UNIT 1 TYPES OF CATEGORICAL PROPOSITIONS: A, E, I, AND O; SQUARE OF OPPOSITION

Contents

1.0 OBJECTIVES

First objective of this unit is to introduce you to the elements of categorical proposition. This is intended to be achieved through the introduction of the nature of terms and their distinction from words. The second objective is to establish an important distinction between sentence and proposition. The last, but not the least, objective is to familiarize you with certain forms of logical relations called immediate inference which should in turn enable you to establish and discover certain other important logical relations.

1.1 INTRODUCTION

As a form of critical thinking, logic has its origin in several ancient civilizations, like Indian, Chinese, Greek, etc. In the Western tradition, logic was systematized by Aristotle and hence he is credited with its origin. Logic, 'the tool for distinguishing between the true and the false' (Averroes), examines the general forms which arguments may take, and distinguishes between valid and invalid arguments. An argument consists of two sets of statements called premise or premises, on the one hand, and the conclusion on the other. The premises are designed to support the conclusion. The presence of this complex relation (also called inference) makes a group of statements an argument and with which logic is concerned. Thus mere collection of propositions does not constitute an argument when this relation is absent. In this unit we shall confine ourselves to an analysis of terms and propositions which are basic to our study of logic and postpone a detailed study of inference to the next unit.

1.2 TERMS AND THEIR KINDS

Logic makes a sharp distinction between 'word' and 'term'. All words are not terms, but all terms are words. Terms refer to specific classes of objects or qualities whereas words refer to none of them. Further, a term may consist of more than one word. Table, planet, etc. are terms which consist of one word only. *The author of Hamlet* is a term which consists of four words. Words in different languages may express the same term; e.g. *tree, vriksha* etc. While there is only one term in this example, there are two words. Traditional logic has recognized different kinds of terms. A brief description of kinds of terms throws some light on the way in which traditional logic understood 'term'.

Positive and negative: Positive terms signify the presence of desirable qualities e.g., light, health, etc.; negative terms signify, generally, undesirable qualities or qualities not desired, rightly or wrongly. The clearest negative terms are those with the negative prefix 'in' (or 'im'), dis, etc. Inefficient, dishonest, etc. are negative terms. However, it is not always the case. For example, immortal, invaluable, discover, to name a few, surely, are not negative. Therefore what constitutes a negative term is, essentially, our attitude in particular and custom in general. In other words, the distinction is not really logical, but it has something to do with value judgment. That is why some words without such prefixes are regarded as negative since they too imply negation: e.g. darkness (absence of light).

Concrete and abstract: Concrete terms are those which refer to perceptible entities; abstract terms are those which do not; e.g. man, animal, tall etc., are concrete terms; mankind, animalism, etc., are abstract terms. However, this classification depends upon use. For example, the word 'humanity' is used not only to mean individual men but also the quality of man. Hence use or meaning determines the class.

Relative and absolute: Relative terms are those which express a relation between two or more than two persons or things, e.g. father, son, etc. Absolute terms do not express such relation, e.g. nationality, cone, etc. Comparative terms are obviously relative: e.g., larger, prettier, etc.

Singular and General: Singular terms denote specific objects. It points to one object only. All proper names are singular terms. Russellian proper names are also counted among singular terms. '*The author of Principia Mathematica, The farthest planet from the sun'*, etc. are singular terms. General terms are just class names. Vegetable, criminal, politician, etc., are examples for general terms.

Univocal, and Equivocal terms: Univocal terms carry only one meaning. Entropy is an example for univocal terms. Equivocal terms are burdened with at least two meanings. Gravity is equivocal; so is astronomical. When natural language becomes the medium of expression, equivocal terms pose hurdles in determining the validity of arguments because such terms cause ambiguous structure of statements. Therefore in our study of logic we must ensure that the arguments consist of only unambiguous terms. Later we will come to know that symbolic logic became indispensable precisely for this reason.

1.3 DENOTATION AND CONNOTATION OF TERMS

By denotation of a term we mean the number of individuals to which the term is applied or extended. For example, 'society' denotes the human society, a philanthropic society, the Society of Jesus, a political society (or a State), etc. Another word used for 'denotation' is extension. By connotation or intension of a term we mean the complete meaning of a term as expressed by the sum total of its essential as opposed to accidental characteristics. For example, consider the same term; 'society' connotes (a) an association of persons and (b) united by a common interest. Crowd does not mean the same as society because it lacks these characteristics. Complete meaning, therefore, excludes accidental and figurative characteristics. The latter is misleading in the sense that it is not a characteristic at all in the strict sense of the term. Denotation and connotation together determine what is called class or set of objects. Therefore every term stands for one class or the other.

In Scholasticism, connotation and denotation are reserved for terms and comprehension and extension for concepts of which terms are signs, or expressions. Note that the word 'connotation' may vary in meaning from time to time. For example, 'politician' may acquire a different meaning in different societies at a given point of time or in the same society at different points of time. Therefore connotation is only conventional.

It is clear that greater the connotation (intension) of a term smaller the denotation (extension) and, conversely, greater the denotation smaller the connotation. For example, the term 'being' connotes simply 'existence' and can be extended to everything that exists (man, animal, plant, stone, etc.). But as soon as I say 'human being', thus increasing the connotation (i.e. 'human' and 'being'), the term includes only human beings; not others. 'Oriental human being' is still less extensive, for the term cannot be applied to Westerners. This is called the law of inverse variation.

1.4 MEANING AND SUPPOSITION OF TERMS

There is a subtle difference between meaning and supposition. Meaning is what convention accepts. Therefore many words have more than one meaning because convention is always inaccurate. Xystus is one such word which has several meanings like covered portico used for exercise by the athletes in antiquity, a garden walk on terrace, etc. Likewise, 'Supposition' of a word is the function or the use of a word which depends upon the intention of the speaker. Therefore meaning is objective whereas supposition is subjective. The Scholastics understood 'supponit' as the one which stands for the concept.

Check Your Progress I

Note: Use the space provided for your answer.

1. Critically examine various classes of terms.

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2. Analyse the meaning of connotation and denotation.

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1.5 PROPOSITIONS

In the previous section we came to know that all terms are words, but all words are not terms. Similarly, all propositions are sentences, but all sentences are not propositions. Only those sentences are propositions which grammar regards as assertive. A proposition is always either true or false, but not both; and no proposition is neither true nor false. This means to say that a proposition is a declarative sentence which gives certain information and it is this information which makes a proposition true or false. It is equally important to note that there is no need to know whether a given proposition is true or false. Further, several sentences may express one proposition. Consider these groups of sentences.

A 1 Jealousy thy name is woman.
2 What is wrong with your car?
3 Copper sulphate is an organic compound.
4 Newton wrote *Optiks*.

B 1 *Cogito Ergo Sum.*
2 I think, therefore I am.

In group A, first two sentences are not assertive sentences (the first sentence is misleading. It appears to be an assertive sentence, but in reality, it is not. Sentences which express emotional outburst are, more or less, exclamatory). The third sentence is false whereas the fourth sentence is true. Group B consists of two sentences which belong to different languages but give the same meaning. Within the same language also it is possible to have two sentences which give the same meaning. Consider this group.

C 1 Rama killed Ravana.
2 Ravana was killed by Rama.

Sentences in B and C groups show that a proposition is the meaning of a sentence. Although several sentences can give one meaning, it is impossible in an unambiguous system to have one sentence with more than one meaning.

A new word is introduced at this point. The *truth-value* of true proposition is said to be true and that of a false proposition is said to be false. Here afterwards we frequently employ this term in our study.

Let us turn to Aristotelian analysis of proposition. A proposition consists of two terms in his system. The term (class) about which the proposition asserts something is called ‘Subject’ (S) and what is said about it is called ‘Predicate’ (P). S and P are to be regarded as S-class and P-class respectively. In a proposition these are related using the verb of the form ‘to be’ called ‘Copula’, which must be always in present tense. According to Aristotle a sentence becomes a proposition only when it meets all these requirements, not otherwise. It is obvious that only the first example considered above (A3) falls within the limits of Aristotelian system. This sort of restriction severely thwarted further progress of logic.

Traditional logic considers two kinds of propositions; categorical (unconditional) and conditional. Assertion is of two types; affirmation and negation. Affirmation or negation is made in the former without stating any condition, whereas in the latter it is stated with condition or conditions. Initially, we shall restrict ourselves to the former kind. Affirmation or negation is possible in this category in two ways; total or partial. If P is affirmed or negated of the whole class of S, then it is total. On the other hand, if affirmation or negation applies to only a part of the class, then it is partial. Consequently, we obtain four kinds of categorical proposition.

1 Universal affirmative (total affirmation)
2 Universal negative (total negation)
3 Particular affirmative (partial affirmation)
4 Particular negative (partial negation)

For the sake of simplicity and brevity (in logic these two are very important) these four kinds are represented symbolically. From *Affirmo,* a Greek, word we choose A and I to represent first and third kinds respectively and similarly, from *Nego,* another Greek word, we choose E and O to represent second and fourth kinds respectively. It is a mere convention to prefix S and suffix P to A, E, I and O. In modern parlance the first letters of S-term and of P-term are used in place of S and P.

Each kind is illustrated below.

S	Copula	P		
1 All (scientific theories)	are	(improvable).	Universal Affirmative	SAI
2 No (celestial bodies)	are	(static).	Universal Negative	CES
3 Some (fruits)	are	(bitter).	Particular Affirmative	FIB
4 Some (chemicals)	are not	(toxic).	Particular Negative	COT

The distinction between universal and particular depends upon what is called quantity and the one between affirmative and negative on what is called quality. Not much discussion is needed to know what quality is. If any negative word like no, not, etc., occurs in the proposition (2 and 3), then quality is negative. Otherwise, it is affirmative. A word of caution is required. Sometimes predicate carries negative force. But it does not make the quality of proposition negative. For example 'dishonest', non-natural, etc. constitute terms in their own right. They have nothing to do with the quality of proposition. Consider these two propositions.

5 Shakuni is *dis*honest.
6 Telepathy is a *non*-natural phenomenon.

These propositions are affirmative only. It means that a proposition is negative only when negative word is a part of copula. However, quantity of proposition needs elaborate explanation which becomes intelligible only after explication of what is called the *distribution* of term.

Distribution of terms is an indispensible concept in Aristotelian logic. A term is said to be distributed if the proposition in which it occurs, either includes or excludes the said term completely. Inclusion or exclusion is complete provided the proposition refers to every member of the class. If so, when is it said to be undistributed? Suppose that n is the number of members in a given class. If the proposition includes or excludes (n – 1) members of that particular class, say S, then S is said to be undistributed.

Let us turn to the pattern of distribution in categorical proposition. Quantity of any proposition is determined by the extension or magnitude of S, i.e., the number of elements in the given set and a term acquires magnitude only when it is a component of a proposition. Only sets have magnitude (this is so as far as logic is concerned). A set is defined as the collection of well-defined elements as its members. A null set or an empty set does not have any element. Let us assume that term is synonymous with set. Then we can accept that a term has magnitude. If magnitude of any term is total in terms of reference, then it is said that the term is distributed. If magnitude is incomplete, then that term is undistributed. It shows that any term is distributed only when the entire set is either included or excluded in such a way that not a single member is left out. This is another way of explicating what complete magnitude means.

All universal propositions distribute S whereas particular propositions do not. Just as distribution is explicated, undistribution also must be explicated. A term is undistributed only when inclusion or exclusion is partial. The meaning of partial inclusion or exclusion is, again, repeated, but in a very different manner. Let us attempt a formal explication of the same. Let the magnitude of S be x. Let S* (to be read s-star) denote that part of S, which is included in or excluded by a proposition. Now the formula, which represents the undistribution of S can be represented as follows:

$$|x| > S^* \geq 1 \quad \ldots\ldots\ldots\ldots.(1)$$

This is the way to read (1): ‘The modulus of x (|x|) is greater than S* greater than or equal to 1’. It is highly rewarding to use set theory here. (1) indicates that S* is a proper subset of S. Therefore its magnitude must be smaller than that of x which is the magnitude of S. However, S* is not a null set. (1) shows that there exists at least one member in S*. In the case of undistribution, therefore, the magnitude of S* varies between 1 and | x-1|. Now it is clear that in A and E, S is distributed while in I and O it is undistributed. Just to complete this aspect, let us state that all affirmative propositions undistribute P whereas all negative propositions distribute P.

A far better way of presenting the distribution of terms was invented by Euler, an 18th C. Swiss mathematician and later, John Venn, a 19th C. British mathematician improvised the representation further. An understanding of the techniques adopted by them presupposes some aspects of set theory.

Let S and P be non-null (non-empty) sets with elements as mentioned below (it is important that the status of set must be mentioned invariably, i.e., null or non-null). The following pairs shall be considered.

1. S = {a,b,c,d,e,f}, P = {g,h,i,j,k}

All letters within parentheses are elements of respective sets. In the first grouping there is no common element. Now, consider following groupings.

2. S = {a,b,c,d,e,f}, P = {a,b,c,d,e,f,g,h,i}
3. S = {a,b,c,d,e,f}, P = {b,c,d,g,h}
4. S = {a,b,c,d,e,f}, S* = {a,b,c}, P = {m,n,g,h}
5. S = {a,b,c,d,e}, P = {a,b,c,d,e}

Fifth group is unique in the sense that these two sets possess exactly the same elements. Therefore the magnitude of these sets also remains the same. Such sets are called identical sets. Identity of elements and also the equality of magnitude make sets identical. In 1908, Zermelo proposed what is called 'Axiomatic Set Theory'. One of the principal axioms of this theory is known as the Axiom of Extension or Extensionality. This Axiom helps us to understand the structure of identical sets. This theory was modified later by A Fraenkal and T. Skolem. Let us call this theory Zermelo – Fraenkel – Skolen theory (ZFS theory). This theory states the above mentioned axiom as follows.

ZFS1: If a and b are non-null sets and if, for all x, $x \in a$ iff $x \in b$, then $a \equiv b$

[Note '$\in$' is read 'element of'; 'iff' is read 'if and only if' and '$\equiv$' is read identical.]

Symbolically, it is represented as follows:

$\{Sa \wedge Sb\} \wedge \{\forall x \ (x \in A \Leftrightarrow x \in b) \Rightarrow a \equiv b$

This is the way to read:

Sa = a is a set

$\wedge$ = and

$\forall$ = for all values of

<=> = if and only if

=> = if …then

The summary of this formula is very simple. Whatever description applies to S (here a) also applies to P (here b). When distribution of terms is examined, the magnitude and elements of sets also are examined. Therefore it is wrong to assert that when S and P are identical sets, P is undistributed in A. Let us designate this type of proposition as A+ (read A cross). Consider these two propositions:

7 All bachelors are unmarried men. (BAU)

8 All spinsters are unmarried women. (SAU)

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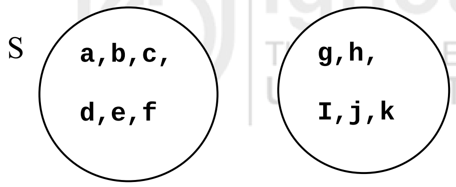
2 SEP

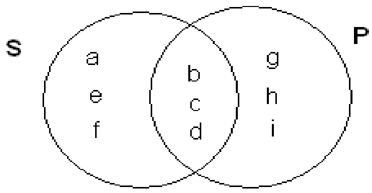


S = {a,b,c,d,e,f}

P = {g,h,i,j,k}

$\therefore S \triangle P$

3 SIP

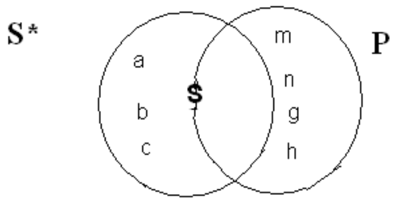


S = {a,b,c,d,e,f}

P = {b,c,d,g,h,i}

S∩P = {b,c,d}

4 SOP



S* = {a,b,c}

P = {m,n,g,h}

Now we are in a position to examine the fourth group. It requires a little explication to understand the status of O with regard to distribution. In this instance S* is incomplete, i.e., undistributed and P is completely excluded by S*. It shows that P is distributed. Let us see how this happens.

1. Let S = {a,b,c,d,e,f}

2. Let S* = {a,b,c}; there is no information on d, e and f.

3. S* $\subseteq$ S (S* $\leq$ S); S* is smaller than or equal to S. It also means that S* is only a subset, not a proper subset, of S.

4. Let S – S* = S** (S** $\geq \Phi$) or S = S* + S**

'Φ' reads phi which stands for null set.

