, bL) \quad a, b > 0 \quad \dots (6.2)$$

where the operator “min” indicates that q is given by the smaller of the two values in parentheses. See that in this formulation capital and labour are used in a fixed ratio. Consequently, its iso-quants are L-shaped (see Figure 6.1(b)).

A firm characterised by this production function will always operate along the ray where the ratio K/L is fixed at b/a and any point of operation other than at the vertex of the iso-quants would be inefficient. Because K/L is a constant, it is easy to see from the definition of the elasticity of substitution that σ must equal 0.

With the form reflected by Equation 6.2, the fixed proportion production function exhibits constant returns to scale since

$$f(mK, mL) = \min(amK, bmL) = (m)\min(aK + bL) = mf(K, L)$$

for any $m > 0$, increasing or decreasing returns can be easily incorporated into the function by using a nonlinear transformation of the functional form.

iii) Cobb-Douglas Production Function

The Cobb-Douglas production function is given by:

$$q = f(K, L) = AK^a L^b, \quad \dots (6.3)$$

where A , a , and b are all positive constants.

Iso-quants resulting from the functional form have convex shape (see Figure 6.1(c)). The Cobb-Douglas function can exhibit any degree of returns scale depending on the values of a and b . Suppose all inputs were increased by a factor of m . Then,

$$\begin{aligned} f(mK, mL) &= A(mK)^a (mL)^b = Am^{a+b} K^a L^b \\ &= m^{a+b} f(K, L). \end{aligned} \quad \dots (6.4)$$

Hence, in the Cobb-Douglas function

- $a + b = 1$ implies constant returns to scale
- $a + b > 1$ implies increasing returns to scale
- $a + b < 1$ implies decreasing returns-to-scale case.

To determine the elasticity of substitution in Cobb-Douglas production

function, let us use Allen's definition $\sigma = \frac{\frac{\partial q}{\partial L} \cdot \left(\frac{\partial q}{\partial K} \right)}{q \cdot \left(\frac{\partial^2 q}{\partial L \cdot \partial K} \right)}$.

When $q = AK^a L^{1-a}$,

$$\sigma = \frac{(1-a) \left(\frac{q}{L} \right) \cdot a \left(\frac{q}{K} \right)}{\frac{q^2 (1-a)(a)}{KL}} = 1$$

Note that the Cobb-Douglas function is linear in logarithms, i.e.,

$$\ln q = \ln A + a \ln K + b \ln L. \quad \dots (6.5)$$

As a result, the constant a in (eqn. 6.5) is the elasticity of output with respect to capital input, and b is the elasticity of output with respect to labour input. To get the result, take

$$e_{q,k} = \frac{\partial q}{\partial K} \cdot \frac{K}{q} = \frac{\partial \ln q}{\partial \ln K} \text{ such that}$$

$e_{q,k} = a$ from Equation 6.5. Similarly, we get $e_{q,L} = b$.

iv) CES Production Function

The constant elasticity of substitution (CES) production function is given by

$$q = \gamma [\delta K^\rho + (1 - \delta)L^\rho]^{\varepsilon/\rho} \quad \dots (6.6)$$

where $\gamma > 0$, $0 \leq \delta \leq 1$, $\rho \leq 1$ and $\varepsilon \geq 0$.

It is important to note that

γ shifts the production function and is called the efficiency parameter;

δ allows K and L to vary and is called a distribution parameter and

ρ is the substitution parameter. In case of higher the elasticity of substitution, ρ is equal to its maximum value of 1. It can be shown that for the constant returns-to-scale case

$$\sigma = \frac{1}{1 - \rho}$$

so that CES function incorporates the linear, fixed-proportions and Cobb-Douglas functions as special cases (for $\rho = 1$, $-\infty$, and 0 respectively). In particular, for $\rho = 0$, CES production function approaches a Cobb-Douglas function of the form

$$q = \gamma K^\delta L^{1-\delta}$$

Further, it explains the use of the term distribution parameter for the parameter δ , since the exponents of the Cobb-Douglas function with constant returns to scale equal competitively determined factor income shares.

It can be shown that CES function exhibits increasing, constant or decreasing returns to scale depending on the parameter $\varepsilon > 1$, $\varepsilon = 1$, or $\varepsilon < 1$:

$$\begin{aligned} f(mK, mL) &= \gamma [\delta(mK)^\rho + (1 - \delta)mL^\rho]^{\varepsilon/\rho} \\ &= \gamma [m^\rho]^{1/\rho} (\delta K^\rho + (1 - \delta)L^\rho)^{\varepsilon/\rho} \\ &= m^\varepsilon f(K, L) \end{aligned} \quad \dots (6.7)$$

for any $m > 0$.