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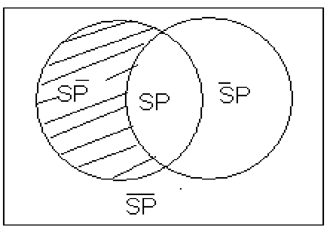

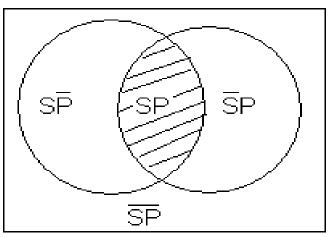

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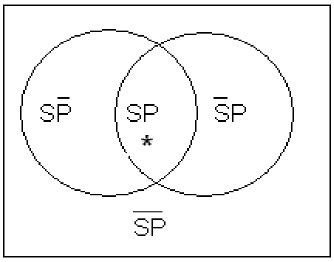
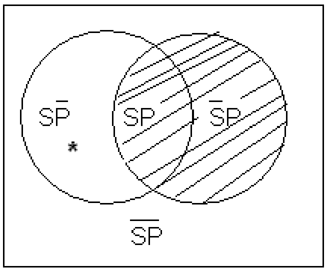




$\bar{P}$

$\bar{P}$

REH: $(\forall x)\ \{x \in R) \Rightarrow (x \notin H)\}$; $\notin$ is read ‘not an element of’

RIH: $(\exists x) \ni \{(x \in R)\ \Lambda\ (x \in H)\}$; $\exists x$ is read ‘there exists at least one x; $\ni$ is read ‘such that’.

ROH: $(\exists x) \ni \{(x \in R)\ \Lambda\ (x \notin H)\}$

$\forall$ is known as universal quantifier and $\exists$ is known as existential quantifier. (x) can also be used in place of $(\forall x)$.

1.6 SQUARE OF OPPOSITION

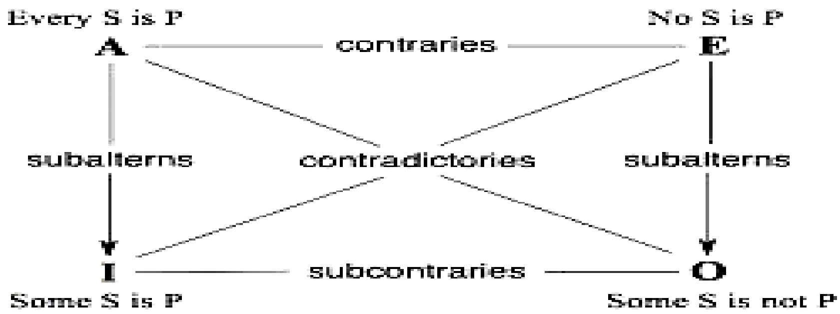
This is one type of immediate inference because in this type of inference conclusion is drawn from one premise only. Eduction is another word used for immediate inference Opposition is a kind of logical relation wherein propositions ‘stand against’ one another in terms of truth-value when they have the same subject and the same predicate, but differ in quantity or quality or both. Traditional logic called this relation square of opposition because these relations are represented by a square. Four such relations are discussed in Aristotelian system.

1. Contradiction: When two propositions differ in both ‘quantity’ and ‘quality’, the relation is called contradiction, e.g. ‘All men are wise’ (A) – ‘Some men are not wise’ (O). It is the most complete form of logical opposition because they are neither true nor false together. If one is true, the other is necessarily false and vice versa. This sort of self - contradiction is due to incompatibility between respective statements. Similarly, the statements, ‘No men are wise’ (E) – ‘Some men are wise’ (I) are contradictory.
2. Contrariety: When two universal propositions differ only in ‘quality’, the opposition is called contrary; e.g. ‘All men are wise’ (A) – ‘No men are wise’ (E). By definition, both contraries can be false – precisely as in the example given – but they cannot be true at the same time. If one of them is true, the other must necessarily be false, but if one is false, the other may be true or false. One kind of proposition called singular proposition (also called simple), whose S is proper name, has no contrary and its contradiction differs only in quality. One example is ‘Jo is bad – Jo is not bad’. Another example is ‘The author of Hamlet, is an Englishman and ‘The author of Hamlet’ is not an Englishman.

3. Subcontrariety: When two particular propositions differ only in ‘quality’, the opposition is called subcontrariety. E.g. ‘Some men are wise’ (I) – ‘Some men are not wise’ (O). Subcontrary propositions can be true together – as in the example given, but they cannot be false at the same time. If one of them is true, the other may be true or false, but if one of them is false, the other must necessarily be true. The inverse order of ‘contrary’ and ‘subcontrary’ propositions is evident.

4 Subalternation: When two propositions differ only in 'quantity' (one is universal and the other is particular), the opposition is called subalternation, e.g. 'All men are wise' (A) - 'Some men are wise' (I). Notice that 'subaltern' propositions can be true together or false together. And this is to say that though from the truth of the universal, one can infer the truth of the particular the reverse order does not hold, namely that from the truth of the particular, one cannot infer the truth of the universal. On the other hand, though from the falsity of the particular, one can infer the falsity of the universal, one cannot infer the falsity of the particular from the falsity of the universal.

This type of relation is expressed in the form of a square.



The following two schemes and one diagram offer visual aid to retain more easily in mind what we have just said about the 'opposition' of propositions:

For the sake of simplicity the truth - relation which holds good between various relations is provided in a nutshell.

Inferences in Subalternation

From truth of universal	→	truth of particular
From truth of particular	/→	truth of universal
From falsity of particular	→	falsity of universal
From falsity of universal	/→	falsity of particular

→: Can infer
/→: Cannot infer

II. Mnemonic Device for remembering the Square of Opposition (Lander University, Greenwood).

A. If you picture God at the top of the square of opposition and the Devil at the bottom of the square and remember the phrase 'both cannot be ...' for contraries and subcontraries, the following mnemonic device might be helpful.

B. The big ‘X’ across the center of the Square represents contradictories with opposite truth – values. This should be very easy to remember.

C. Since God (or truth) is at the top of the diagram, both contraries ‘cannot be true.’

D. Since the Devil (or falsity) is at the bottom of the diagram, both subcontraries ‘cannot be false’.

E. With subalternation, God can send truth down, but we cannot know what it means for God to send falsity down (hence this would be indeterminate).

But, the Devil can send falsity up (since this is what Devils are good at), and we cannot know what it means for the Devil to send truth up. So this relation is indeterminate.

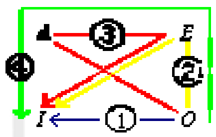
III. ‘Bouncing Around the Square of Opposition.’

Suppose we know that O (Some **S** is not **P)** is false. In how many ways can we determine the truth - value of **I** (‘Some **S** is **P**’)?

There are four ways of determining the truth-value. These four ways consist in travelling between different points (here the propositions are points). The four routes are as follows.

(Notice that we could set an itinerary of our journey along the selected four routes. The ‘reason,’ given below, is, so to speak, our ‘inference ticket’ for travel Cf. Lander University, Greenwood).

	Originating Point	Through	Terminating Point
1	SOP	Direct	SIP
2	SOP	SEP	SIP
3	SOP	SAP to SEP	SIP
4	SOP	SEP and SAP	SIP



Route 1: **O** to **I**

Statement of	**Reason**	**Truth -Value**
1. Some **S** is not **P**.	Given	false
2. Some **S** is **P**.	subcontrariety	true

Route 2: **O** to **I through E**

Statement	**Reason**	**Truth - Value**
1. Some **S** is not **P**.	given	false
2. No **S is P.**	subalternation	false

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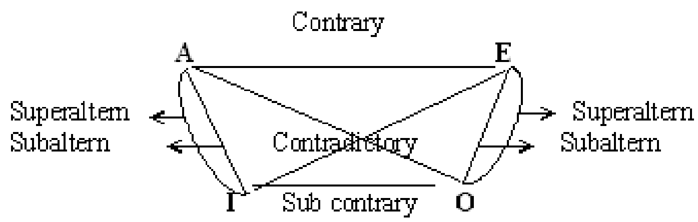

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At this stage, it is important to become familiar with two other types of relation called conversion and obversion. They are also known as equivalent relation because the truth-value of both the premise and the conclusion remains the same, i.e. if the premise is true, the conclusion is true and if the premise is false, the conclusion is also false. When there is a change in the structure of sentences, on some occasions meaning remains unchanged. It only means that the very same information is provided in different ways. Recognition of this simple fact helps us in testing accurately the validity of arguments and also in avoiding confusions. There are two primary forms of equivalent relation; conversion and obversion. The conclusion in conversion is called converse and in obversion obverse. The processes of conversion and obversion are quite simple. These operations deserve a close scrutiny.

Conversion: This is governed by three laws.

1st Law: S and P must be transposed.

After transposition P becomes subject and S becomes predicate. This is the 1st stage.

2nd Law: Quality of propositions should remain constant. If the premise is affirmative, the conclusion must be affirmative. If the premise is negative, the conclusion must be negative.

3rd Law: A term, which is undistributed in the premise, should remain undistributed in the conclusion. It can be stated in another way also. A term can be distributed in the conclusion only if it is distributed in the premise. However, a term, which is distributed in the premise, may or may not be distributed in the conclusion. The following examples illustrate these rules.

10 All philosophers are kings	PAK
Converse: ∴ Some kinks are philosophers.	KIP
11 No vegetables are harmful.	VEH
Converse: ∴ No harmful things are vegetables.	HEV
12 Some women are talkative.	WIT
Converse: ∴ Some talkative people are women.	TIW

There are three aspects to be noted. Conversion of A is conversion by limitation because the quantity is reduced from universal to particular after conversion. Secondly, conversion of E and I is simple because in these cases S and P are just transposed and no other change takes place. Thirdly, while A, E and I have conversion, O does not have conversion. What happens when A undergoes simple conversion and O is converted? In these cases conversion leads to a fallacy called fallacy of illicit conversion. Fallacy in formal logic arises when a rule is violated. In both these cases conversion violates a rule or rules.

Consider these statements.

13 All Europeans are white.

∴ All white people are Europeans.

14 Some gods are not powerful.

∴ Some powerful beings are not gods.

Conversion in these two cases is invalid because the terms, 'white' and 'gods' are distributed in the respective conclusions while they are undistributed in the respective premises. This type of conversion violates the third law. The terms 'white' and 'gods' remain undistributed in the premises since the former is the predicate of an affirmative premise while the latter is the subject of a particular premise. If we obtain affirmative converse from a negative premise in order to undistibute predicate term, then we violate the second law of conversion. It only means that when A undergoes simple conversion and when O is converted, in the case of A the third law is violated and in the case of O second or third law of conversion, as the case may be is violated. Therefore A becomes I after conversion and 'O' has no conversion.

Obversion: This is one technique of preserving the meaning of a statement after effecting change of quality. The procedure is very simple; change the quality of the premise and simultaneously replace the predicate by its complementary. We apply this law to the premises (A, E, I, and O) to obtain the conclusions. The conclusion is called obversion.

15	All players are experts.	PAE
	∴ No players are non-experts.	$PE\bar{E}$
16	No musicians are novelists.	MEN
	∴ All musicians are non-novelists.	$MA\bar{N}$
17	Some scholars are women.	S I W
	∴ Some scholars are not non-women.	$SO\bar{W}$
18	Some strangers are not helpful.	SO H
	∴ Some stranger are non-helpful.	$SI\bar{H}$

Check Your Progress II

Note: Use the space provided for your answers.
1 Give symbolic representation of propositions? What do the symbols stand for?
....
....
....
....
2. Determine all possible product classes of the following terms and their complements.

a) players and experts b) philosophers and kings c) fruits and vegetables d) actors and directors
....

1.7 LET US SUM UP

The basic units of argument are terms and proposition. All words are not terms; all terms are words. All sentences are not propositions; all propositions are sentences. Subject and predicate are the constituents of categorical proposition according to Aristotle. There are four kinds of categorical proposition. Distribution of a term means total extension. Euler and Venn interpreted distribution diagrammatically. Square of opposition, conversion and obversion are three kinds of immediate inference.

1.8 KEY WORDS

Supposition: A 'supposition' of a word is the function or the use of a word has in a presupposition depending on the intention of the speaker.

Term: Any word or group of words that stands for the subject or the predicate of a proposition.

Proposition: A statement affirming or denying something of somebody.

Categorical proposition: It is a proposition in which the predicate is affirmed or denied unconditionally of all or part of the subject.

Copula: A 'copula' joins the subject and the predicate of the preposition. Normally it is the verb 'is' or 'is not'.

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UNIT 2 CATEGORICAL SYLLOGISM

Contents:

2.0 OBJECTIVES

This unit introduces you to the essence of Aristotelian logic. Since syllogism is the most important form of inference, you ought to have a background of the nature of inference and various issues associated with it. One objective of this unit is to give a brief explanation of the nature of deductive inference and contrast it with inductive inference. Another objective is to analyze different kinds of syllogism to enable you to understand variety in syllogism.

2.1 INTRODUCTION

Categorical syllogism is the essence of traditional logic. This form of inference is called mediate inference because the conclusion is drawn from two premises. Further, this is called categorical because all propositions involved are categorical. Since syllogistic inference is nearly identical with deductive inference, an exhaustive analysis of inference is required as a prelude to a proper understanding of syllogism.

2.2 REASON AND INFERENCE: MEANING AND OBJECTIONS

Reasoning consists, essentially, in the employment of intellect, in its ability to 'see' beyond, and within as well, what is available to senses. Reasoning, therefore, is a sort of bridge which connects 'unknown' with 'known'. While reasoning is regarded so, inference is regarded as the process involved in extracting what is unknown from what is known. Reasoning is essentially a psychological process which is, undoubtedly, not the concern of logic. Therefore some logicians thought it proper to replace reasoning with inference. However, this replacement did not improve matters much. The reason is obvious. If all human beings stop thinking, then there will be nothing like inference. This dependence shows that inference is as much a psychological activity as reasoning is. What is psychological is necessarily subjective. Logic, in virtue of its close association with knowledge, has nothing to do with anything that is subjective. Therefore it was imperative for logicians to discover an escape route.

Cohen and Nagel for this particular reason chose to use 'implication' instead of 'inference'. The difference in kind can be understood easily when we look at the usage. Statements always 'imply' but do not 'infer'. Therefore implication is in the nature of relation between statements. On the other hand, I 'infer', but I do not 'imply'. This clearly shows that inference is an activity of mind. Salmon fell in line with Cohen and Nagel when he said that the very possibility of inference depends upon reasoning. Despite the fact that inference is subjective, logicians like Copi, Carnap, Russell, etc., chose to retain the word inference. But, all along, they only meant implication. Therefore keeping these restrictions in our mind let us use freely the word 'inference'.

Though the use of the word 'reason' is not much rewarding, the word 'reasonableness' has some weight. We often talk about reasonableness of the conclusion. In this context reasonableness means 'grounds of acceptability'. Surely, in this restricted sense, reasonableness is objective just as inference is.

2.3 KINDS OF INFERENCE

In a broad sense, there are two kinds of inference; deductive and inductive. Deductive inference regards the form or structure as primary and therefore it is called formal logic (inference and logic are interchangeable). It remains to be seen what form means. Inductive logic regards matter or content of argument as primary. Some logicians, like Cohen and Nagel, did not regard induction as logic at all. Without considering the merits and demerits of their arguments, let us consider briefly the characteristics of these two kinds.

Our study of formal logic begins with the distinction between truth and falsehood on the one hand, and validity and invalidity on the other. This particular distinction is very prominent. Only statements are true (or false) whereas only arguments are valid (or invalid). This distinction will take us to this table.

Table 1:

	Statements	Arguments
1)	True	Valid
2)	True	Invalid
3)	False	Valid
4)	False	Invalid

This table helps us to understand the following possibility. a) A valid argument (1 and 3) may consist of completely true statements or completely false statements. b) An invalid argument (2 and 4), similarly, may consist of statements in exactly the same manner mentioned above. It shows that truth and validity, on the one hand, and falsity and invalidity, on the other, do not coincide always. Similarly, we have to distinguish between material truth and logical truth. Material truth is what characterizes matter of fact. Logical truth is determined by the structure of argument. We shall consider examples which correspond to four combinations (see table1). Let

us call premises p1, p2, etc. and conclusion q.
Arg1:

p1: No foreigners are voters.
p2: All Europeans are foreigners.
q: ∴ No Europeans are voters.

Arg2:

p1: Some poets are literary figures.
p2: All play writers are literary figures.
q: ∴ Some play writers are poets.

Arg3:

p1: All politicians are ministers.
p2: Medha Patkar is a politician.
q: ∴ Medha Patkar is a minister.

Arg4: p1: 3 is the cube root of – 27.
p2: - 27 is the cube root of 729.
q: ∴ 3 is the cube root of 729.

These four arguments apply to arguments 1, 2, 3, and 4 of Table 1 respectively. First and third arguments have a definite structure in virtue of which they are held to be valid. Second and fourth arguments have a different structure which makes them invalid. When an argument is valid, the premise or premises imply the conclusion. If there is no implication, then the argument is invalid. Validity is governed by a certain rule which is represented in a tabular form. [Let us designate 'true' by '1' and 'false' by '0' as a matter convention].

	p	q	
1)	T(1)	T(1)	Valid
2)	F(0)	F(0)	Valid
3)	F(0)	T(1)	Valid
4)	T(1)	F(0)	Invalid

We can also say that the premises necessitate the conclusion and when they necessitate the conclusion there is implication. In this case, necessity is of a particular kind, viz., logical necessity. Therefore, when there is implication, conclusion is necessarily true and *vice versa*. Very often, deductive logic is identified with mathematical model. It is generally admitted that in both these disciplines information provided by the conclusion is the same as the one provided by the premises. It means that both are characterized by material identity. Deductive logic, therefore, is an example for tautology. This characterization is highly significant and is in need of some elaboration.

If, one can ask, the conclusion does not go beyond premises (it may go below or well within) and no new information is acquired in the process, then why argue and what is the use of arguments? The answer is very simple. Knowledge is not the same as mere acquisition of information. In other words novelty is not a measure of knowledge. The legend is that Socrates extracted a geometrical theorem from a slave purported to be totally ignorant of mathematics. The moral is that knowledge is within, not in the sense in which brain or liver is within. Knowledge is the outcome of critical attitude. It is discovered, not invented and so goes an ancient Indian maxim: eliminate ignorance and become enlightened. If what is said is not clear, then consider this path. Deductive argument helps us to know what is latent in the premises, i.e., the meaning of the premises. It is an excursion into the analysis of their meaning or meanings. And the conclusion is an expression of the same. If so, it is easy to see how the denial of the conclusion in such a case amounts to denying the meaning or meanings of the premises which were accepted earlier. What is called self-contradiction is exactly the same as the combination of the denial of the conclusion and the acceptance of the premises. Therefore we say that a valid deductive argument is characterized by logical necessity. If so a deductive argument is always true. This is the meaning of tautology.

At this stage, two terms are introduced: analytic and a priori. Consider this example: 'all men with no hair on their heads are bald'. We know that this statement is true in virtue of the meaning of the word 'bald'; not otherwise. Such a statement is called analytic. In such statements the predicate term (here 'bald') is contained in the subject term (here 'men with no hair on their heads'). Knowledge obtained from an analytic statement is necessarily a priori, meaning knowledge prior to sense experience. In philosophical parlance, all analytic statements are necessarily a priori. Deductive logic provides knowledge a priori, though the premises and the conclusion considered independently are not analytic. It is the knowledge of the relation between the premises and the conclusion which is a priori. Therefore deductive argument and analytic statement share a common characteristic; in both the cases the denial leads to self-contradiction.

How can we say that deductive logic provides a priori knowledge? Consider an example.

Arg. 5: All saints are pious.
All philosophers are saints.
∴ All philosophers are pious.

Evidently, there is no need to examine saints and philosophers to know that the conclusion is true. Indeed, it is not even necessary that there should be saints who are pious as well as philosophers. This being the case, arg. 5 takes the following form without leading to any distortion of meaning.

Arg. 5a: If all saints are pious and all philosophers are saints, then all philosophers are pious.

The argument is transformed into a statement which involves relation. Implication (the present relation is one such) is such that without the aid of sense experience, but with the laws of formal logic alone, it enables us to derive the conclusion. Thus like an analytic statement, a valid deductive argument provides a priori knowledge and hence it is devoid of novelty. It is this sort of relation that precisely describes the relation between the premises and the conclusion in deductive inference. This does not mean that deductive argument is absolutely certain. This is because necessity is a logical property whereas certainty is a psychological state. The former is objective and the latter is subjective.

When sense experience takes back seat, reason becomes the prime means of acquiring knowledge. Following the footsteps of Descartes, who is regarded as the father of rationalism, we can conclude, somewhat loosely, that deductive logic is rational. So we have sketched three characteristics; logical necessity, a priori and rational. There is a thread which runs through these characteristics. Therefore one character presupposes another.

Deductive argument is characterized by qualitative difference in opposition to quantitative difference, i.e. the difference between valid and invalid arguments is only in kind but not in degree. Further, validity is not a matter of degree. Let us make matters clear: a valid argument cannot become more valid in virtue of the addition of premise or premises. On the other hand, if any one premise is taken out of a valid argument, then the argument does not become 'less valid'; it simply becomes invalid. So an argument is either valid or invalid. A valid argument is always satiated. In other words, the premises in a valid argument constitute the necessary and sufficient conditions to accept the conclusion. An argument is invalid due to a 'missing link' in the class of premises. Deductive argument, therefore, is regarded as demonstrative argument. Acceptance of premises leaves no room for any reasonable or meaningful doubt.

We have learnt that validity is an important facet of deductive logic. Any account of validity is incomplete without considering Strawson's analysis of the nature of deductive logic. Strawson lists three aspects of formal logic: generality, form and system. Generality is distinguishable, clearly, from matter. Generality means that individual is not the subject matter of logic. Formal logic concerns only with the relation between statements, but not objects. It is futile to embark upon a study involving objects because such a study has only beginning but no end. Consider two examples,

Arg. 6:

p1: The author of Abhijnana Shakuntala was in the court of king Bhoja.

p2: Kalidasa is the author of Abhijnana Shakuntala.

q : $\therefore$ Kalidasa was in the court of king Bhoja.

Arg. 7:

p1: The author of Monadology was in the court of the queen of Prussia.

p2: Leibniz is the author of Monadology.

q: $\therefore$ Leibniz was in the court of the queen of Prussia.

It is easy to decide prima facie that the structure of these arguments is identical. The difference consists in subject matter only and it is possible to construct, at least theoretically, countless arguments having an identical structure. Obviously, this is not a profitable exercise. The essence of formal logic consists in saying that p1 & p2 together imply q or that q follows from or entails p1 and p2 together. Only implication and entailment are relevant here. Strawson has made this aspect very clear. Implication or entailment is independent of subject matter. Therefore it is impossible to identify the subject matter in virtue of recognition of implication. This point can be further clarified with the help of variables. Let us represent Abhijnana Shakunthala or Monadology with x, Kalidasa or Leibniz with y and queen of Prussia or King Bhoja with z. Now the argument takes this form.

Arg 7a : p1: The author of x was in the court of z.

p2: y is the author of x.

q : $\therefore$ y was in the court of z.

In this particular context, without knowing the contents of x, y, and z we can know that p1 and p2 together imply q. Therefore it is possible to determine the validity or invalidity of an argument without knowing the contents of the argument.

Let us call such forms logical forms. A logical form has two components: variables and constants. x, y, z etc are variables. In the case of categorical proposition the words *all*, *some*, *no* and *not* are constants. In the final analysis, the structure of an argument is determined by constants, but not variables. The dependence of the laws of an argument on constants is illustrated in this way. In life science the classification of animals is an important topic. The anatomical features of birds and aquatic creatures differ and there is difference in the function of those organs. Just as birds have some organs in common, aquatic creatures have certain other organs in common. These common organs correspond to constants and individual creatures correspond to variables. Similarly, every class of argument has definite constants. Just as the structure of birds is different from the structure of aquatic creatures, the structure of one class of arguments is different from the structure of some other class of arguments. The laws which explain the function of the organs of birds are different from the laws which explain the function of the organs of aquatic creatures. Similarly, in the case of arguments when the structure of an argument differs from that of another, the corresponding laws also differ from one another.

Integration of rules is another characteristic of formal logic. The structures of argument and rules are mutually dependent. If it is possible to decide the structure of an argument and also different classes of arguments, then is possible to achieve what is called formalization or systematization because formalization enables us to make a complete list of rules and also classify them so as to correlate them with respective arguments..

On the contrary, induction, in the first place, stands for any non-demonstrative argument where the premises, irrespective of their number, do not and cannot constitute conclusive evidences for the conclusion. The word ‘induction’ is the translation of what Aristotle called ‘epagoge’. C.S. Peirce used the term ‘ampliative’ for epagoge because in this type of argument the conclusion always goes beyond the premises and the premises offer, at best, reasonable grounds to ‘believe’ such conclusion. Belief is not the same as proof, a distinction which was, more often than not, completely ignored by the protagonists of induction. Nor is it a measure of proof. This is one difference. Secondly, uncertainty and sense experience characterize inductive argument. Let us consider the latter first. Inductive inference begins with sense experience. The premises, therefore, can be called ‘observation-statements’ which directly result from experience. However, the conclusion is not an observation-statement because it overshoots the material provided by observation– statements, which is why they cannot justify the conclusion. No matter how many black crows I have seen, they cannot prove that ‘all crows are black.’

At the stage, it is necessary to dispel a widespread and deep-rooted misconception. It is claimed erroneously that inductive argument always produces universal statement. On the contrary, what it provides is a statement which simply depends upon experience for further verification, but in itself is not an experiential statement. On some occasions, experience vouches for the conclusion, but on some other occasions, it does not. For example, considering the fact that, today I observed 5384 black crows, I may conclude that ‘tomorrow I will observe the same number of black crows’. This type of conclusion is characterized by a sort of leap, leap from ‘observed to unobserved or unobservable’. This is called inductive leap or simply generalization. But this is not a universal statement as understood by traditional logic. It shows that induction is just inconceivable in the absence of generalization though universal proposition is not necessary for an inference to become inductive. It is possible to construct a universal statement within the limits of sense experience without involving generalization, for example, when I conclude after close scrutiny that every book in the library is a hardback edition. This has nothing to do with induction. Therefore inductive inference may or may not yield universal proposition though it has to yield necessarily generalization.

The examples considered above are future-oriented and in principle, they are verifiable. However, inductive inference need not be so always. It can also be past-oriented which is surely, 'unverifiable'. History, Anthropology, Geology, etc. consist of arguments which are past-oriented. But the mechanism, involved in both the cases is exactly the same. Therefore the prime characteristic of induction is that the conclusion does not necessarily follow from the premises and that experience precedes inference which means that inductive inference is dubitable and *a posteriori*. Whatever knowledge we acquire 'after experience', or whatever depends upon experience is called *a posteriori* as opposed to a priori.

Uncertainty or dubious nature and a posteriori knowledge provided by inductive logic entitle it to be called empirical- again loosely- a characteristic disputed by Popper. The uncertainty of inductive conclusion brought in another term basic to the philosophy of science, viz. 'probability'. According to some inductivists all inductive conclusions are only probable. It is important to distinguish validity and probability. As mentioned earlier, validity is not a matter of degree, whereas probability is a matter of degree. Therefore an inductive inference may be less probable or highly probable.

2.4 DEDUCTIVE REASONING AND SYLLOGISM

In Aristotelian sense, syllogism is the kind of logical form to which every deductive inference is reducible. On most of the occasions, when people reason, they reason in methods in which some logical pattern runs as undercurrent. It is only logicians who discover these undercurrents because they are capable of critical examination of these undercurrents. The so-called logical pattern is extracted from a lay-man's method. A system is evolved by formalizing apparently disparate arguments. The difference in these methods is clearly perceptible. The difference is that a logician determines the standard-form which such argument or arguments take whereas a lay-man is unaware of such standard-form. A logician's method generalizes various arguments and it helps in discovering the common form to which all such arguments subscribe. It is important remember that the process of generalization is an important characteristic of formal logic. Otherwise, logic will be looked upon as a mere rhetoric and therefore with no practical value. If this is the way logic is evaluated, then anyone will conclude that it is far removed from the way people, as a matter of fact, talk and argue which is, no doubt, far from truth.

Logical analysis of syllogism

To make clear what we have just said, let us contrast these methods.

A Lay-man's method: 'Does God exist? Of course, he does not! No one has ever seen him, heard him, talked to him; has any one?'

B Logician's method:

8	All bodies which exist are perceivable.	B AP
	God is not perceivable.	G E P
	∴ God is not a body which exists.	∴G E B

A Lay-man's method: 'Was the Neanderthal a man? Yes he was. In fact we have proof to assert that he made tools, could paint, lived in groups etc.'

B Logician's method:

9 All beings who make tools, can paint, live in groups, etc. are men. B A M
The Neanderthal was a being who made tools, could paint, lived in groups, etc. N A B
∴ The Neanderthal was a man. ∴ NAM

What do we notice in these arguments? We notice that these arguments consist of three propositions (each with an S and a P). The statement to be proved is found in the last place in *logical sense*, and hence its technical name is 'conclusion'; the other two propositions function as reasons. Hence their logical name is 'premise'. Premises are found at the very beginning, again in *logical sense.* The order of the statements, therefore, is immaterial. Suppose that the conclusion appears at the end, as it happens generally. Then the conclusion is immediately preceded by words like *therefore, as a result, hence, consequently,* etc. It indicates that the 'consequentia' (the inference itself, as distinct from the 'consequence' which is another word for 'conclusion') is valid. The conclusion can as well appear at the very beginning in which case it is immediately succeeded by words like *because, for* etc. Any of these words in italics functions as a bridge connecting the premises with the conclusion. Further, we notice that at least one evidence is in the form of general principle which is invariably a universal proposition ('For somebody to exist….. in the first example and 'A man is one…' in the second example) and also that it is applied it to a particular case. Consequently, syllogism is invalid in the absence of universal proposition.

A close look at arguments considered above reveals an interesting aspect. Though there are three propositions, there are only three terms. Each term occurs twice in the arguments. These terms are named as follows. S and P of the conclusion are called the minor term (S) (or simply minor) and major term (P) (or simply major) respectively. The premise in which the minor occurs is called the minor premise and the premise in which the major occurs is called the major premise. One term is common to both the premises. This is called the middle term (M). In the first example 'God' is minor , 'bodies which exist' is major and 'perceivable' is middle and in the second example 'Neanderthal' is minor, 'man' is major and 'beings who…groups' is middle. Again, order of premises does not matter though, generally, major finds the first place.

Aristotle had convincing reason to choose these names. While the major has maximum extension, minor has minimum extension. The middle is so called because its extension varies between the limits set by the minor and the major. Aristotle argued that our inference proceeds from minor to major through middle. This explains the meaning of mediate inference.

Check Your Progress I

Note: Use the space for your answers.

1. Compare and contrast deduction and induction. ....
....

....

....
2. In a syllogism how do you relate the major, minor and middle terms?
....

....

....

2.5 KINDS OF SYLLOGISM

Syllogism is a class name with several subclasses. The classification is determined by constants. The types of constants vary from one class of syllogism to another class. Categorical syllogism is an important subclass. In this subclass propositions with their constituent terms are variables and quality and quantity of propositions are constants. Variables, i.e., propositions and their constituent terms do not determine the logical status, i.e., validity or invalidity of arguments because change of propositions does not affect the logical status as long as quality and quantity remain the same. Only the latter determine the logical status of arguments. This is an important aspect of formal logic. Let us first assume that every letter stands for a unique term and then examine the following arguments.

1 Categorical syllogism:

10 All X are Y.	11 All P are Q.	12 All M are N.
All Y are Z.	All Q are S.	All N are O.
∴ All X are Z.	∴ All P are S.	∴ All M are O.

The logical status of 10, 11, and12 remains unchanged though terms differ. If terms are different, then propositions also are different. In these arguments ‘All’ and ‘are’ are constants. Suppose that ‘All’ is replaced ‘No’ in both the premises of 10. Then the argument becomes invalid though variables remain unchanged. Even when the argument remains valid, its structure may vary. This will become evident in the following example.

13 Some X are Y.
All Y are Z.
∴ Some X are Z.

The structure of 10 and 13 are different. It shows that the axioms which determine the logical status of syllogism deal with quantity and quality of propositions and in turn distribution of terms. ‘Some’ and ‘not’ are other constants. Constants mentioned above determine the structure of categorical syllogism.

Before we turn to other subclass of syllogism called conditional syllogism we should consider an important aspect. Modern logic makes a distinct classification of propositions; simple, general and compound. If a grammatical sentence expresses one and only one proposition, then it is simple. Categorical proposition is called general in modern logic and conditional proposition, which is called compound in modern logic, is a combination of two or more than two propositions of any kind. Those propositions which constitute a compound proposition are components of such proposition. Several propositions are compounded using constants. Each constant determines the species which belongs to this subclass. Let us restrict ourselves to conditional syllogism and postpone further discussion of categorical syllogism to a later stage.

There are three kinds of conditional syllogisms which are discussed briefly.