Many-Input Cases

As we have stated earlier, the two-input cases of production functions can be generalized to many-input ones. In the following, we present Cobb-Douglas and CES formulations along with Generalized Leontif and Translog production functions. In all of these examples, β 's are nonnegative parameters, and the n inputs are represented by $X_1 \dots X_n$.

Cobb-Douglas

A many-input Cobb-Douglas production function is given by:

$q = \prod_{i=1}^n X_i^{\beta_i}$ where X_i for $i = 1, 2, \dots, n$ represents input and parameter β_i , the non-negative input coefficient.

The function would exhibit constant returns to scale if $\sum_{i=1}^n \beta_i = 1$. Any degree of increasing returns to scale can be incorporated into this function depending on $\sum_{i=1}^n \beta_i$.

CES

The many-input CES production function is given by

$$q = [\sum \beta_i X_i^\rho]^{1/\rho} \quad \rho \leq 1.$$

If $\rho = 1$, the function is under constant returns to scale while for $\rho > 1$ it would be under increasing returns to scale.

Generalised Leontief

The many-input Leontief production function is given by

$$q = \sum_{i=1}^n \sum_{j=1}^n \beta_{ij} \sqrt{X_i X_j} \quad \beta_{ij} = \beta_{ji}.$$

We need to impose the symmetry condition of $\beta_{ij} = \beta_{ji}$ to ensure the fulfillment of second-order partial derivatives.

The function exhibits constant returns to scale and increasing returns to scale can be incorporated into the function by using the transformation

$$q' = q^\epsilon, \epsilon > 1.$$

Translog

The translog production function is written as

$$\log q = \beta_0 + \sum_{i=1}^n \beta_i \ln X_i + 0.5 \sum_{i=1}^n \sum_{j=1}^n \beta_{ij} \ln X_i \ln X_j \quad \text{with } \beta_{ij} = \beta_{ji} \dots (6.8)$$

The condition is $\beta_{ij} = \beta_{ji}$ is required to assure the equality of cross-partial derivatives.

The important features of the function are

- flexibility of capturing any degree of returns to scale. If $\sum_{i=1}^n \beta_i = 1$ and $\sum_{i=1}^n \beta_{ij} = 0$ for all i , it exhibits constant returns to scale
- depiction of Cobb-Douglas function as a special case if $\beta_0 = \beta_{ij} = 0$ for all i, j, n

6.2.1 Technical Progress

Using the concept of production function, it is possible to assess improvement in the methods of production over time. In other words, we can examine if the same level of output is produced with less input or higher levels of output are obtained with the same level of inputs. Essentially our concern is to derive the

change in output per unit of input within the framework of production function over time. The possible variations in the production that may be observed are called technical progress.

Note that the technical progress attempts to evaluate the rate of growth of output over time, which might have exceeded the growth rate that can be attributed to the growth in conventionally defined inputs. To evaluate such a feature, the production function is written in a modified form as follows:

$$q = A(t) f(K, L) \quad \dots (6.9)$$

Look at the term $A(t)$ in the above function. It represents influences that go into the determination of q besides the inputs K and L . Changes in A over time represent technical progress.

Differentiating Equation 6.9 with respect to time gives:

$$\begin{aligned} \frac{dq}{dt} &= \frac{dA}{dt} \cdot f(K, L) + A \cdot \frac{df(K, L)}{dt} \\ &= \frac{dA}{dt} \cdot \frac{q}{A} + \frac{q}{f(K, L)} \left[\frac{\partial f}{\partial K} \cdot \frac{dK}{dt} + \frac{\partial f}{\partial L} \cdot \frac{dL}{dt} \right] \end{aligned} \quad \dots (6.10)$$

Dividing by q , we get

$$\frac{dq/dt}{q} = \frac{dA/dt}{A} + \frac{\partial f / \partial K}{f(K, L)} \cdot \frac{dK}{dt} + \frac{\partial f / \partial L}{f(K, L)} \cdot \frac{dL}{dt} \quad \dots (6.11)$$

$$\text{or, } \frac{dq/dt}{q} = \frac{dA/dt}{A} + \frac{\partial f}{\partial K} \cdot \frac{K}{f(K, L)} \cdot \frac{dK/dt}{K} + \frac{\partial f}{\partial L} \cdot \frac{L}{f(K, L)} \cdot \frac{dL/dt}{L}$$

For any variable x , $(dx/dt)/x$ is the rate of growth of x per unit of time (G_x). We write Equation 6.11 as

$$G_q = G_A + \frac{\partial f}{\partial K} \cdot \frac{K}{f(K, L)} \cdot G_K + \frac{\partial f}{\partial L} \cdot \frac{L}{f(K, L)} \cdot G_L \quad \dots (6.12)$$

Since $\frac{\partial f}{\partial K} \cdot \frac{K}{f(K, L)} = \frac{\partial q}{\partial K} \cdot \frac{K}{q}$ = elasticity of output with respect to capital

input

= $e_{q,K}$

and

$\frac{\partial f}{\partial L} \cdot \frac{L}{f(K, L)} = \frac{\partial q}{\partial L} \cdot \frac{L}{q}$ = elasticity of output with respect to labour input

= $e_{q,L}$.