2. Pure Hypothetical Syllogism (P. H. S.): In this subclass of syllogism all propositions are hypothetical. They are called hypothetical because they express a condition. The words *if* …. *then* constitute the condition and also constant because in the absence of this particular constant the proposition ceases to be hypothetical. The statement which appears immediately after *if* is called antecedent and the statement which appears immediately after *then* is called consequent. P. H. S. is governed by one rule which says that one statement must be common to two premises. If quality is constant, then it should appear in one premise as antecedent and in another as consequent. The common statement can appear as antecedent in one and as consequent in another provided it is affirmative in one and negative in another. In the latter case the conclusion becomes negative.
P. H. S. is illustrated below.

14 If this party wins, then we shall have a good government.	If A, then C.
If we shall have a good government, then we shall prosper.	If C, then B.
∴ If this party wins, then we will prosper.	∴ If A, then B.

A and C constitute the components of the first premise, C and B constitute the components of second and A and B constitute the components of the conclusion.

2 Mixed Hypothetical Syllogism (M. H. S.): If the major premise (usually the first one) alone is hypothetical, then the syllogism is called M. H. S.. Second premise and the conclusion are simple or general. M. H. S. is illustrated below.

15 If I do my duty, then I shall be happy.	If A, then B.
I do my duty.	A
∴ I shall be happy.	∴ B

In these kinds, there is no 'middle term'. However, middle term is replaced by a proposition which is common to both the premises. In 14 'we shall have a good government' is common to both the premises and in 15 'I do my duty' is common to both the premises. Hence we shall introduce a new word; middle proposition. An important limitation should be noted at this stage itself. It is fallacious to affirm B, in the minor premise instead of A and thereby affirm A in the conclusion instead of B. It is a fallacy because it violates a rule of M. H. S. which states that antecedent and consequent must be affirmed in the minor and the conclusion respectively. The only legitimate alternative is to deny the consequent and the antecedent in the minor and the conclusion respectively. In terms of prohibition it only means that the consequent and the antecedent should not be affirmed in the minor and the conclusion respectively. When antecedent and consequent are affirmed in the legitimate manner, the structure (technically known as mood) of the argument is identified as *Modus Ponendo Ponens* (in brief *Modus Ponens*). When the consequent and the antecedent are denied in the minor and the conclusion respectively, then the structure is identified as *Modus Tollendo Tollens* (in brief *Modus Tollens*). When we undertake a study of symbolic logic, we will come to know the importance of these moods which are called the Rules of Inference. If antecedent is denied in the minor instead of affirming, then the fallacy committed is called the *fallacy of denying the antecedent*. If the consequent is affirmed in the minor instead of denying, then the fallacy committed is called, the *fallacy of affirming the consequent*. *Modus Ponendo Ponens* is illustrated by 15. The rest of the structures (both valid and invalid) are given below.

Modus Tollendo Tollens

16 If C, then D.
not-D
∴ not-C

Fallacy of denying the antecedent	*Fallacy of affirming the consequent*
17 If E, then F.	18 If G, then H.
not- E.	H.
∴ not-F.	∴ G.

4. Disjunctive Syllogism (D.S.): In this subclass of syllogism the major premise (usually the first one) expresses alternatives connected by connectives, '*either… or*'. So they are constants too. Such a proposition is called disjunctive proposition. In disjunctive proposition the connective *either* is implicit many times. Therefore its presence or absence does not alter the structure of the proposition. Second premise and the conclusion are simple or general. In D.S. itself there are two types. While regarding these two types the emphasis is on the connective *or* because as mentioned above *either* is implicit many times. One use of *or* is called inclusive and another is called exclusive. *Or* is used in inclusive sense if both alternatives are admissible and it is used in exclusive sense when the alternatives are mutually exclusive and totally exhaustive and the acceptance of one alternative excludes the other. In the proposition 'either he is stupid or stubborn' *or* is used in inclusive sense because the same person may be both stupid and stubborn. However, in the proposition 'either he is generous or miser' *or* is used in exclusive sense because no one can be both generous and miser at the same time. In order to bring both usages under one class a rule is devised which says that one of the alternatives must be denied in the minor so that the remaining alternative is affirmed in the conclusion. The following argument illustrates the explanation.

19 Either he is stupid or stubborn.	Either A or B .
He is not stupid.	Not A.
∴ He is stubborn.	∴ B.

Here again there is no middle term. However, one component (A) appears in the first premise in affirmative mode and in the second in negative mode. This occurrence corresponds to the affirmative mode of common component in one premise and its negative mode in another in PHS. A disjunctive argument with this structure is identified as *Modus Tollendo Ponens.*

In a disjunctive proposition the components are commutable, i.e., ‘either A or B’ means the same as ‘either B or A’. Therefore in the minor premise any component can be denied. Affirming of a component in the minor premise is not permissible. If this rule is violated, then the fallacy committed is called the *fallacy of Modus Ponendo Tollens.* The following example illustrates this fallacy.

20 Either I or J

I

∴ not-J

However, this is not a fallacy if the alternatives are mutually exclusive and totally exhaustive.

Check Your Progress II

Note: Use the space provided for your answer.

1. Describe the structure of Pure Hypothetical Syllogism with an example.

....

....

....

2. Describe the structure of Disjunctive Syllogism with an example.

....

2.6 LET US SUM UP

Inference and implication are the essence of logic. Inference is psychological and implication is logical. Deduction and induction are two forms of logic. Deduction is formal and induction is material. Logical necessity characterizes the former and uncertainty characterizes the latter. Categorical syllogism, P. H. S., M. H. S. and D.S. are the kinds of syllogism accepted by traditional logic.

2.7 KEY WORDS

Inference: It is an operation of reason by which from some known truth we arrive at unknown truth.

Major term: The term occurring in the predicate of the conclusion in a categorical syllogism.

Middle term: The term occurring in both the major and the minor premises of a standard-form categorical syllogism.

Minor term: is the subject of the conclusion.

Major premise: The premise of a categorical syllogism that contains an instance of the major term and in conditional syllogism the conditional proposition.
Minor premise: The premise of a categorical syllogism that contains the minor term.

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UNIT 3 FIGURE, MOOD AND THE POSSIBLE TYPES OF SYLLOGISMS

Contents

3.0 OBJECTIVES

This unit proposes to introduce a very interesting aspect of syllogism, viz. figures and moods. Through a study of figures and moods you will be in a position to gain an insight into the intricacies of categorical syllogism. This is the main objective of this unit. Second objective is to introduce you to the abridged and extended versions of syllogism.

Another equally important objective is to bring out the features of dilemma which is a sort of pseudo- syllogism so that you will be in a position to contrast a genuine argument like syllogism with a pseudo-argument. Thereby another objective is also served. Your acumen to evaluate the logical significance is further sharpened. This is the most invaluable gift of logic.

3.1 INTRODUCTION

Arguments are of complex nature. It is not possible to bring all arguments, even arguments of one class, under a common head. A detailed analysis of syllogism reveals the hidden complexities of the same. Such a study consists in the discussion of the structure of syllogism which leads to figures and moods. A clear understanding of the structure of syllogism exposes the wealth of syllogistic argument. As usual, the premises have to be taken as true, whether or not they are factually true.

3.2 MOODS OF CATEGORICAL SYLLOGISM

In the previous unit a brief reference was made to what is known as 'mood'. It is not possible to fully appreciate the role played by moods in the study of syllogism without prior discussion of what is known as *figure*. Figure and mood together determine the structure of syllogism. An appraisal of the significance of structure in deductive inference in general and syllogism in particular is made much easier when we deal with ‘figures and moods’ of syllogism. An analysis of the structure of argument in deductive inference is a pre-requisite to the classification of arguments into good (valid) and bad (invalid). Since the very function of logic is to distinguish arguments in the aforesaid manner, a study of figure and mood occupies an important position in our study of syllogism. In order to simplify the task, let us state the arguments in what is called standard-form. Accordingly, the major premise is stated first followed by the minor premise and ending with the conclusion. The following example illustrates what standard-form means:

1 All humans are mortal.
Joseph is a human.
∴ Joseph is mortal.

Although arguments in ordinary language appear in several forms, it is not at all difficult to restate them in standard-form. First we identify the conclusion which is to be placed in the final position. Whichever premise contains the predicate term of the conclusion automatically occupies the first place because the major premise should be stated first (Kemerling 2010). We notice that ‘mortal’ is the predicate of the conclusion which appears in the first place in the argument followed by the minor premise. Therefore this type of arrangement subscribes to standard-form.

The Mood of a Syllogism

As mentioned earlier, there are four types of categorical proposition; universal affirmative (A), universal negative (E), particular affirmative (I), and particular negative (O). Since a syllogistic argument consists of three categorical propositions, they may occur in any order in the arguments. What is more interesting is the fact that the very same type of proposition may occur thrice. There is no restriction on the number of occasions on which a particular type of proposition occurs in an argument. For example, all three propositions in an argument may be A only. Or they may be I only. Briefly said, the mood of a syllogism is simply a combination of categorical propositions (A, E, I, or O) which the argument comprises of. Suppose that only O proposition comprises of an argument, then the mood of the argument is said to be OOO. Similarly, a syllogistic argument with a mood of OAO has an O proposition as its major premise, an A proposition as its minor premise, and another O proposition as its conclusion; and EIO has an E as its major premise, and an I as the minor premise, and an O as the conclusion; etc. (Kemerling 2010).

Let us consider another example.

2 A: All rocks are hard things.
E: No rocks are liquid.
I: ∴ Some liquid things are not hard.

The mood of this argument is AEI. This shows that every letter states symbolically the quantity and quality of propositions and every letter occurs in the very same order in which the propositions occur in the argument. Therefore the order in which the three letters occur specifies the mood of the syllogism. Consider the following syllogistic argument.

3 E: No women named Deepti are outer island Yapese women.

A: All outer island Yapese women are weavers of the baskets.
O: ∴ Some weavers of the baskets are not women named Deepti.

In the above syllogism the minor term (subject of the conclusion) is 'weavers of the baskets', the major term (predicate of the conclusion) is 'women named Deepti' and the middle term is 'outer island Yapese women'. Therefore the first premise is the major, second is the minor and the third is the conclusion.

The structure of these arguments is considered for the purpose of illustration. While symbolizing the propositions, let us use the first letter of the term. The letter which appears in the middle stands for the quality and quantity of propositions.

1 Major premise:	All H are M.	HAM	2 All R are H.	RAH
Minor premise:	J is H.	JAH	No R are L.	REL
Conclusion	∴ J is M.	JAM	∴ Some L are not H.	∴LOH

3 Major premise:	No W is Y.	W EY
Minor premise:	All Y is B.	Y AB
Conclusion	∴ Some B is not W.	∴B OW

One question remains to be answered. How many moods can we list? For the time being, let us restrict ourselves to an incomplete answer. Accordingly, we can list 64 Moods. (At this stage, let us not restrict ourselves to valid Moods). There is no need to list all these 64 Moods. But what is needed is to know how we arrive at this figure because the number is not fixed arbitrarily. There are four kinds of propositions which have to take three positions in such a manner that any proposition can occur in any one of the four different ways; 0, 1, 2 and 3. When we compute all possible arrangements, we arrive at 64. There are two important aspects. First, we have discovered a certain number of structures in which syllogistic arguments can be constructed, and secondly, which we notice later, not all structures to which arguments subscribe are valid. It is in this sense that the logical status of an argument is determined by the structure of that particular argument.

3.3 FIGURES OF SYLLOGISM:

It is easy to understand the meaning and significance of figure. The 'figure' of a syllogism is determined by the position of 'middle term'. We have said that the 'middle term' appears both in the major and in the minor premises. Therefore its possible positions in premises result in four different configurations. A schematic representation is preferable to verbal description.

Figure 1	Figure 2	Figure 3	Figure 4
M – P	P – M	M – P	P – M
S – M	S – M	M – S	M – S
S – P	S – P	S – P	S – P

From this scheme it is clear that neither P nor S determines the figure of syllogism. History has recorded that Aristotle accepted only the first three figures. The origin of the fourth figure is disputed. While Quine said that Theophrastus, a student of Aristotle, invented the fourth figure, Stebbing said that it was Gallen who invented the fourth figure. This dispute is not very significant. But what Aristotle says on the first figure is significant.

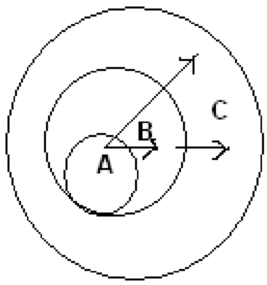
Aristotle regarded the first figure as most 'scientific'. It is likely that by 'scientific' he meant 'satisfactory'. One of the reasons, which Aristotle has adduced in defence of his thesis, is what the nature of laws of mathematics and physical sciences suggest. According to him these sciences establish laws in the form of the first figure. Second reason is that reasoned conclusion or reasoned fact is generally found, according to Aristotle, in the first figure. Aristotle believed that only universal affirmative conclusion can provide complete knowledge and universal affirmative conclusion is possible only in the first figure. Aristotle quotes the fundamental principle of syllogism. 'One kind of syllogism serves to prove that A inheres in C by showing that A inheres in B and B in C'. This principle can be expressed in this form:

Minor: A inheres in B

Major: B inheres in C

∴ A inheres in C

Evidently, this argument satisfies transitive relation. This is made clear with the help of this diagram:



Let us consider four examples, which correspond to four figures.

I

	M P	
Major Premise:	All artists are poets.	AAP
	S M	
Minor Premise:	All musicians are artists.	MAA
Conclusion:	∴All musicians are poets.	MAP

S P

II

	P M	
Major Premise:	All saints are pious.	SAP
	S M	
Minor Premise:	No criminals are pious.	CEP
Conclusion:	∴ No criminals are saints.	CES
	S P	

III

	M P	
Major Premise:	All great works are worthy of study.	GAW
	M S	
Minor Premise:	All great works are epics.	GAE
Conclusion:	∴ Some epics are worthy of study.	EIW
	S P	

IV

	P M	
Major Premise:	No soldiers are traitors.	SET
	M S	
Minor Premise:	All traitors are sinners.	TAS
Conclusion:	∴ Some sinners are not soldiers.	SOS
	S P	

We will consider figures in conjunction with moods. Then only knowledge of the 'figure of syllogism' permits us to compute the total number of possible moods. Mood is determined by quality and quantity of propositions, which constitute syllogism. Since there are four figures, in all two hundred and fifty six ways of arranging categorical propositions is possible. These are exactly what we mean by moods. However, out of two hundred and fifty-six, two hundred and forty-five moods can be shown to be invalid by applying the rules and corollaries. So we have only eleven valid moods. Even this is not sufficient to have a clear picture. There is no figure in which all eleven moods are valid. Within the framework of traditional logic, in any given figure only six moods are valid. They are as follows:

I AAA, *AAI,* EAE, *EAO,* EIO and AII

II AEE, *AEO*, EAE, *EAO*, EIO and AOO

III ***AAI,*** AII, IAI, ***EAO,*** EIO and OAO

IV ***AAI***, IAI, AEE, *AEO*, ***EAO***, and EIO

In all these cases, first letter stands for the major premise, second for the minor and third for the conclusion. Moods are represented above in three ways. Moods in italics and bold form are called strengthened moods, and moods in mere italics are called weakened moods. All other moods are represented in normal form. It is important to know the difference between the first two types. When the laws of syllogism permit two universal premises to yield logically only particular conclusion, then such moods are called strengthened moods. On the other hand, if we deduce particular conclusion from two universal premises, even when the laws of syllogism permit two universal premises to yield logically a universal conclusion, then such moods are called weakened moods.

In this scheme, we notice that EIO is valid in all the figures. Interestingly, IEO is invalid in all the figures. The only difference between EIO and IEO is that the minor and the major premises are only transposed which clearly shows that the position of premises, which is a part of the structure, determines the validity of argument. Though EIO is valid in more than one figure it is one mood in one figure and some other in another figure. Likewise, AEE is valid in the second and the fourth figures. But it is one mood in the second figure and a different mood in the fourth figure.