Thus, Equation 6.11 can be shown as

$$G_q = G_A + e_{q,K} G_K + e_{q,L} G_L \quad \dots (6.13)$$

It is not difficult to see that the rate of growth in output is decomposed into the sum of two components: growth attributed to changes in inputs (K and L) and other "residual" growth due to technical progress clubbed together to changes in A .

6.2.2 Classifying Technical Progress

When technical progress enters the production, we get more output from a given combination of inputs. With the on set of such a process, the technical change factor $A(t)$ can have three possible effects, viz.,

- 1) *Neutral technical progress*: The change affects all the inputs equally.
- 2) *Capital augmenting technical progress*: The change affects only capital.
- 3) *Labour augmenting technical progress*: The change affects only labour.

Check Your Progress 1

- 1) What is the basic difference between Cobb-Douglas and CES production functions?

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- 2) How do you determine the returns to scale of a Cobb-Douglas production function?

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- 3) How would you measure technical progress?

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6.3 COST FUNCTIONS

The total cost function depicts the functional relationship set of input costs for any output level. We present three important cost functions often used in empirical estimation. The following discussion starts with two-input case of Cobb-Douglas and CES; generalizes these to many-input framework and adds the translog cost function.

Cobb-Douglas

The Cobb-Douglas cost function for two inputs (K and L) is given by

$$TC = (q)(r^\alpha w^\beta) \text{ where } \alpha + \beta = 1.$$

The elasticity of substitution between K and L of the function is measured by

$$s = \frac{\partial(K/L)}{\partial(w/r)} \cdot \frac{(w/r)}{(K/L)} = \frac{\partial \ln(K/L)}{\partial \ln(w/r)}$$

CES

The CES cost function for two inputs is given by:

$$TC = q[\delta^\sigma r^{1-\sigma} + (1-\delta)^\sigma w^{1-\sigma}]^{1/(1-\sigma)}.$$

The elasticity of substitution of the function is the same and constant for all values of K and L as found for its production function.

Translog

The translog cost function is given as

$$\ln TC = \ln q + \beta_0 + \beta_1 \ln w + (1 - \beta_1) \ln r + \beta_2 (\ln w)^2 + \beta_3 (\ln r)^2 + \beta_4 \ln w \ln r.$$

See that it is formulated from an approximation to any arbitrary cost function. It reduces to Cobb-Douglas cost function if $\beta_2 = \beta_3 = \beta_4 = 0$ and homogeneous of degree 1 in w and r, with $\beta_2 + \beta_3 + \beta_4 = 0$.

Cost Functions with Many-Inputs

The three cost functions above can be generalized to many inputs ($X_1, \dots, X_n$) with prices $w_1, \dots, w_n$.

i) **Cobb-Douglas:** $TC = q \prod_{i=1}^n w_i^{\beta_i}$ where $\sum_{i=1}^n \beta_i = 1$

ii) **CES:** $TC = q \left[\sum_{i=1}^n \beta_i w_i^{1-\sigma} \right]^{1/(1-\sigma)}$

iii) **Translog:** $\ln TC = \ln q + \beta_0 + \sum_{i=1}^n \beta_i \ln w_i + 0.5 \sum_i \sum_j \beta_{ij} \ln w_i \ln w_j$

where $\sum \beta_i = 1$ and $\beta_{ij} = \beta_{ji}$.

The partial elasticity of substitution (s_{ij}) is computed from this cost function as

$$s_{ij} = \frac{TC \cdot TC_{ij}}{TC_i \cdot TC_j}$$

The advantage of the above formulation of s_{ij} is that for the Cobb-Douglas cost function $s_{ij} = 1$ and for that of CES, $s_{ij} = \sigma$ a constant.

a) cost shares for the many-input translog cost function are given by

$$s_i = \frac{w_i X_i}{TC} = \beta_i + \sum_{j=1}^n \beta_{ij} \ln w_j.$$

b) For the translog case, the partial elasticity of substitution is given by

$$s_{ij} = (\beta_{ij} + s_i s_j) / s_i s_j.$$

Check Your Progress 2

- 1) A cost function is given by $TC = .2 q w^{-5} r^{-5}$. Name which cost function it describes. Why do you think so?

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- 2) Write a CES cost function. Find its elasticity of substitution.