Since Aristotle argued that the first figure is the perfect figure, he felt the need to transmute all valid arguments in II and III figures to I figure so that if the transmuted mood is valid in I figure, then the corresponding mood in any figure other than the first is also valid. Transmutation from fourth figure to the first figure must have been evolved by the inventor of the former. Reduction is the tool to test the validity of arguments. In the thirteenth century, one logician by name Pope John XXI, devised a technique to remember the method of reducing arguments from other figures to the first figure. This technique is known as mnemonic verses. Accordingly, each mood, excluding weakened moods, was given a special name:

I.	Fig:	AAA	BARBARA	III.	Fig:	AAI	DARAPTI
		EAE	CELARENT			IAI	DISAMIS
		AII	DARII			AII	DATISI
		EIO	FERIO			EAO	FELAPTON
						OAO	BOCARDO
						EIO	FERISON
II.	Fig:	EAE	CESARE	IV.	Fig:	AAI	BRAMANTIP
		AEE	CAMESTRES			AEE	CAMENES

EIO	FESTINO	IAI	DIMARIS
AOO	BAROCO	EAO	FESAPO
		EIO	FRESISON

The method is like this. If the names begin with C, then the syllogism has to be reduced to the first figure which begins with a C. For example, CESARE (a syllogism of the second figure) has to be reduced to CELARENT. Other consonants of the name have also their significance; ‘s’ (like in CESARE) signifies that the preceding ‘E’ needs to undergo simple conversion; ‘p’ signifies that the preceding proposition has to be converted by ‘limitation’; ‘t’ signifies that the order of the premises has to be changed; ‘st’ indicates that two operations, viz., simple conversion and transposition of the proposition represented by the preceding vowel are required to be carried out. BAROCO and BOCARDO are reduced in a different manner. O propositions in both the moods have to be obverted first and then follow the relevant path to effect reduction.

However, the situation in modern logic is very different. The logicians proved that from universal propositions alone particular proposition cannot be derived and vice versa. Accordingly, both strengthened and weakened moods become invalid. Thus in the new scheme the number of valid moods reduces to fifteen.

Check Your Progress I

Note: Use the space provided for your answers.

1. What are the factors which determine the mood of a syllogism?

....

....

....

2. Discuss the significance of the ‘figure’ of categorical syllogism.

....

....

....

3.4 INCOMPLETE SYLLOGISM AND COMPOUND SYLLOGISM

1. **Enthymeme**: Enthymeme is called an incomplete syllogism in which one or the other proposition is not stated explicitly. As a matter of fact, such an incomplete syllogism is closer to the way we generally argue in everyday life. If standard–form is the criterion, then it is not logically valid unless what is implicitly understood is taken into consideration. That is, it must be formally completed.

Examples: 1 You have hurt your neighbour.
Therefore you have sinned against God.
(Major premise implicitly understood: Those who hurt their neighbours sin against God).
2 Those who hurt their neighbours sin against God.
Therefore you have sinned against god.
(Minor premise implicitly understood: You have hurt your neighbour).
3 Those who hurt their neighbour sin against God.
And you have hurt your neighbour.
(Conclusion implicitly understood: Therefore you have sinned against God).

When the major premise is implicitly understood, enthymeme is regarded as the first-order enthymeme. When the minor premise is implicitly understood, enthymeme is regarded as the second-order enthymeme. When the conclusion is implicitly understood, enthymeme is regarded as the third-order enthymeme. A question may arise in this context. If two propositions are adequate to convey the information, where is the need to have full-fledged syllogism? This question can be answered in two ways. When we deal with learned or well-informed persons or with ourselves, enthymeme will surely serve the purpose. A full – fledged syllogism is needed when we have to educate not so well – informed, if not ill – informed persons. We should not fail to notice close similarity between enthymeme and *svarthaanumana* and *paraarthaanumana* (inference for self and inference for others). The question can be answered in this way also. Syllogism is formal and enthymeme is informal. Choice is subjective.

2 Sorites: If an argument consists of three or more than three premises, then such an argument is called sorites. It is also called polysyllogism. There are two kinds of sorites: Aristotelian sorites and Goclenian sorites. The primary rules which govern sorites are the rules of the categorical Syllogism only.

Let us begin with the structure of sorites. In Aristotelian sorites the first premise is minor and the last premise is major. In consecutive premises M is predicate in the first premise and in the next premise subject. In sorites there are two or more than two conclusions which are implicit. Every such hidden conclusion functions as the premise. Therefore a sorites consists of at least three syllogistic arguments and hence it consists of a chain of syllogisms which are interrelated. In order to arrive at the final conclusion these hidden conclusions also must be reckoned.
Consider this example.
1

Premises	Hidden conclusions (a and b)
1. All A are B.	a. All A are C.
2. All B are C.	
3. All C are D.	3 All C are D.
	__________
	b. All A are D.
4. All D are E	All D are E
∴ All A are E.	

It is easy to understand this structure. From (1) and (2) we have derived (a). This is hidden because at no point of time is this expressed. When this is conjoined with (3), (a) becomes a premise. So is the case with b. This shows that every hidden conclusion is, in fact, the premise of next argument. In this argument 'a' and 'b' are hidden conclusions which become premises at subsequent stages. In Aristotelian sorites, the subject of the first premise is also the subject of the conclusion and the predicate of the last premise is also the predicate of the conclusion. In the set of hidden conclusions also the same pattern can be noticed. This pattern shows that in Aristotelian sorites the first premise is the minor and the last premise is the major.

Let us consider the rules of Aristotelian sorites.
1 Only major premise (last premise) can be negative.

2 only minor premise (first premise) can be particular.

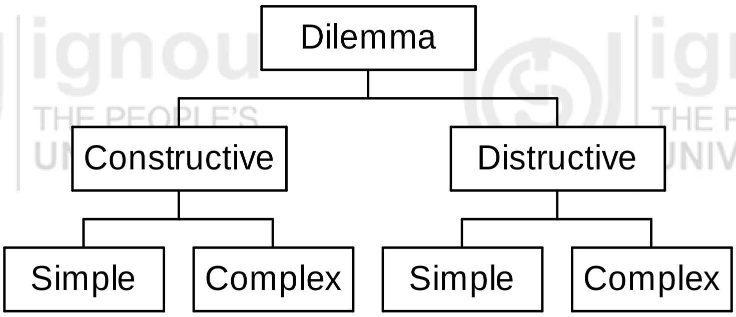
In Goclenian sorites the order is reversed. Consider this example.
2

Premises	Hidden conclusion (a and b)
1 All A are B.	a All C are B.
2 All C are A.	
3 All D are C.	<u>3 All D are C.</u>
	b All D are B.
4 All E are D.	4 All E are D.
∴ All E are B.	

In this kind the predicate of the conclusion is the predicate of the first premise. Therefore the first premise is major. The subject of the conclusion is the subject of the last premise. Therefore the last premise is the minor. The rules of this kind are as follows.
1 Only the first premise (major) can be negative.
2 Only the last premise (minor) can be particular.
One point should become clear at this stage. One kind of sorites is the reversal of the other. If we disregard the positions of premises, then the difference between these two kinds becomes insignificant.

3.5 DILEMMA

The dilemma consists of three propositions of which two constitute premises and third one is the conclusion. One of the premises is a conjunction of two hypothetical propositions and the other one is disjunctive. The conclusion is either disjunctive or simple. Since the dilemma consists of two hypothetical propositions conjoined by the word 'and', it is possible that two different propositions are found in place of antecedents and two different propositions are found in place of consequents. But it is not necessary that it should be so. It is likely that both propositions have a common consequent. If such consequent becomes the conclusion, then, the conclusion is a simple proposition.


Dilemma
Constructive
Distructive
Simple
Complex
Simple
Complex

has structure similar to that of p2 of CDD. Now, we can make a list of common features of different kinds of dilemma.

	Dilemma	Common Features
1.	Constructive	Different antecedents
2.	Destructive	Different consequents
3.	Complex	Disjunctive conclusion
4.	Simple	Simple conclusion

3.6 AVOIDING DILEMMA

Use of dilemma is restricted to some situations. When neither unconditional affirmation of antecedent nor unconditional denial of consequent is possible, logician may use this route. It indicates either ignorance or shrewdness. When we face dilemma, we only try to avoid, but not to refute. There are three different ways in which we can try to avoid dilemma. All these ways only reflect escapist tendency. Only an escapist tries to avoid a problematic situation. Therefore, in logic they do not carry much weight.

1. Escaping between the horns of dilemma: Two consequents mentioned may be incomplete. If it is possible to show that they are incomplete then we can avoid facing dilemma. This is what is known as 'escaping between the horns of dilemma'. It should be noted that even when third consequent is suggested it does not mean that this new consequents is actually true. In other words, the new consequent also is questionable.

2. Taking the dilemma by horns: In this method of avoiding dilemma, attempts are made to contradict the hypothetical propositions, which are conjoined. A hypothetical proposition is contradicted when antecedent and negation of consequent are accepted. However, in this particular case it is not attempted at all. Moreover, since the major premise is a conjunction of two hypothetical propositions, the method of refutation is more complex. (The negation of conjunction will be introduced at a later stage. For the time being it is enough to know that in this particular instance there is no such attempt.)

3. Rebuttal of dilemma: It appears to be the contradiction of dilemma. But, in reality, it is not. In all these cases, the dilemma becomes a potent weapon to mislead the opponent in debate. Therefore none of these methods amounts to the contradiction of opponent's view.

We will consider examples for four kinds, which can be used to illustrate these methods.

i). Complex Constructive Dilemma (CCD):

p

p_1: **If** (any government wages war to acquire wealth), **then** (it becomes a

q r

rogue government) and **if** (it wages war to expand its territory), **then** (it

s

becomes colonial).

p r

p_2: (Any government wages war either to acquire wealth) **or** (to expand its

territory) q s

q: It (becomes a rogue government) **or** (colonial).

ii). Simple Constructive Dilemma (SCD):

p q

p_1: **If** (taxes are reduced to garner votes), **then** (the government loses revenue).

r

and **if** (taxes are reduced in order to simplify taxation), **then** (the

q

government loses revenue).

p r

p_2: (Taxes are reduced either to garner votes) **or** (to simplify taxation)

q

q : ∴ (The government loses its revenue).

iii). Complex Destructive Dilemma (CDD):

p q

p_1: **If** (the nation wages war), **then** (there will be no problem of

r

unemployment) and **if** (the nation does not revise her industrial policy),

s

then (it will lead to revolution).

not- q not - s

p_2: The (problem of unemployment remains unsolved) **or** (there will not be any revolution).

not - p not - r

q : ∴ (The nation does not wage war) **or** (the nation will revise her industrial policy).

iv). Simple Destructive Dilemma (SDD):

p q

p_1: **If** (you are in the habit of getting up early), **then** (you are a

p r

theist) and **if** (you are in the habit of getting up early), **then** (you are a labourer).

not - q not - r

p_2: (you are not a theist) or (you are not a labourer).

not - p

q : ∴ (you are not in the habit of getting up early).

The first way of avoiding the dilemma, i.e., escaping between the horns of dilemma can be illustrated using 1 (CCD). It is possible to argue that, when the government wages war, the motive is neither to acquire wealth nor to expand its territory in which case, the government is neither rouge nor colonial. The motive may be to spread its official religion or personal vendetta or it may be to protect its interests. If the last one is the motive, then, it becomes difficult to find fault with such government. Any one of the proposed alternatives or all alternatives to disjuncts may be false. There is no way of deciding what the situation is. The reader can select remaining examples to illustrate this method. Likewise, consider fourth argument to illustrate the second method. I may concede that a person gets up early only because he wants to maintain health. So the purpose is not to worship God. Nor is he a labourer. Again, this is also an assumption.

Rebutting of dilemma requires a different type of example. Consider this one:

i).

p q

p_1: If (teacher is a disciplinarian), then (he is unpopular among students) and

¬ p r

if (he is not a disciplinarian), then (his bosses do not like him).

p ¬ p

p_2: (Teacher is a disciplinarian) or (he is not a disciplinarian).

q v r

q : ∴ (Teacher is unpopular among students) or (his bosses do not like him).

A witty teacher may respond in this way.

ii).

¬ p q

p_1 : If (teacher is not a disciplinarian), then (he is popular among students)

p r

and if (he is a disciplinarian) then (his bosses will like him.)

¬p p

p_2 : (Teacher is not a disciplinarian) or (he is a disciplinarian)

q ¬ r

q : ∴ (Teacher is popular among students) or (his bosses will like him)

Only a student of logic discovers that these conclusions of i and ii are not contradictories (you will learn about it in the forthcoming units) in the strict sense of the term. Hence, there is really no rebuttal.

Further, the dilemma, which an individual faces in day-to-day life, is very different. For example, moral dilemma has nothing to do with the kinds of dilemma which we have discussed so far.

Since the dilemma is a medley of both types of conditional propositions, i. e., hypothetical and disjunctive, it should follow the basic rules of hypothetical and disjunctive syllogisms. It should affirm disjunctively the antecedents in the minor or deny disjunctively the consequents in the minor. The dilemma is powerful if in the major there is a strong cause-effect relationship between the antecedent and the consequent and in the minor the alternatives are exhaustive and mutually exclusive. Again, the former is debatable.

Check Your Progress II

Note: Use the space provided for your answer.

1. What are the characteristics of dilemma?

....

....

....

2. What are the methods of avoiding dilemma?

....

....

....

3.7 LET US SUM UP

The structure of syllogism is determined by figures and moods. The position of the middle term determines the figure to which syllogism belongs. There are four figures and eleven valid moods. Strengthened and weakened moods are not valid according to modern logic. The dilemma is a shrewd way of getting out of trouble. Escaping between the horns of dilemma, taking the dilemma by horns and rebuttal of dilemma are the ways of avoiding dilemma. Dilemma is not a sound logical way of arguing.

3.8 KEY WORDS

Figure: 'figure' of a syllogism is determined by 'middle term'.

Mood: 'mood' of a syllogism is determined by the 'quantity' and 'quality' of the three propositions.

Dilemma: A dilemma in logic means an argument that presents an antagonist with a choice of two or more alternatives, each of which appears to contradict the original contention and is inconclusive. The dilemma is a powerful instrument of persuasion and a devastating weapon in controversy.

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UNIT 4 VALIDITY, INVALIDITY AND LIST OF VALID SYLLOGISMS

Contents

4.0 OBJECTIVES

This unit brings out the most important part of your study of categorical syllogism. You will be introduced to the rules which determine the validity of arguments. While this is the most important objective, the icing on the cake is the variety of the methods of determining the validity of arguments. Both traditional and modern methods of testing the validity receive due recognition in this unit. Therefore contribution of both John Venn and George Boole find place in this unit. This particular study enables you to grasp the relation between logic and set theory which is brought to the fore in this unit.

4.1 INTRODUCTION

In the second and third units we learnt two important aspects of categorical syllogism, viz., figures and moods. However, we did not develop the technique of distinguishing valid from invalid arguments. Consequently, we could not know under what conditions a mood becomes valid and what is still worse, we could not understand why a certain arrangement or configuration of propositions in one figure is legitimate (only a legitimate combination of propositions yields valid mood) and in some other figure illegitimate yielding only invalid moods, and conversely, why a certain configuration of propositions is illegitimate in some figures and legitimate in some other figure or figures. In other words, the question what makes an argument valid was not raised at all. The point is that the validity of an argument depends on whether or not the conclusion is a conclusion in the strict sense of the word, i.e. whether or not it logically follows from the premises. This brings us to the vital aspect of our study. Just as application or non-application of rules makes a game legitimate or illegitimate, mere application or non-application of rules makes an argument valid or invalid. Application of rules demands knowledge of rules. Therefore we must focus on the question what rules are there which determine the validity of syllogism.

4.2 THE RULES OF CATEGORICAL SYLLOGISM

Classical Logic lists eight rules of valid categorical syllogism; four of them concern the terms, and four of them concern the propositions. These rules are not provable. They have to be either accepted or rejected. If they are rejected, syllogism is not possible. Therefore what is given is only an explication of the rules. Classical logic classified these rules under *rules of structure*, *rules of distribution of terms*, *rules of quality*, and *rules of quantity*.

I. Rules of structure

1. Syllogism must contain three, and only three, propositions

Syllogism is defined as a kind of mediate inference consisting of two premises which together determine the truth of the conclusion. This definition shows that if the number of propositions is more than two, then it ceases to be syllogism. Therefore by definition syllogism must consist of two premises and one conclusion. Therefore together they make up for three propositions.

2. Syllogism must consist of three terms only

A proposition consists of two terms. However, three propositions consist of only three terms because each term occurs twice. Suppose that there are four terms. Then there is no middle term, a term common to two premises. In such a case the violation of rule results in a fallacy called fallacy of four terms. Such a fallacy is never committed knowingly because knowing fully well the fixed number of terms, we do not choose four terms. But we do it unknowingly. It happens when an ambiguous word is used in two different senses on two different occasions. Then there are really four terms, not three terms. If an ambiguous word takes the place of middle term, then the fallacy committed is known as fallacy of ambiguous middle. Similarly, if an ambiguous term takes the place of the major or the minor term, then the fallacy of ambiguous major or ambiguous minor, as the case may be, is committed. The following argument illustrates the fallacy of ambiguous middle.