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- 3) Write a translog cost function and show the condition in which it will be homogenous of degree 1.

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6.4 PROFIT MAXIMISATION

A profit-maximising firm chooses both its inputs and output such that it reaps maximum economic profits. In order to see the underlying process, let us start with profit as the difference between revenue earned from selling output and costs incurred in the process of its production.

$$\begin{aligned} \text{So, } \pi &= TR(q) - TC(q), \\ &= p(q)q - TC(q). \end{aligned}$$

To maximise π , we differentiate with respect to q so that $MR = MC$.

The second order condition requires that

$$\begin{aligned} \frac{d^2\pi}{dq^2} \Big|_{q=q^*} \\ = \frac{d\pi(q)}{dq} \Big|_{q=q^*} < 0 \end{aligned}$$

i.e., marginal profit must be decreasing at the optimal level of q .

6.4.1 Profit Maximisation and Input Demand

Until now we have considered firm's decision to maximise profit from a given output level. Since output level itself is determined by the inputs chosen through a production function, say $q = f(K, L)$, it seems appropriate to include inputs in the profit maximisation. So, we write

$$\begin{aligned}\pi(K, L) &= pq - TC(q) \\ &= pf(K, L) - (rK + wL).\end{aligned}$$

The profit maximisation exercise, therefore, is firm's decision problem of selecting appropriate levels of capital and labour inputs. Let us assume that the producer is a price taker so that the output and input prices are parameters in the maximisation problem.

The first order conditions (FOCs) of the maximisation problem are:

$$\frac{\partial \pi}{\partial K} = p \frac{\partial f}{\partial K} - r = 0$$

$$\frac{\partial \pi}{\partial L} = p \frac{\partial f}{\partial L} - w = 0$$

Solving these two equations, we get

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

The second order conditions of the maximisation need to satisfy such that:

$$\pi_{KK} < 0, \pi_{LL} < 0 \text{ and } \pi_{KK}\pi_{LL} - \pi_{KL}^2 > 0$$

6.4.2 Supply Function

The supply function shows the quantity of output produced by the firm at various prices for its output as well as input costs. Thus, from the first order condition of profit maximisation, we solve for optimal inputs functions of the parameter p , r and w . If

$$K^* = K^*(p, r, w) \text{ and}$$

$L^* = L^*(p, r, w)$, these inputs can be included in the production $q = f(K, L)$ to obtain the profit-maximising output (q^*) to be chosen. So,

$$\begin{aligned}q^* &= f(K^*, L^*) \\ &= f[K^*(p, r, w), L^*(p, r, w)] \\ &= q^*(p, r, w)\end{aligned}$$

Thus, the profit function shows that firm's profit depends on market prices, p , r and w . We will derive the supply function for the firm, given the profit function. To understand the underlying reasoning, consider the case of a single output produced through a single input. In this case

$$\pi = pf(x) - wx, \text{ the FOC is}$$

$$\frac{d\pi}{dx} = p \frac{df(x)}{dx} - w = 0 \quad \dots (6.14)$$

The factor demand function $x(p, w)$ must satisfy the FOC. Therefore, we write the profit function as

$$\pi(p, w) = pf(x(p, w)) - wx(p, w)$$

Differentiating the profit function with respect to w , we have

$$\begin{aligned}\frac{\partial \pi}{\partial w} &= p \frac{\partial f(x(p, w))}{\partial x} \frac{\partial x}{\partial w} - w \frac{\partial x}{\partial w} - x(p, w) \\ &= \left[p \frac{\partial f(x(p, w))}{\partial x} - w \right] \frac{\partial x}{\partial w} - x(p, w)\end{aligned}$$

Substituting from (1), we derive

$$\frac{\partial \pi}{\partial w} = -x(p, w) \quad \dots (6.15)$$

The minus sign indicates that increasing price of an input leads to decrease in profit.

Return to the equation above and see that change in the input price has two part-effects on the changed profit. First, there is a direct effect of increasing profit because of the price increase even when the firm produces the same level of output. Secondly, there is an indirect effect induced by the increased output price; the firm moves to change its level of output by a small amount. However, the change in profits due to an infinitesimal change in output must be zero since production is already in the profit-maximising level.

6.4.3 Hotelling Lemma

Equation (6.15) can be related to the method used in the Hotelling Lemma to derive firm's net supply function from the profit function by differentiating it.

Let $y_i(p)$ be the firm's net supply function for good i . Then

$$y_i(p) = \frac{\partial \pi(P)}{\partial p_i} \text{ for } i = 1, 2, \dots, n$$

assuming that the derivative exists and that $p_i > 0$.