Fallacy of ambiguous middle:

1

All *charged* particles are electrons.
Atmosphere in the college is *charged*.
$\therefore$ Atmosphere in the college is an electron.

The word in italics is ambiguous. The other two fallacies are hardly committed. Therefore there is no need to consider examples for them. The moral is that all sentences in arguments must be unambiguous. This is possible only when all terms are unambiguous in the given argument. We must also consider the inversion of ambiguous middle. Suppose that synonymous words are used in place of middle term. Then apparently there are four terms. But, in reality, there are three terms. For example *starry world* and *stellar world* are not two terms. Such usages also are uncommon. Hence they deserve to be neglected.

II. Rules of distribution of terms:

1. Middle term must be distributed at least once in the premises. If this rule is violated, then the argument commits the fallacy of undistributed middle. One example will illustrate this rule.

2

All circles are geometrical figures.
All squares are geometrical figures.
∴ All circles are squares.

2. In the conclusion, no term may be taken in a more 'extensive' sense than in the premises. It also means that a term which is distributed in the conclusion must remain distributed in the respective premise. This rule can be stated this way also. A term which is undistributed in the premise must remain undistributed in the conclusion. However, it is not necessary that a term, which is distributed in the premise, must be distributed in the conclusion.

Suppose that the major term violates this rule. Then the argument commits the *fallacy of illicit major*. When the minor term violates this rule, *fallacy illicit minor is committed*. The following arguments illustrate these fallacies.

3

All philosophers are thinkers.
No ordinary men are philosophers.
∴ No ordinary men are thinkers.

4

All aquatic creatures are fish.
All aquatic creatures swim.
∴ All those which swim are fish.

First argument illustrates the fallacy of undistributed middle; second illustrates the fallacy of illicit major and the third illustrates the fallacy of illicit minor.

III. *Rules of quality:*

1. From two negative premises, no conclusion can be drawn. It only means that at least one premise must be affirmative.
2. If both premises are affirmative, the conclusion cannot be negative. Negatively, it only means that a negative conclusion is possible only when one premise is negative.

IV *Rules of quantity:*

If both premises are particular, no conclusion can be drawn or the conclusion must always follow the weaker part. Here weaker part is particular. This rule shows that at least one premise must be universal.

If one premise is particular, then the conclusion must be particular only. It means that universal conclusion is possible only when both premises are universal. In practice, last three sets of rules play an important role in determining the validity of categorical syllogism.

Check Your Progress I

Note: Use the space provided for your answers.

Examine the following arguments.

1) All kings are thinkers.
   Some ordinary men are not kings.
   ∴ No ordinary men are thinkers.

....
....
....

2) All stars are bright.
   All bright objects are attractive.
   ∴ All attractive objects are stars.

....
....

3) Some radicals are good men.
   Some good men are honest.
   ∴ Some radicals are honest.

....
....
.... 4

4) The monkey is nonhuman.
   Some of those who are of capable of laughter are humans.
   ∴ The monkey is not capable of laughter.

....
………………………………………………………………………………………

4.3 SPECIAL APPLICATIONS OF THE GENERAL RULES

In the previous unit we learnt that a certain arrangement of categorical propositions is legitimate in one figure and illegitimate in some other figure, the only exception being EIO. For the purpose of contrast we should recognize that its reversal, IEO, is invalid in all the figures. One way of recognizing valid or invalid arguments is the use of rules listed above. We have another method known as 'special rules of figures'. These rules are called special rules because they apply to only that particular figure but not to others. These rules are dependent upon general rules. Therefore it is possible to give proofs to these rules.

I Figure

a) The minor must be affirmative.
b) The major must be universal.

$$\begin{array}{c} M - P \\ \underline{S - M} \\ S - P \end{array}$$

a) If the minor is negative, then the conclusion must be negative. In negative conclusion P is distributed while it is undistributed in the major premise. This goes against the rule which asserts that a term undistributed in the premise should remain undistributed in the conclusion. Therefore minor must be affirmative.
b) That the major must be universal is clear from the fact that if the minor is affirmative, M in it is undistributed and therefore the major must be universal if M must be distributed in it.

Now we shall apply these special rules to know how or why a certain mood is valid and certain other moods invalid in a figure, a point which we discussed in the previous unit. Let us omit weakened moods.
The valid moods of I figure are listed below.

I Fig:	AAA	BARBARA
	EAE	CELARENT
	AII	DARII
	EIO	FERIO

II Figure:
a) One premise must be negative.
b) The major must be universal.

P – M
S – M
―――――
S- P

a) One premise must be negative. Otherwise, M remains undistributed in both the premises. b) The major must be universal because P is distributed in negative conclusion and hence it must be distributed in the major.

The valid moods of II figure are listed below.

EAE	CESARE
AEE	CAMESTRES
EIO	FESTINO
AOO	BAROCO

III Figure:
a) The 'minor' must be affirmative.
b) The conclusion must be Particular.

M – P
M – S
―――――
S – P

a) Minor must be affirmative because negative minor gives only negative conclusion in which case P is distributed in the conclusion. P can be distributed in major only if it is negative. Negative minor results in negative major which is not allowed. Therefore minor must be affirmative.
b) The conclusion must be particular. Otherwise S becomes distributed in the conclusion while it remains undistributed in affirmative minor.

The valid moods are listed below.

III	**Fig:**	AAI	DARAPTI
		IAI	DISAMIS
		AII	DATISI
		OAO	BOCARDO
		EIO	FERISON
		EAO	FELOPTON

IV Figure:

a) If the 'major' is affirmative, the 'minor' must be universal.
b) If the minor is affirmative, the conclusion must be particular.
c) If the conclusion is negative, the major must be negative.

P – M
M – S
_____
S – P

The valid moods are listed below.

AAI	DARAPTI
AEE	CAMENES
EAO	FESAPO
IAI	DIMARIS
EIO	FRESISON

It would be good logical exercise for the student to take up these special rules and try to deduce them from the general ones. This is the reason why we have left the special rules of figure 4 unexplained.

4.4 REDUCTION OF ARGUMENTS TO I FIGURE

Reducing arguments from other figures to the first figure is one of the techniques developed by Aristotle and one of his followers to test the validity of arguments. After reduction, if the argument is valid in the first figure, then it means that the original argument in the corresponding figure is valid. This technique is quite mechanical. So we are only required to know what exactly is involved in this method. We will learn this only by practice. Strengthened moods are included for the sake of exercise though they are not required from the point of view of modern logic. There is no need to consider weakened moods separately when the technique involved is reduction. What is required is replacement of universal by its corresponding subaltern in the conclusion.

1

II Figure

II Figure		I Figure
CESARE		CELARENT
P E M	→ Conversion→	M E P
S A M		S A M
S E P		S E P

No politicians are poets.	→ Conversion →	No poets are politicians.
All girls are poets.		All girls are poets.
∴No girls are politicians.		∴No girls are politicians.

In CESARE 'S' after 'E' indicates simple conversion. It shows that 'E' (major premise) must undergo simple conversion.

2

II Figure

II Figure	I Figure
CAMESTRES	CELARENT
P A M	M E S
S E M → conversion M E S	P A M
∴S E P	∴P E S conversion→S E P

'S' and 'T' after 'E' show that 'E' (minor premise) should undergo simple conversion and both premises be transposed. 'S' after second 'E' shows that this 'E' (conclusion) also should undergo simple conversion. [The student is advised to construct arguments for this and subsequent reductions.]

3

II Figure

II Figure		I Figure
FESTINO		FERIO
P E M	→ Conversion →	M E P
S I M		S I M
S O P		S O P

FESTINO becomes FERIO when the major premise undergoes simple conversion. The kind of reduction of the above mentioned moods is known as direct reduction. BAROCO becomes

FERIO through the process of indirect reduction. Indirect reduction includes, in addition to conversion, obversion also.

4

II Figure

II Figure — I Figure
BAROCO — FERIO
P A M → obversion P E $\overline{M}$ → Conversion → $\overline{M}$ E P
S O M → obversion S I $\overline{M}$ — S I $\overline{M}$
S O P — S O P

5

III Figure

III Figure — I Figure
DARAPTI — DARII
M A P — M A P
M A S → Conversion→ S I M
S I P — S I P

6

DATISI — DARII
M A P — M A P
M I S → Conversion→ S I M
S I P — S I P

7

FELAPTON — FERIO
M E P — M E P
M A S → Conversion→ S I M
S O P — S O P

'P' which follows 'A' in DARAPTI and FELAPTON shows that conversion by limitation applies to 'A'.

8

FERISON — FERIO
M E P — M E P
M I S → Conversion→ S I M
S O P — S O P

9

DISAMIS — DARII
M I P — M A S — M A S
M A S — M I P → Conversion → P I M
S I P — P I S
↓ Conversion
S I P

While the reduction of the above-mentioned moods is direct, next one is indirect.

10

BOCARDO

M O P → Obversion → M I $\bar{P}$ → Conversion → $\bar{P}$ I M → M A S

M A S   M AS   M A S → $\bar{P}$ I M

S O P

$\bar{P}$ IS

↓Conversion

S I $\bar{P}$

↓Obversion

S O P

When BOCARDO undergoes reduction, conversion, obversion and transposition are required to complete the process. Here OAO becomes AII. Further, when we consider obverted conclusion of AII, we obtain AIO. This is, surely, a paradox.

11

IV Figure	IV Figure
	I Figure
BRAMANTIP	Weakened mood
P A M	M A S
M A S	P A M
S I P	P A S → Conversion →S I P

12

CAMENES	CELARENT
P A M	M E S
MES	PAM
S E P	S E P

13

DIMARIS	DARII
P I M	M A S
M A S	P I M
S I P	S I P

14

FESAPO		FERIO
P E M	→ Conversion→	M E P
M A S	→ Conversion→	S I M
S O P		S O P

As usual ‘S’ stands for simple conversion of ‘E’ (major Premise) and ‘P’ stands for conversion by limitation of ‘A’ (minor premise). This process is similar to the one applied for first and third moods of III figure.

15

FRESISON		FERIO
P E M	→ Conversion→	M E P
M I S	→ Conversion→	S I M
S O P		S O P

From reduction technique one point becomes clear. Originally, there were twenty-four valid moods. Later weakened and strengthened moods were eliminated on the ground that particular proposition (existential quantifier) cannot be deduced from universal propositions (universal quantifier) only, and the number was reduced to fifteen. Now after reduction to first figure the number came down to four. Strawson argues that reduction technique is superior to axiomatic technique to which he referred in the beginning of his work ‘Introduction to Logical Theory’. He regards the moods as inference-patterns. He argues that the path of reduction should be an inverted pyramid. Strawson also maintains that in addition to equivalence relation, we require opposition relation also to effect reduction. What we gain in the process is economy in the number of moods.

4.5 ANTILOGISM OR INCONSISTENT TRIAD

This technique was developed by one lady by name, Christin Lad Franklin. This technique applies only to fifteen moods. The reason is, again, impropriety of deriving existential from universals only. The method is very simple. Consider Venn’s results for all propositions. Replace the conclusion by its contradiction. This arrangement constitutes antilogism. If the corresponding argument should be valid, then antilogism should conform to certain structure. It must possess two equations and one inequation. A term must be common to equations. It should be positive in one equation and negative in another. Remaining two terms ought to appear only in inequation. Consider one example for a valid argument.

	Venn’s Results	**Antilogism**
All Indians are Asians.	$I\bar{A} = \emptyset$	$I\bar{A} = \emptyset$
All Hindus are Indians.	$H\bar{I} = \emptyset$	$H\bar{I} = \emptyset$
∴All Hindus are Asians.	$H\bar{A} = \emptyset$	$H\bar{A} \neq \emptyset$

In this case, antilogism satisfies all the requirements. ‘I’ is common to equations; in one equation it is positive and in another negative. There is only one inequation. Remaining terms appear in inequation. In all cases, this is the method to be followed. If any one of these characteristics is absent in antilogism, then the corresponding mood is invalid.

Now antilogism can be easily constructed for the remaining fourteen moods.

1 Fig.

1) CELARENT

Contradiction

M E P		$MP = \emptyset$
S A M		$S\overline{M} = \emptyset$
S E P →	SIP	$SP \neq \emptyset$

2) DARII

M A P		$M\overline{P} = \emptyset$
S I M		$SM \neq \emptyset$
S I P →	SEP	$SP = \emptyset$

3) FERIO

M E P		$MP = \emptyset$
S I M		$SM \neq \emptyset$
S O P →	SAP	$S\overline{P} = \emptyset$

II Fig.

4) CESARE

P E M		$PM = \emptyset$
S A M		$S\overline{M} = \emptyset$
S E P →	SIP	$SP \neq \emptyset$

5) CAMESTRES

P A M		$P\overline{M} = \emptyset$
S E M		$SM = \emptyset$
S E P →	SIP	$SP \neq \emptyset$

6) FESTINO

P E M		$PM = \emptyset$
S I M		$SM \neq \emptyset$
S O P →	SAP	$S\overline{P} = \emptyset$

7) BAROCO

P A M		$P\overline{M} = \emptyset$
S O M		$S\overline{M} \neq \emptyset$
S O P →	SAP	$S\overline{P} = \emptyset$

III Fig.

8) DISAMIS

M I P		$MP \neq \emptyset$
M A S		$M\overline{S} = \emptyset$
S I P →	SEP	$SP = \emptyset$

9) DATISI

M A P		$M\bar{P} = \emptyset$
M I S		$MS \neq \emptyset$
S I P →	SEP	$SP = \emptyset$

10) BOCARDO

M O P		$M\bar{P} \neq \emptyset$
M A S		$M\bar{S} = \emptyset$
S O P →	SAP	$S\bar{P} = \emptyset$

11) FERISON

M E P		$MP = \emptyset$
M I S		$MS \neq \emptyset$
S O P →	SAP	$S\bar{P} = \emptyset$

IV Fig.

12) CAMENES

P A M		$P\bar{M} = \emptyset$
M E S		$MS = \emptyset$
S E P →	SIP	$SP \neq \emptyset$

13) DIMARIS

P I M — $PM \neq \emptyset$

M A S — $M\bar{S} = \emptyset$

S I P → SEP — $SP = \emptyset$

14) FRESISON

P E M — $PM = \emptyset$

M I S — $MS \neq \emptyset$

S O P → SAP — $S\bar{P} = \emptyset$

Now consider a weakened mood.

II Fig.

Weakened mood:

P A M — $P\bar{M} = \emptyset$

S E M — $SM = \emptyset$

S O P → SAP — $S\bar{M} = \emptyset$

There is no inequation in this antilogism. Hence, corresponding argument is invalid. It can be shown that any other strengthened or weakened mood is invalid.

4.6 VENN DIAGRAM TECHNIQUE

Let us extend our knowledge of Venn diagram to the testing of arguments. If two terms yield four product classes, then three terms should yield eight product classes according to the formula $2^x = n$, where x stands for the number of terms and n stands for the number of product classes. Since syllogism consists of three terms, we have eight product classes. Let us begin with a valid mood and list these product classes.

1 BARBARA

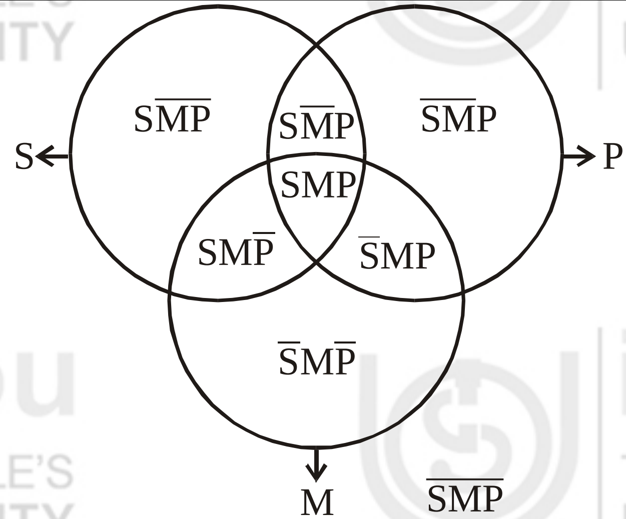
p1: All M are P. — $M\bar{P} = \Phi$

p2: All S are M. — $S\bar{M} = \Phi$

q: ∴ All S are P — $S\bar{P} = \Phi$

The product classes are as follows: -

SMP
$\overline{SMP}$
$SM\overline{P}$
$S\overline{MP}$
$\overline{S}\,\overline{M}P$
$\overline{S}MP$
$\overline{S}M\overline{P}$
$S\overline{M}P$



While listing product classes, sufficient care should be taken to ensure that no product class is repeated. It is always advisable to make a list of product classes with diagrams and mark classes accurately to avoid confusion.

Now let us use diagram to represent the propositions. The procedure is as follows. null sets are shaded and non-null sets are starred. We should also note that product of null set and non-null set is a null set. It is like saying that 4 x 0 = 0. But the union, i.e., addition of a non-null set and null set is a non-null set. Remember 4 + 0 = 4.

Since $M\overline{P}$ is a null set, not only $SM\overline{P}$, but also $\overline{S}M\overline{P}$ is a null set. It does not mean that there are two null sets. There is only one null set. $S\overline{M}$ is also a null set. Therefore not only the product of $S\overline{M}$ & P, but also $S\overline{M}$ and $\overline{P}$ is a null set. Now we shall shade relevant subsets, which are null.

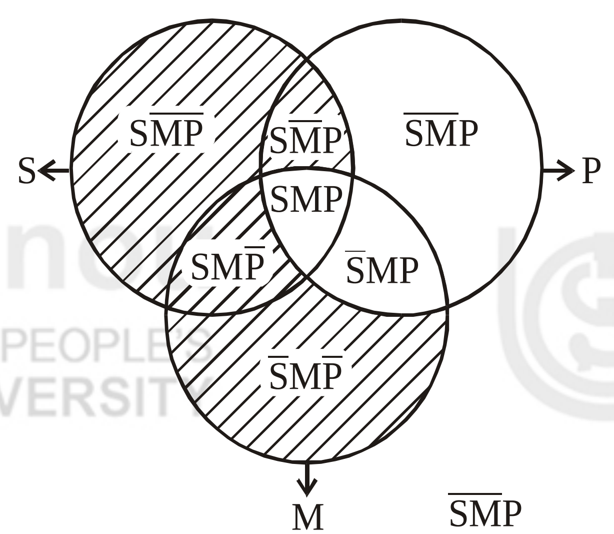
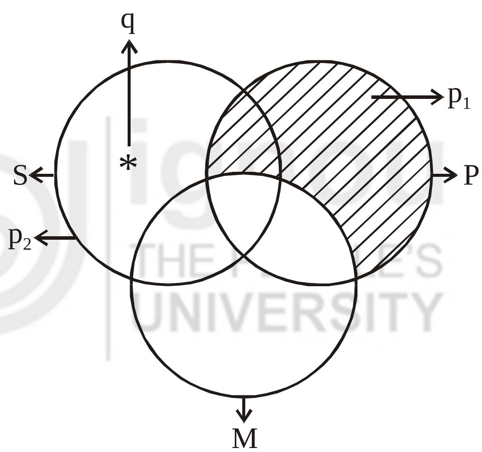


Fig. 2.

p_1 and p_2 show that: $\bar{S} M \bar{P} = SM\bar{P} = S\,\overline{MP} = S\,\bar{M}\,P = Ø$. The conclusion shows that $S\bar{P}$ also is a null set. We did not specially shade $S\bar{P}$. Shading of $M\bar{P}$ and $S\bar{M}$ included naturally the shading of $S\bar{P}$ segment. This is what actually happens in the case of valid arguments. Marking of premises naturally includes the conclusion. It is not marked separately. In other words marking, of conclusion is inclusive. When we adopt Venn diagram technique, this important condition should be borne in mind. Secondly, when any premise is particular, the segment, which corresponds to the universal premise, should be shaded first. This is the initial step to be followed. Now we shall consider some moods. Others are left for the student as an exercise. [In all cases all product classes should be identified by the student even if there is no need. This is a good exercise.]

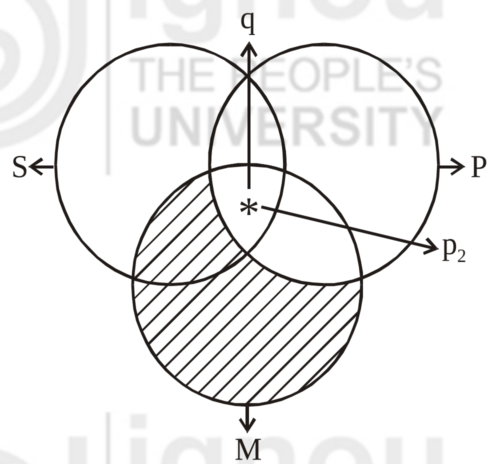
2 BAROCO

p1: All P are M.	$P\bar{M} = Ø$
p2: Some S are not M.	$S\bar{M} \neq Ø$
q: ∴ Some S are not P.	$S\bar{P} \neq Ø$



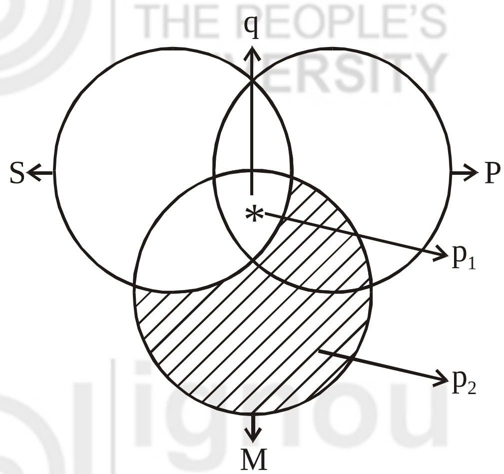
3 DATISI

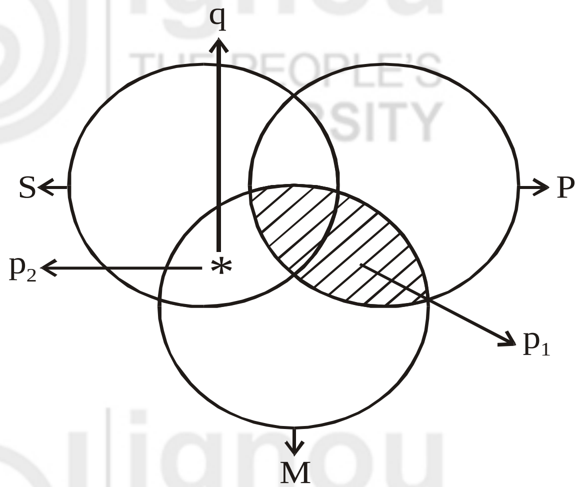
p1: All M are P.	$M\bar{P} = Ø$
p2: Some M are S.	$M S \neq Ø$
q: ∴ Some S are P.	$S P \neq Ø$



4 DISAMIS

p1: Some M are P.	$M P \neq Ø$
p2: All M are S.	$M \bar{S} = Ø$
q: ∴ Some S are P.	$S P \neq Ø$





5 FERISON

p1: No M are P. $\quad$ $M P = \emptyset$

p2: Some M are S. $\quad$ $M S \neq \emptyset$

---

q: Some S are not P. $\quad$ $S \bar{P} \neq \emptyset$

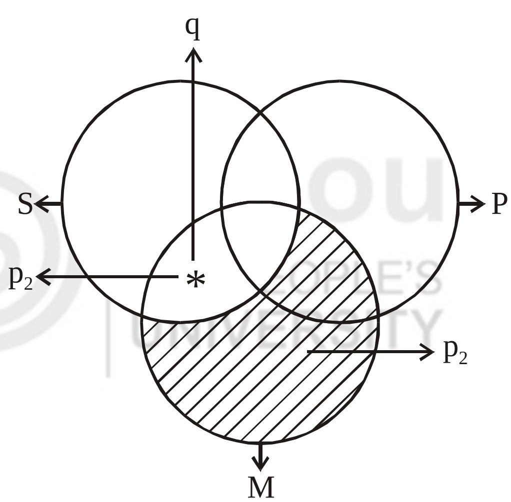
6 BOCARDO

p1: Some M are not P. $\quad$ $M \bar{P} \neq \emptyset$

p2: All M are S. $\quad$ $M \bar{S} = \emptyset$

---

q: $\therefore$ Some S are not P. $\quad$ $S \bar{P} \neq \emptyset$



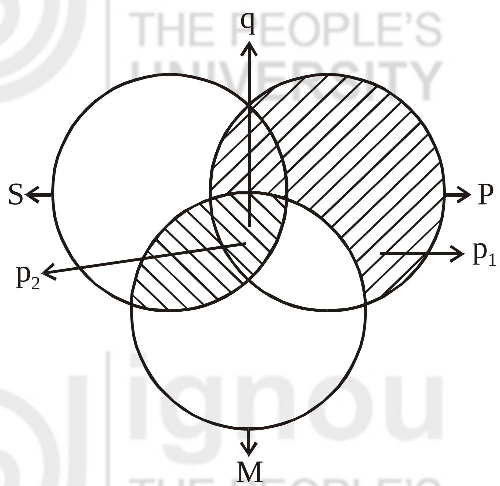
7 CAMENES

p1: All P are M. $P\overline{M} = \emptyset$

p2: No M are S. $MS = \emptyset$

---

q: No S are P. $SP = \emptyset$



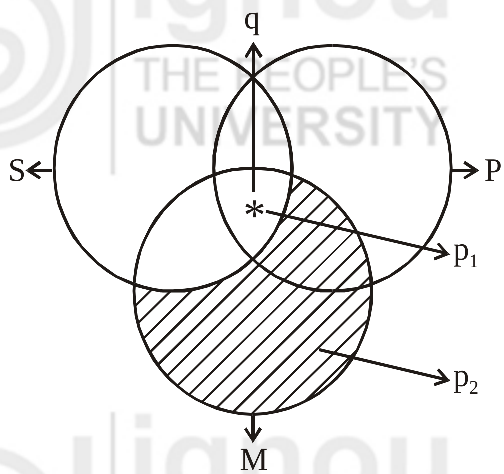
8 DIMARIS

p1: Some P are M. $PM \neq \emptyset$

p2: All M are S. $M\overline{S} = \emptyset$

---

q: ∴ Some S are P. $SP \neq \emptyset$



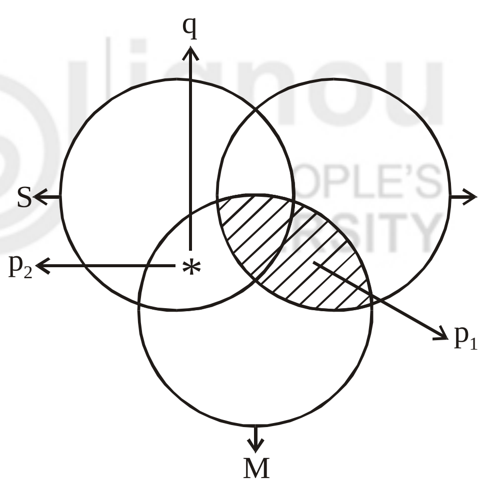
9 FRESISON

p1: No P are M. $\quad PM = \emptyset$

p2: Some M are S. $\quad MS \neq \emptyset$

---

q: Some S are not P. $\quad S\bar{P} \neq \emptyset$



Let us examine a few weakened and strengthened moods using Venn's diagram.

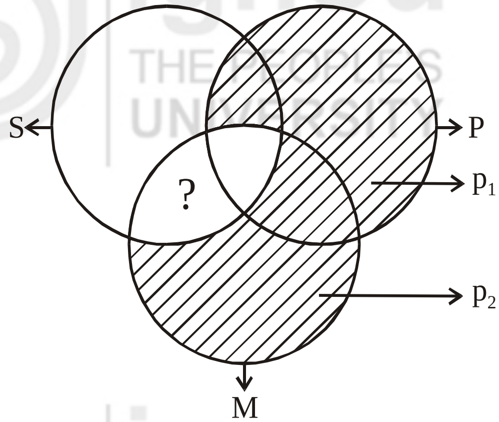
10 BRAMANTIP

p1: All P are M. $\quad P\bar{M} = \emptyset$

p2: All M are S. $\quad M\bar{S} = \emptyset$

---

q: Some S are P. $\quad SP \neq \emptyset$



No information on S M $\overline{P}$ and SMP is available after the premises are diagrammed. Therefore BRAMANTIP is invalid. Now consider a weakened mood.

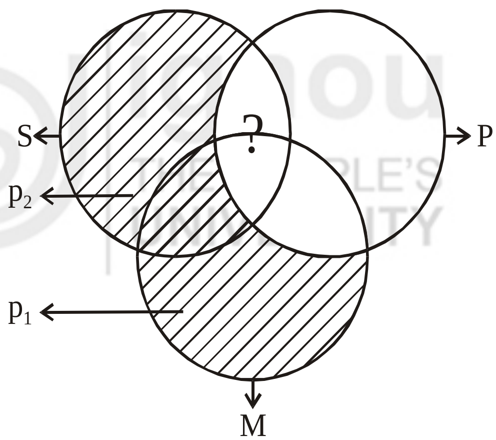
AAI

p1: M A P. $\quad\quad$ $M\overline{P} = Ø$

p2: S A M. $\quad\quad$ $S\overline{M} = Ø$

---

q: ∴ S I P. $\quad\quad$ $SP \neq Ø$



In this case also no information is available on S $\overline{M}$ P and S M P after the premises are diagrammed. Hence AAI is invalid.

4.7 BOOLEAN ANALYSIS:

George Boole published his work *The Mathemaical Analysis of Logic* in 1847. This work provided not only the required breakthrough to logic but also a new direction to its development. This analysis is known as *The Boolean Algebra of Classes*. It is a rewarding exercise to understand this approach.

Boolean analysis presupposes some axioms. Basson and O'connor list thirteen axioms while Alexander considers seven. However, for our purpose only four of them are sufficient to understand this analysis. Let us begin with these axioms.

1 Law of multiplication: a) the product of a universal set and a non-null set(S) is a non-null set.
b) The product of null set and a non-null set is null set.

$1 \times S = S$ (where 1 is the universal set.) 1a
$\Phi \times S = \Phi$ 1b
2 Law of addition: The addition of complementary sets is universal set.
$S + \overline{S} = 1$ 2
3 Law of Commutation for a) addition and b) multiplication: Transposition of two or more than two sets is equivalent to original structure.

a) $(S+ P + M) = (S+ M +P) = (P + M + S) = (M+ S +P)$ ….. 3a
b) $(S PM) = (SMP) = (MSP) = (MPS)$……… 3b

(Instead of addition and multiplication we can also use union and product respectively.)

4 Law of distribution: The multiplication of a l set on the one hand and the addition of two non-null sets on the other is equivalent to the addition of the product of two sets.

$S(P+M) = SP+PM$……. 4

Some valid moods are worked out and the rest are left as exercises for the student.

1) BARBARA

p1:	All M are P.	$M\overline{P} = \Phi$
p2:	All S are M.	$S\overline{M} = \Phi$
q:	$\therefore$ All S are P	$S\overline{P} = \Phi$

Boolean analysis begins with the expansion of statements. The first stage of the expansion of major premise is as follows.

$$\begin{aligned} M\overline{P} &= M\overline{P} \times 1 && \text{Rule 1b} \\ &= M\overline{P}\left(S + \overline{S}\right) = M\overline{P} && \text{Rule 2} \\ &= M\overline{P}S + M\overline{P}\,\overline{S} = M\overline{P} && \text{Rule 4} \end{aligned}$$

Now we shall pass on to the second stage.

$$a) \left.\begin{aligned} S \times M\overline{P} = SM\overline{P} = \Phi \\ \overline{S} \times M\overline{P} = \overline{S}M\overline{P} = \Phi \end{aligned}\right\} \qquad \text{Rule 1}$$

$$M\overline{P} = SM\overline{P} + \overline{S}M\overline{P} = \Phi$$

The last line corresponds to the expansion of major premise. While expanding these lines, we must obtain the addition or union of the product of all relevant sets and their complements as well. On these lines, we shall expand remaining lines.

b) $\left.\begin{array}{l} \mathrm{P} \times \mathrm{S\overline{M}} = \mathrm{PS\overline{M}} = \Phi \\ \mathrm{\overline{P}} \times \mathrm{S\overline{M}} = \mathrm{\overline{P}S\overline{M}} = \Phi \end{array}\right\}$ Rule 1b

$\left.\begin{array}{l} \mathrm{PS\overline{M}} = \mathrm{S\overline{M}P} = \Phi \\ \mathrm{\overline{P}S\overline{M}} = \mathrm{S\overline{M}\overline{P}} = \Phi \end{array}\right\}$ Rule 3b

$\mathrm{S\overline{M}} = \mathrm{S\overline{M}P} + \mathrm{S\overline{M}\overline{P}} = \Phi$

The last line corresponds to the expansion of minor premise.

c) $\left.\begin{array}{l} \mathrm{S\overline{P}} \times \mathrm{M} = \mathrm{S\overline{P}M} = \Phi \\ \mathrm{S\overline{P}} \times \mathrm{\overline{M}} = \mathrm{S\overline{P}\overline{M}} = \Phi \end{array}\right\}$ Rule 1b

$\left.\begin{array}{l} \mathrm{S\overline{P}M} = \mathrm{SM\overline{P}} = \Phi \\ \mathrm{S\overline{P}\overline{M}} = \mathrm{S\overline{M}\overline{P}} = \Phi \end{array}\right\}$ Rule 3b

$\mathrm{S\overline{P}} = \mathrm{SM\overline{P}} + \mathrm{S\overline{M}\overline{P}} = \Phi$

$\mathrm{a} + \mathrm{b} = \mathrm{SM\overline{P}} + \mathrm{\overline{S}M\overline{P}} + \mathrm{S\overline{M}P} + \mathrm{S\overline{M}\overline{P}} = \Phi$

Since the union of four product classes is null set any set in this group is null set. Consider the union of relevant sets.