6.4.4 The Envelope Theorem

The envelope theorem shows the change in the optimal value of a particular function when a parameter of the function changes. In the context of our present discussion, we would like to see the change in profit function due to change in input prices. The envelope theorem provides a short route to solve the problem. It states that the change in the optimal value of a function $f(x, a)$ with respect to a parameter (a) of that function can be derived by partially differentiating the objective function while holding the optimal value of x constant. To see the underlying process, take $f(x, a)$ as the function of both x and a .

Let a be the parameter in the function above. We choose x to maximise the function. However, for each value of a , there will be a different optimal choice of x . Therefore, in sufficiently regular cases, it will be possible to write the function $x(a)$, which gives us the optimal choice of x for each different value of a .

There are two equivalent ways to solve the optimisation problem, provided by the envelope theorem. First, define a value function $M(a) = f(x(a), a)$ which gives the optimal value of f for different choices of a .

So with optimised value of x , we write

$$M(a) \equiv f(x(a), a) \quad \dots (6.16)$$

Differentiating both the sides of the identity (6.16), we have

$$\frac{dM(a)}{da} = \frac{\partial f(x(a), a)}{\partial x} \cdot \frac{\partial x(a)}{\partial a} + \frac{\partial f(x(a), a)}{\partial a}$$

Since $x(a)$ is the choice of x that maximises f , we know that

$$\frac{\partial f(x(a), a)}{\partial x} = 0.$$

$$\text{So } \frac{dM(a)}{da} = \frac{\partial f(x(a), a)}{\partial a}$$

$$\text{or, } \frac{dM(a)}{da} = \frac{\partial f(x, a)}{\partial a} \Big|_{x=x(a)}$$

which shows that the derivative is taken holding x fixed at the optimal value $x(a)$.

Note that the envelope theorem states: the total derivative of the value function with respect to the parameter is equal to the partial derivative when the derivative is evaluated at the optimal choice. This result is based on the idea that there would be two effects direct and indirect of change in a .

The change in a directly affects f . Again the change in a affects x which in turn affects f . Since x is chosen optimally, a small change in x has a zero effect on f . So the indirect effect is eliminated and the direct effect remains.

Example:

$$\text{Let } f(x, a) = \log x - ax \quad \dots (6.17)$$

To get the optimal value of x , differentiate, $f(x, a)$ with respect to x , so that

$$f'(x, a) = \frac{1}{x} - a$$

$$\text{For maximum, } \frac{1}{x} - a = 0$$

$$\text{or, } \frac{1}{x} = a$$

$$\text{or, } x = \frac{1}{a}$$

Thus, the optimal value of x , say, x^* is $\frac{1}{a}$.

Using this result write the value function

$$\begin{aligned} M(a) &= f(x(a), a) \\ &= \log\left(\frac{1}{a}\right) - a \cdot \frac{1}{a} \\ &= \log(a) - 1 \end{aligned}$$

$$\text{Now, } \frac{dM(a)}{da} = -\frac{1}{a}$$

In the second method, take

$f(x, a) = \log x - ax$ and get

$$\frac{df(x, a)}{da} = -x$$

Setting x to its optimal value, which is $\frac{1}{a}$, obtain

$$\frac{df(x, a)}{da} = -\frac{1}{a}$$

Let us return to our profit maximisation problem with one-input and one-output and apply the result of envelope theorem.

The profit maximisation problem is:

$$\pi(p, w) = pf(x) - wx.$$

The parameter a in the envelope theorem we saw can be extended to either p or w in the profit function above. So

$$\frac{\partial \pi(p, w)}{\partial p} = f(x) \Big|_{x=x(p, w)}$$

$= f(x(p, w))$, which gives the profit maximising, supply of the firm at prices (p, w) . Similarly,

$$\frac{\partial \pi(p, w)}{\partial w} = -x \Big|_{x=x(p, w)}$$

gives the profit maximising net supply of the factor x .

Check Your Progress 3

- 1) Take an example and solve it according to the method of envelope theorem.

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- 2) Describe Hotelling Lemma.

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- 3) What is the first order condition for profit maximisation?

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6.5 COST MINIMISATION AND PROFIT

Continuing with two inputs, capital and labour, the total cost of the firm can be written as

$TC = wL + rK$ where L and K represent labour and capital with prices w and r . Assuming that the firm produces one output, its total revenue is given by price (p) of its product and quantity of output (q), produced by the production function $q = f(K, L)$. Then profit (π) of the firm is given by the difference between total revenue (TR) and total costs (TC) for an output level q^0 .

Thus, $\pi = TR - TC = pq - wL - rK$.