$\mathrm{SM\overline{P}} + \mathrm{S\overline{M}\overline{P}} = \Phi$

Since this is equivalent to what we have obtained from the conclusion, the argument is valid. This shows that the expansion of conclusion must be equal to or less than the union of premises if the argument is valid. Hence this conclusion is not repeated further while dealing with some arguments which are valid. Since we follow this method throughout, we should bear in our mind all these details.

2 CELARENT

p1: No M are P.	$\mathrm{MP} = \Phi$
p2: All S are M.	$\mathrm{S\overline{M}} = \Phi$
q: $\therefore$ No S are P.	$\mathrm{SP} = \Phi$

Expansion of major premise:

a: $\mathrm{SMP} + \mathrm{\overline{S}MP} = \Phi$

Expansion of minor premise:

b: $\mathrm{S\overline{M}P} + \mathrm{S\overline{M}\overline{P}} = \Phi$

Expansion of conclusion:

c: $\mathrm{SMP} + \mathrm{S\overline{M}P} = \Phi$

$\mathrm{SP\overline{M}} = \mathrm{S\overline{M}P}$

$$\therefore \quad SMP + SP\overline{M} = SMP + S\overline{M}P = \Phi$$

$$a + b \quad \Rightarrow \quad SMP + \overline{S}MP + S\overline{M}P + S\overline{M}\,\overline{P} = \Phi$$

$$c = SMP + S\overline{M}P = \Phi$$

$$a + b = c$$

3) DARII

p1: All M are P.	$M\overline{P} = \Phi$
p2: Some S are M.	$SM \neq \Phi$
q: $\therefore$ Some S are P.	$SP \neq \Phi$

Expansion of major premise:

a: $SM\overline{P} + \overline{S}M\overline{P} = \Phi$

Expansion of minor premise:

b: $SMP + SM\overline{P} \neq \Phi$

Expansion of conclusion:

c: $SMP + S\overline{M}P \neq \Phi$

$$a + b \Rightarrow SM\overline{P} + \overline{S}M\overline{P} + SMP + SM\overline{P} \neq \Phi$$

$$SM\overline{P} = \overline{S}M\overline{P} = \Phi$$

$$\Phi + SMP \neq \Phi$$

$$SMP \neq \Phi$$

$$SMP + S\overline{M}P \neq \Phi$$

$$a + b = SMP + S\overline{M}P \neq \Phi$$

Since SMP is a non-null set, its union with null set yields a non-null set.

$$c = SMP + S\overline{M}P \neq \Phi$$

$$a + b = c$$

4) FERIO

p1: No M are P.	$MP = \Phi$
p2: Some S are M.	$SM \neq \Phi$
q: $\therefore$ Some S are not P.	$S\overline{P} \neq \Phi$

Expansion of major premise:

a: $SMP + \overline{S}MP = \Phi$

Expansion of minor premise:

$b: \quad SMP + SM\bar{P} \neq \Phi$

Expansion of conclusion:

$c: \quad SM\bar{P} + S\bar{M}\bar{P} \neq \Phi$

$a + b \quad \Rightarrow \quad SMP + \bar{S}MP + SMP + SM\bar{P} \neq \Phi$

$= \Phi + SM\bar{P} \neq \Phi$

$SM\bar{P} \neq \Phi$

$SM\bar{P} + S\bar{M}\bar{P} \neq \Phi$

$c = SM\bar{P} + S\bar{M}\bar{P} \neq \Phi$

$a + b = c$

5) CESARE

p1: No P are M.	$PM = \Phi$
p2: All S are M.	$S\bar{M} = \Phi$
q: $\therefore$ No S are P.	$SP = \Phi$

Expansion of major premise:

$a: \quad SMP + \bar{S}MP = \Phi$

Expansion of minor premise:

$b: \quad S\bar{M}P + S\bar{M}\bar{P} = \Phi$

Expansion of conclusion:

$c: \quad SMP + S\bar{M}P = \Phi$

$a + b \quad \Rightarrow \quad SMP + \bar{S}MP + S\bar{M}P + S\bar{M}\bar{P} = \Phi$

$\therefore \; SMP + S\bar{M}P = \Phi$

$c = SMP + S\bar{M}P = \Phi$

$a + b = c$

6) CAMESTRES

p1: All P are M.	$P\bar{M} = \Phi$
p2: No S are M.	$SM = \Phi$
q: $\therefore$ No S are P.	$SP = \Phi$

Expansion of major premise:

$a: \quad S\bar{M}P + \bar{S}\bar{M}P = \Phi$

Expansion of minor premise:

$b: \quad SMP + SM\overline{P} = \Phi$

Expansion of conclusion:

$: c: \quad SMP + S\overline{M}P = \Phi$

$a + b \quad \Rightarrow \quad S\overline{M}P + \overline{S}\,\overline{M}P + SMP + SM\overline{P} = \Phi$

$= SMP + S\overline{M}P = \Phi$

$c = SMP + S\overline{M}P = \Phi$

$a + b = c$

7) FESTINO

p1: No P are M.	$PM = \Phi$
p2: Some S are M.	$SM \neq \Phi$
q: $\therefore$ Some S are not P.	$S\overline{P} \neq \Phi$

Expansion of major premise:

$a: \quad SMP + \overline{S}MP = \Phi$

Expansion of minor premise:

$b: \quad SMP + SM\overline{P} \neq \Phi$

Expansion of conclusion:

$c: \quad SM\overline{P} + S\overline{M}\,\overline{P} \neq \Phi$

$a + b \quad \Rightarrow \quad SMP + \overline{S}MP + SMP + SM\overline{P} \neq \Phi$

$= \Phi + SM\overline{P} \neq \Phi$

$\therefore SM\overline{P} \neq \Phi$

$c = SM\overline{P} + S\overline{M}\,\overline{P} \neq \Phi$

$a + b = c$

8) BAROCO

p1: All P are M.	$P\overline{M} = \Phi$
p2: Some S are not M.	$S\overline{M} \neq \Phi$
q: $\therefore$ Some S are not P.	$S\overline{P} \neq \Phi$

Expansion of major premise:

$a: \quad S\overline{M}P + \overline{S}\,\overline{M}P = \Phi$

Expansion of minor premise:

$b: \quad S\overline{M}P + S\overline{M}\,\overline{P} \neq \Phi$

Expansion of conclusion:

$c: \quad SM\bar{P} + S\bar{M}\bar{P} \neq \Phi$

$a + b \quad \Rightarrow \quad S\bar{M}P + \bar{S}\bar{M}P + S\bar{M}P + S\bar{M}\bar{P} \neq \Phi$
$\quad = \Phi + S\bar{M}\bar{P} \neq \Phi$

$\therefore S\bar{M}\bar{P} \neq \Phi$

$\therefore SM\bar{P} + S\bar{M}\bar{P} \neq \Phi$

$a + b = c$

9) DISAMIS

p1:	Some M are P.	$MP \neq \Phi$
p2:	All M are S.	$M\bar{S} = \Phi$
q:	$\therefore$ Some S are P.	$SP \neq \Phi$

Expansion of major premise:
$a: \quad SMP + \bar{S}MP \neq \Phi$
Expansion of minor premise:
$b: \quad \bar{S}MP + \bar{S}M\bar{P} = \Phi$
Expansion of conclusion:
$c: \quad SMP + S\bar{M}P \neq \Phi$
$a + b \quad \Rightarrow \quad SMP + \bar{S}MP + \bar{S}MP + \bar{S}M\bar{P} \neq \Phi$
$\quad = SMP + \Phi \neq \Phi$

$\therefore SMP \neq \Phi$

$\therefore SMP + S\bar{M}P \neq \Phi$

$a + b = c$

10) DATISI

p1:	All M are P.	$M\bar{P} = \Phi$
p2:	Some M are S.	$MS \neq \Phi$
q:	$\therefore$ Some S are P.	$SP \neq \Phi$

Expansion of major premise:
$a: \quad SM\bar{P} + \bar{S}M\bar{P} = \Phi$
Expansion of minor premise:

b: $SMP + SM\bar{P} \neq \Phi$

Expansion of conclusion:

c: $SMP + S\bar{M}P \neq \Phi$

$$a + b \Rightarrow SM\bar{P} + \bar{S}M\bar{P} + SMP + SM\bar{P} \neq \Phi$$
$$= \Phi + SMP \neq \Phi$$

$\therefore SMP \neq \Phi$

$\therefore SMP + S\bar{M}P \neq \Phi$

$a + b = c$

11) BOCARDO

p1: Some M are not P. $M\bar{P} \neq \Phi$

p2: All M are S. $M\bar{S} = \Phi$

---

q: $\therefore$ Some S are not P. $S\bar{P} \neq \Phi$

Expansion of major premise:

a: $SM\bar{P} + \bar{S}M\bar{P} \neq \Phi$

Expansion of minor premise:

b: $\bar{S}MP + \bar{S}M\bar{P} = \Phi$

Expansion of conclusion.:

c: $SM\bar{P} + S\bar{M}\bar{P} \neq \Phi$

$$a + b \Rightarrow SM\bar{P} + \bar{S}M\bar{P} + \bar{S}MP + \bar{S}M\bar{P} \neq \Phi$$
$$= \Phi + SM\bar{P} \neq \Phi$$

$\therefore SM\bar{P} \neq \Phi$

$\therefore SM\bar{P} + S\bar{M}\bar{P} \neq \Phi$

$a + b = c$

12) FERISON

p1: No M are P. $MP = \Phi$

p2: Some M are S. $MS \neq \Phi$

---

q: $\therefore$ Some S are not P. $S\bar{P} \neq \Phi$

Expansion of major premise:

$a: \quad SMP + \overline{S}MP = \Phi$

Expansion of minor premise:

$b: \quad SMP + SM\overline{P} \neq \Phi$

Expansion of conclusion:

$c: \quad SM\overline{P} + S\overline{M}\overline{P} \neq \Phi$

$$a + b \quad \Rightarrow \quad SMP + \overline{S}MP + SMP + SM\overline{P} \neq \Phi$$
$$= \Phi + SM\overline{P} \neq \Phi$$

$\therefore SM\overline{P} \neq \Phi$

$\therefore SM\overline{P} + S\overline{M}\overline{P} \neq \Phi$

$a + b = c$

13) CAMENES

$$\begin{array}{ll} p1: \text{All P are M.} & P\overline{M} = \Phi \\ p2: \text{No M are S.} & MS = \Phi \\ \hline q: \therefore \text{No S are P.} & SP = \Phi \end{array}$$

Expansion of major premise:

$a: \quad S\overline{M}P + \overline{S}\,\overline{M}P = \Phi$

Expansion of minor premise:

$b: \quad SMP + SM\overline{P} = \Phi$

Expansion of conclusion:

$c: \quad SMP + S\overline{M}P = \Phi$

$$a + b \quad \Rightarrow \quad S\overline{M}P + \overline{S}\,\overline{M}P + SMP + SM\overline{P} = \Phi$$
$$= S\overline{M}P + SMP = \Phi$$

$a + b = c$

14) DIMARIS

$$\begin{array}{ll} p1: \text{Some P are M.} & PM \neq \Phi \\ p2: \text{All M are S.} & M\overline{S} = \Phi \\ \hline q: \therefore \text{Some S are P.} & SP \neq \Phi \end{array}$$

Expansion of major premise:

$a: \quad SMP + \overline{S}MP \neq \Phi$

Expansion of minor premise:

$b: \quad \overline{S}MP + \overline{S}M\overline{P} = \Phi$

Expansion of conclusion:

$c: \quad SMP + S\bar{M}P \neq \Phi$

$a + b \quad \Rightarrow \quad SMP + \bar{S}MP + \bar{S}MP + \bar{S}M\bar{P} \neq \Phi$

$= SMP + \Phi \neq \Phi$

$\therefore SMP \neq \Phi$

$\therefore \; c = SMP + S\bar{M}P \neq \Phi$

$a + b = c$

15) FRESISON

p1: No P are M.	$PM = \Phi$
p2: Some M are S.	$MS \neq \Phi$
q: $\therefore$ Some S are not P.	$S\bar{P} \neq \Phi$

Expansion of major premise:

$a: \quad SMP + \bar{S}MP = \Phi$

Expansion of minor premise:

$b: \quad SMP + SM\bar{P} \neq \Phi$

Expansion of conclusion:

$c: \quad SM\bar{P} + S\bar{M}\bar{P} \neq \Phi$

$a + b \quad \Rightarrow \quad SMP + \bar{S}MP + SMP + SM\bar{P} \neq \Phi$

$= \Phi + SM\bar{P} \neq \Phi$

$\therefore SM\bar{P} \neq \Phi$

$\therefore \; SM\bar{P} + S\bar{M}\bar{P} \neq \Phi$

$a + b = c$

Let us examine an invalid mood which is regarded as valid in traditional framework.

16) AAI

p1: All M are P.	$M\bar{P} = \Phi$
p2: All S are M.	$S\bar{M} = \Phi$
q: $\therefore$ Some S are P.	$SP \neq \emptyset$

Expansion of major premise:

$a: \quad SM\bar{P} + \bar{S}M\bar{P} = \Phi$

Expansion of minor premise:

$b: \quad S\bar{M}P + S\bar{M}\bar{P} = \Phi$

Expansion of conclusion:

$c: \quad SMP + S\bar{M}P \neq \Phi$

$$a + b = SM\bar{P} + \bar{S}M\bar{P} + S\bar{M}P + S\bar{M}\bar{P} = \Phi$$

$$a + b \neq c$$

This is so because from equations alone it is not possible to obtain inequation.

Antilogism, Venn Diagram Technique and Boolean Analysis have one distinct advantage. They do away with the concept of distribution of terms which is a cumbersome to apply. What is required is only the application of some elements of set theory.

Apply these techniques for the following arguments to test their validity.

1 All dogs have four legs.
All animals have four legs.
$\therefore$ All dogs are animals.

2 All dogs have four legs.
All chairs have four legs.
$\therefore$ All dogs are chairs.

3 No bats are cats.
No rats are bats.
$\therefore$ No rats are cats.

4 No fish are birds.
No golden plovers are fish.
$\therefore$ No golden plovers are birds.

5 All Indians are people.
John is a person.
$\therefore$ John is an Indian.

6 Some readers are philosophers.
Chanakya is a philosopher.
$\therefore$ Chanakya is a reader.

7 No human being is perfect.
Some human beings are presidents.
$\therefore$ Some presidents are not perfect.

8 All matter obeys wave equations.
All waves obey wave equations.
$\therefore$ All matter is waves.

9 All human action is conditioned by circumstances.
All human action involves morality.

∴ All that involves morality is conditioned by circumstances.

10 All that is good is pleasant.
All eating is pleasant.
∴ All eating is good.

11 All patriots are voters.
Some citizens are not voters.
∴ Some citizens are not patriots.

12 All potatoes have eyes.
John's head has eyes.
∴ John is a potato head.

Check Your Progress II

Note: Use the space provided for your answers.

1. EIO is valid and IEO is invalid in all the figures. Explain.

....

....

....

2. If both premises are universal, then the conclusion must also be universal. Explain.

....

....

....

4.8 LET US SUM UP

General rules apply to all figures whereas special rules apply to specific figures. Special rules indirectly depend upon general rules only. Antilogism, Venn diagram technique and Boolean analysis do away with the concept of distribution of terms. According to the last three methods weakened and strengthened moods become invalid though traditional logic regards them as valid.

4.9 KEY WORDS

Mood: By the 'mood' of a syllogism is meant that kind of a syllogism which is determined by the 'quantity' and 'quality' of the three propositions.

Figure: By the 'figure' of a syllogism is meant that kind of syllogism which is determined by the function the 'middle term' plays in the syllogism.

4.10 FURTHER READINGS AND REFERENCES

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