6.5.1 Cost-Minimising Inputs

The objective of a firm if it behaves rationally is to maximise profit. Therefore, cost of production has to be minimised by selecting appropriate inputs and without violating the requirements of production function. In the process, the firm has to solve a constrained cost minimisation problem. In other words, we seek to minimise total costs, given $q = f(K, L) = q^0$. Setting up the Lagrangian format

$$\ell = wL + rK - \lambda [q^0 - f(K, L)]$$

the first order conditions for a constrained minimum are:

$$\frac{\partial \ell}{\partial L} = w - \lambda \frac{\partial f}{\partial L} = 0$$

$$\frac{\partial \ell}{\partial K} = r - \lambda \frac{\partial f}{\partial K} = 0$$

$$\frac{\partial \ell}{\partial \lambda} = q^0 - f(K, L) = 0$$

For a solution, divide the first two equations to get

$$\frac{w}{r} = \frac{\partial f / \partial L}{\partial f / \partial K} = \text{RTS (L for K), i.e.,}$$

the cost-minimising firm should equate the RTS for the two inputs to the ratio of their prices.

6.5.2 Cost-Minimising Factor Demand Functions

From the FOC we can get the *cost-minimizing factor demand functions* or *conditional factor demand functions*. To derive these, let us consider from the three equations of FOC above K , L and λ as three unknowns. Given any parameter values for r , w , and q , we can solve these equations for the unknowns. It may be necessary to note that these functions are different from the factor demand functions derived from the profit maximisation problem where output was produced keeping total cost fixed.

Shephard's Lemma

There is an alternative method of deriving the input demand from the cost minimisation objective of a firm. The compensated demand functions can be computed directly from the expenditure function for inputs by partial differentiation of the expenditure with respect to input prices, which is known as the Shephard's Lemma.

Since output is held constant in the cost minimisation problem and the firm is a price taker, so that r and w are given; from the constrained optimisation problem

$\ell = wL + rK - \lambda [q^0 - f(K, L)]$, we get from the method offered by envelope theorem,

$$\partial \ell / \partial w = L \text{ and}$$

$$\partial \ell / \partial r = K.$$

Again, the total cost function yields

$$\partial(TC)/\partial w = L \text{ and } \partial(TC)/\partial r = K. \text{ Therefore,}$$

$$\partial(TC)/\partial w = \partial \ell / \partial w = L \text{ and } \partial(TC)/\partial r = \partial \ell / \partial r = K.$$

The input demand functions are also constant output demand functions.

6.5.3 Dual Problem: Output Vs Cost in Profit Maximisation

Output Maximisation

The result discussed above can also be obtained by considering the dual formulation of firm's primal cost-minimisation problem, that is, it can maximise the level of output for a given total cost of inputs (TC_1). Mathematically, the Lagrangian expression for this problem is

$$\ell = f(K, L) - \lambda (wL + rK - TC_1)$$

Differentiating with respect to K and L for output maximisation, the necessary FOCs can be derived as under:

$$MP_K - \lambda r = 0$$

$$MP_L - \lambda w = 0$$

$$wL + rK - TC_1 = 0$$

By solving the first two equations we get

$MP_K / r = MP_L / w$, which is identical to the condition that was necessary for cost minimisation.

Cost Minimisation and Profit Maximisation Duality

If we compare the FOCs for the profit-maximisation problem with the FOCs for the cost minimisation problem, we can see that they will give the same solution values for K and L if the value of the Lagrange multiplier is $\lambda = p$. It is not difficult to get this condition in the presence of competitive market conditions as in the equilibrium there is $MC = p$. Consequently, λ can be related to MC through cost optimising conditions.

Check Your Progress 4

- 1) What is the first order condition of cost minimising output?

- 2) How would you establish the duality between cost minimisation and output maximisation of a firm?

- 3) Derive the Shephard's Lemma from a cost equation.

6.6 LET US SUM UP

In this unit, we have discussed the production related issues delineated by microeconomic theory. The important types of production functions usually employed by empirical studies have been pointed out by considering two- and multi-input cases. The scope of substitution between inputs and returns to scale in each production function has been highlighted. The technical change over time, which is responsible for producing more output with a given set of inputs

or yielding the same level of output with fewer inputs, is pointed out. Some of the important cost functions estimated in empirical studies are discussed after the production functions. The profit maximising conditions, which highlight the MR and MC equality are derived as a constrained optimisation exercise. Since profit maximisation has a bearing on the choice of input demand through cost minimisation for a given level of output, we have discussed the conditions of compensated input demand function. The use of results offered by Envelope Theorem, Hotelling Lemma and Shephard's Lemma in choosing the optimal input demand, deriving supply functions from profit functions and input demand from cost functions has been pointed out. The last part of the unit is devoted to establish the duality between cost minimisation and profit maximising output decisions.

6.7 KEY WORDS

Cobb- Douglas production Function: A production function of the form $Q = aL^b K^c$ where c are constants, Q is output, and K are the Quantities of input employed.

Compensated Demand Curve: Curve showing relationship between the price of a good and the quantity consumed while holding real income (or utility) constant. denoted by $h(P^x, P^y, U)$.

Duality: The relationship between any constrained maximisation problem and its related "dual" constrained minimisation problem.

Envelope Theorem: a mathematical result: the change in the maximum value of a function brought about by a change in a parameter of the function can be found by partially differentiating the function with respect to the parameter (when all other variables take on their optimal values).

Fixed Proportions production Process: A production process characterised by a substitution right angles.

Homogeneous Production Function: A special case of homothetic production function where a proportionate change in input causes output to change by a proportion which does not vary with changes in the input bundle.

Input Demand Function: Function showing the firm's demand for an input (say, labor) that depends on put costs (w, v) and on the level of output (a):

$$L = L(w, v, q).$$

Marginal Cost: The cost of an additional unit of output. Graphically, marginal cost is the slope of the total cost curve.

Marginal Rate of Substitution: The rate at which one commodity can be substituted for another without changing total utility. The marginal rate of substitution or x or y is equal to the absolute value of the slope of the indifference curve.

Marginal Rate of Technical Substitution: The rate at which one input can be substituted for another input in the production process without affective total output. Graphically, the marginal rate of technical substitution is equal to the absolute value of the slope of the isoquant. Mathematically, it is equal to the ratio of marginal products for the two inputs.

Marginal Revenue Product: The change in total revenue resulting from a one-unit increase in input usage, ceteris paribus.

Profit Function: The relationship between a firm's maximum profits (π^*) and the output and input prices it faces:

$$\pi^* = \pi^*(P, v, w).$$

6.8 SOME USEFUL REFERENCES

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Varian, Hal (1992), *Microeconomic Analysis*, W.W. Norton & Company, Inc., New York.

6.9 ANSWER OR HINTS TO CHECK YOUR PROGRESS

Check Your Progress 1

- 1) Elasticity of substitution is one in C-D and constant in CES
- 2) Take the powers of the inputs and add.
- 3) Insert a term with time, say $A(t)$, in the production function.

Check Your Progress 2

Hint Q1) C-D. Consider its elasticity of substitution.

Hint Q2) Same as in CES production function

Q3) Sum of coefficients associated with the square and interactive terms of input prices = 0

Check Your Progress 3

- 1) Do Yourself
- 2) Do Yourself
- 3) $\frac{MP_L}{MP_K} = \frac{w}{r}$

Check Your Progress 4

- 1) $\frac{w}{r} = \frac{\partial f / \partial L}{\partial f / \partial K} = \text{RTS (L for K)}.$
- 2) The order conditions of both would be the same.
- 3) Do yourself by reading Sub-section 6.5.2.

6.10 EXERCISES

- 1) A BPO firm in Delhi operates with a production function

$f = 10e^{.05t}k^{.5}L^{.5}$. It succeeds in increasing its production by 5% Rs. time period t_0 using the same level of inputs. obtain the type of technical change this firm experiences.

Hints: In (-1) the native technical change is indistinguishable.

- 2) You are given a production function of a firm represented by

$$q = \beta_0 + \beta_1 (KL)^5 + \beta_2 K + \beta_3 L \text{ with } 0 : \beta_i : 1 \text{ and } i=1, 2, 3.$$

If this function is supposed to be experiencing constant returns to scale, what restriction you should impose on the parameters $\beta_0, \dots, \beta_3$?

Use Allen's formula for elasticity of substitution to calculate c . Is c constant?

(Hints $\beta_0 = 0$, c is not constant)

- 3) The total cost function of a firm is given as

$$TC = [.5r + (rw)^5 + .5w]f$$

- a) Get the output demand function for K and L using shepherd's Lemma.

- b) Derive the underlying production function.

Hint:
$$L = .5f \left[1 + \left(\frac{r}{w} \right)^5 \right]$$

$$K = .5f \left[1 + \left(\frac{w}{r} \right)^5 \right]$$

$$f = [.5K + .5L]^{-1}$$

- 4) The production function of a firm producing polished diamond products is given by $f=2L^{.5}$. The firm is a price taker in the market.

Derive the supply function of the product. If the prices of output and labour are P and W. Show that such a function is homogenous of degree zero in P and W

Hints: $f = \frac{2P}{W}$

NOTES

It will be useful to see that, TC and TVC (in the short-run) have the same slope at each level of output. TC is simply TVC displaced upward by the constant amount of TFC. This is shown in Figure 5.11. Since the slopes are the same, MC is given by the slope of either curve.

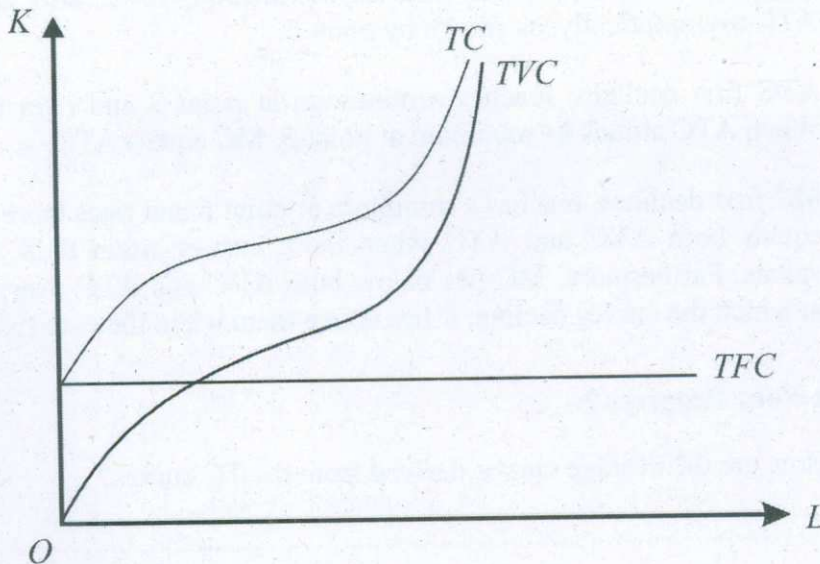


Fig. 5.11: TC, TVC, TFC

In panel A of Figure 5.8, the slope of the ray OS gives the minimum point on AVC, which is also tangent to TVC. Hence, it gives MC at this point. Thus, $MC=AVC$, when the latter attains its minimum value.

Similarly, in panel A of Figure 5.10, the slope of the ray ON gives the minimum ATC. The ray being tangent to TC, its slope gives MC. Consequently, $MC=ATC$, when the latter attains its minimum value.

The properties of the average and marginal cost curves as derived in the previous section can be illustrated the short-run cost curves (Figure 5.12). These are summarised in the following.

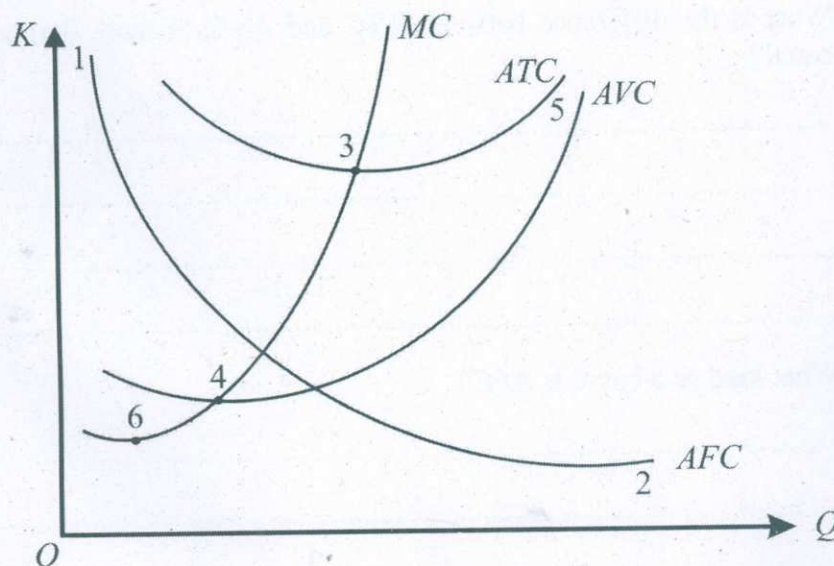


Fig. 5.12: Short-run Average Cost Curves

AFC curve declines continuously approaching both axes asymptotically, as shown by points 1 and 2 in Figure 13. AFC is a rectangular hyperbola.

- i) AVC first declines, reaches a minimum at point 4 and rises thereafter. When AVC attains its minimum at point 4, MC equals AVC. As AFC approaches the horizontal axis, asymptotically, AVC also approaches ATC asymptotically, as shown by point 5.
- ii) ATC first declines, reaches a minimum at point 3 and rises thereafter. When ATC attains its minimum at point 3, MC equals ATC.
- iii) MC first declines, reaches a minimum at point 6 and rises thereafter. MC equals both AVC and ATC when these curves attain their minimum values. Furthermore, MC lies below both AVC and ATC over the range in which the curves decline; it lies above them when they are rising.

Check Your Progress 2

- 1) How are the average curves derived from the TC curves?

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- 2) Can the MC be derived from the ATC and AVC?

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- 3) What is the difference between ATC and AVC, if their derivatives are equal?

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- 4) What kind of a curve is AFC?

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